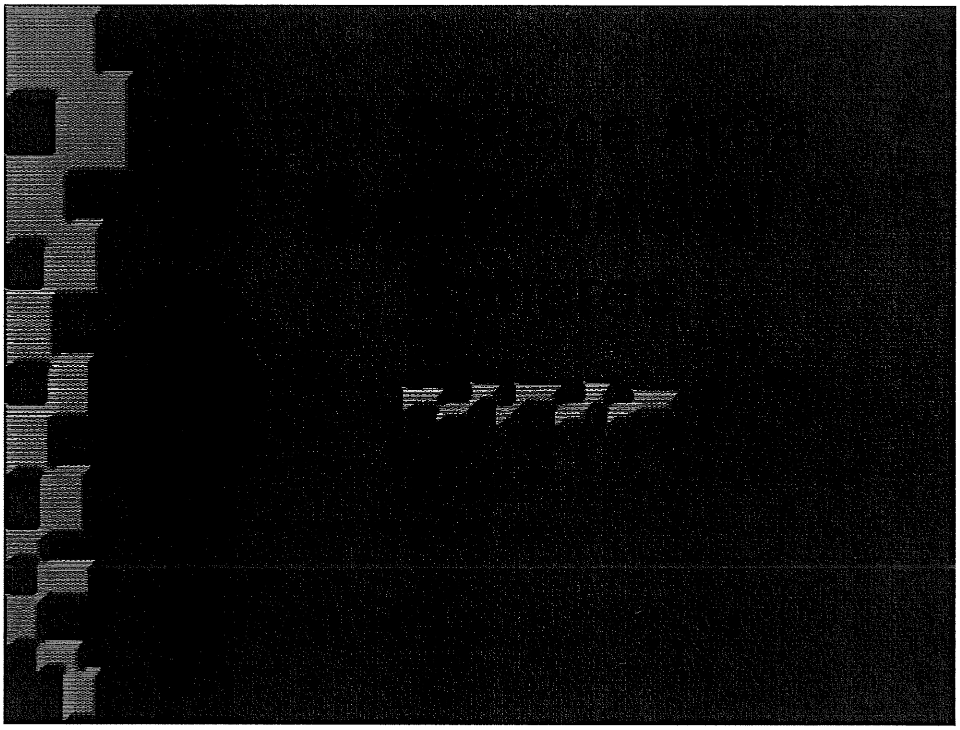
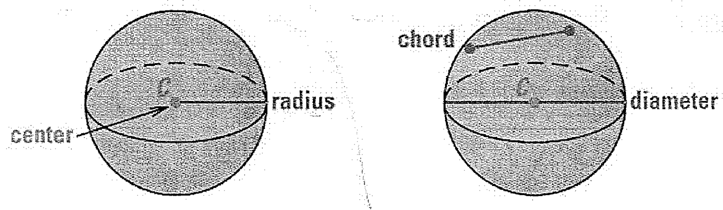


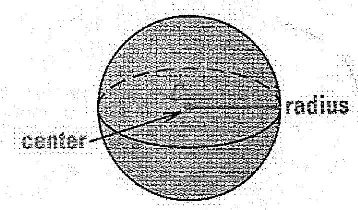
Finding the Surface Area of a Sphere

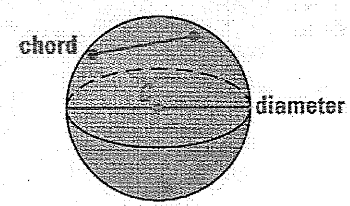
- In Lesson 6.1, a circle was described as the set of all points in a plane that are equidistant from a given point. (center).
- A sphere is the set of all points in space that are equidistant from a given point. (center)



Finding the Surface Area of a Sphere

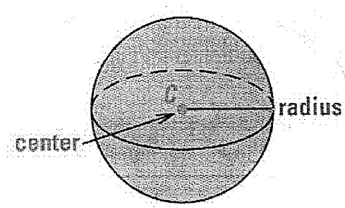
- A radius of a sphere is a segment from the center to a point on the sphere.
- A chord of a sphere is a segment whose endpoints are on the sphere.

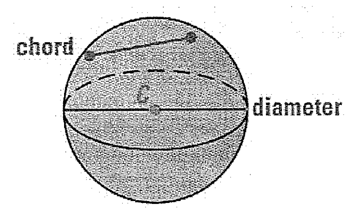




Finding the Surface Area of a Sphere

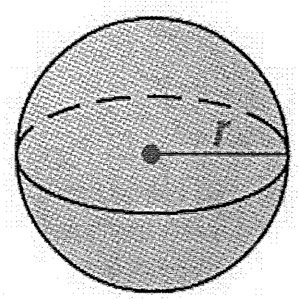
- A diameter is a chord that contains the center. As with all circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.





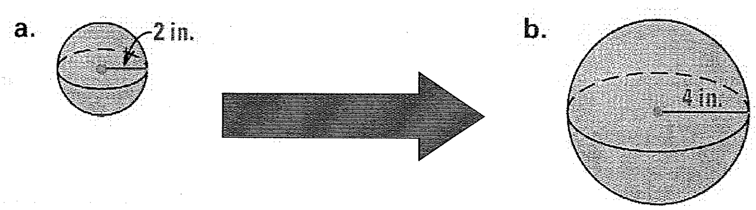
Theorem 6.22: Surface Area of a Sphere

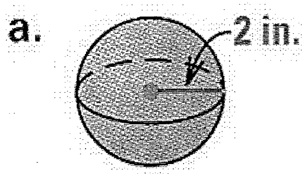
- The surface area of a sphere with radius r is $S = 4\pi r^2$.

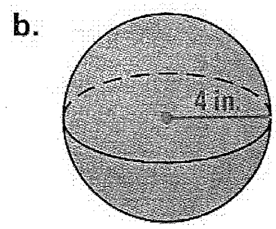


Ex. 1: Finding the Surface Area of a Sphere

- Find the surface area. When the radius doubles, does the surface area double?





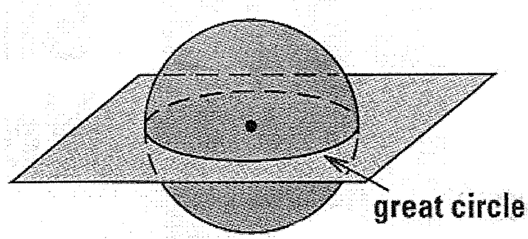


$$S = 4\pi r^2 \quad\quad S = 4\pi r^2$$
$$= 4\pi 2^2 \quad\quad = 4\pi 4^2$$
$$= 16\pi \text{ in.}^2 \quad\quad = 64\pi \text{ in.}^2$$

The surface area of the sphere in part (b) is four times greater than the surface area of the sphere in part (a) because $16\pi \cdot 4 = 64\pi$

☞So, when the radius of a sphere doubles, the surface area DOES NOT double.

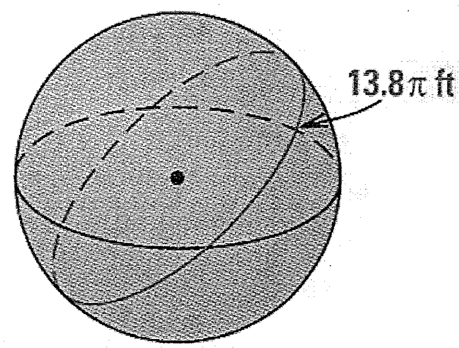
More . . .



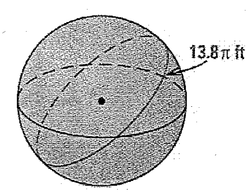
- If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a great circle of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called hemispheres.

Ex. 2: Using a Great Circle

- The circumference of a great circle of a sphere is 13.8π feet. What is the surface area of the sphere?



Solution:



Begin by finding the radius of the sphere.

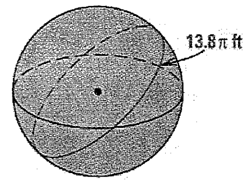
$C = 2\pi r$

$13.8\pi = 2\pi r$

$\dfrac{13.8\pi}{2\pi} = r$

$6.9 = r$

Solution:



Using a radius of 6.9 feet, the surface area is:

$S = 4\pi r^2$

$= 4\pi(6.9)^2$

$= 190.44\pi$ ft.^{2}

So, the surface area of the sphere is 190.44 π ft.^{2}

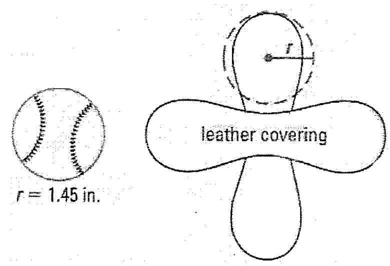
Ex. 3: Finding the Surface Area of a Sphere

- Baseball. A baseball and its leather covering are shown. The baseball has a radius of about 1.45 inches.

a. Estimate the amount of leather used to cover the baseball.

b. The surface area of a baseball is sewn from two congruent shapes, each which resembles two joined circles. How does this relate to the formula for the surface area of a sphere?

Ex. 3: Finding the Surface Area of a Sphere



SOLUTION

a. Because the radius r is about 1.45 inches, the surface area is as follows:

$S = 4\pi r^2$ — Formula for surface area of sphere

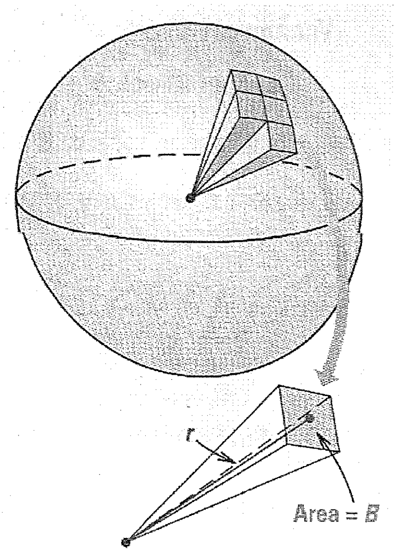
$\approx 4\pi(1.45)^2$ — Substitute 1.45 for r.

$\approx 26.4 \text{ in.}^2$ — Use a calculator.

b. Because the covering has two pieces, each resembling two joined circles, then the entire covering consists of four circles with radius r. The area of a circle of radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the same as the formula for the surface area of a sphere.

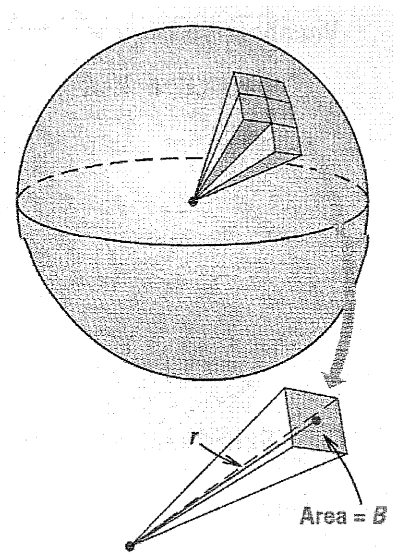
Finding the Volume of a Sphere

- Imagine that the interior of a sphere with radius r is approximated by n pyramids as shown, each with a base area of B and a height of r, as shown. The volume of each pyramid is 1/3 Br and the sum is nB.



Finding the Volume of a Sphere

- The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows:

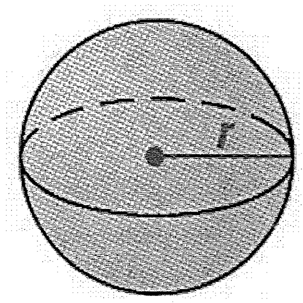


More . . .

$V \approx n(1/3)Br$	Each pyramid has a volume of 1/3Br.
$= 1/3\ (nB)r$	Regroup factors.
$\approx 1/3(4\pi r^2)r$	Substitute $4\pi r^2$ for nB.
$=4/3\pi r^3$	Simplify.

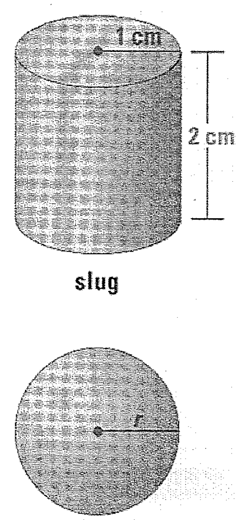
Theorem 6.23: Volume of a Sphere

- The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.



Ex. 4: Finding the Volume of a Sphere

- Ball Bearings. To make a steel ball bearing, a cylindrical slug is heated and pressed into a spherical shape with the same volume. Find the radius of the ball bearing to the right:



Solution:

- To find the volume of the slug, use the formula for the volume of a cylinder.

$V = \pi r^2 h$

$= \pi(1^2)(2)$

$= 2\pi \text{ cm}^3$

To find the radius of the ball bearing, use the formula for the volume of a sphere and solve for r.

More . . .

$V = 4/3\pi r^3$	Formula for volume of a sphere.
$2\pi = 4/3\pi r^3$	Substitute 2π for V.
$6\pi = 4\pi r^3$	Multiply each side by 3.
$1.5 = r^3$	Divide each side by 4π.
$1.14 \approx r$	Use a calculator to take the cube root.

So, the radius of the ball bearing is about 1.14 cm.