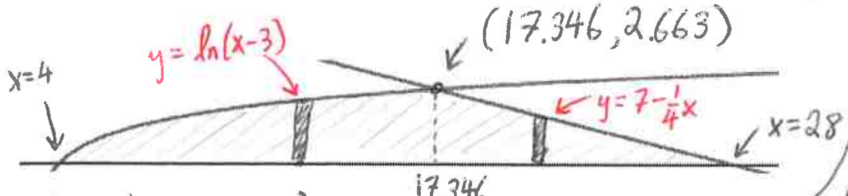
Key

A.P. Calculus AB Chapter 7.1-7.2 Area & Volume Unit Review WS #3

1) Given the region below enclosed by $f(x) = \ln(x-3)$, the line $y = 7 - \frac{1}{4}x$, and the x – axis.



Method 1: (Top-Bottom) *2 separate integrals*

$$Area = \int_{4}^{17.346} \ln(x-3) - 0\,dx + \int_{17.346}^{28} 7 - \tfrac{1}{4}x - 0\,dx$$

$$Area = 24.864 + 14.188 = \boxed{39.052 \text{ units}^2}$$

Method 2: (Right-Left)

$y = \ln(x-3)$; $y = 7 - \frac{1}{4}x$

$e^y = e^{\ln(x-3)}$; $(y - 7 = -\frac{1}{4}x)\cdot(-4)$

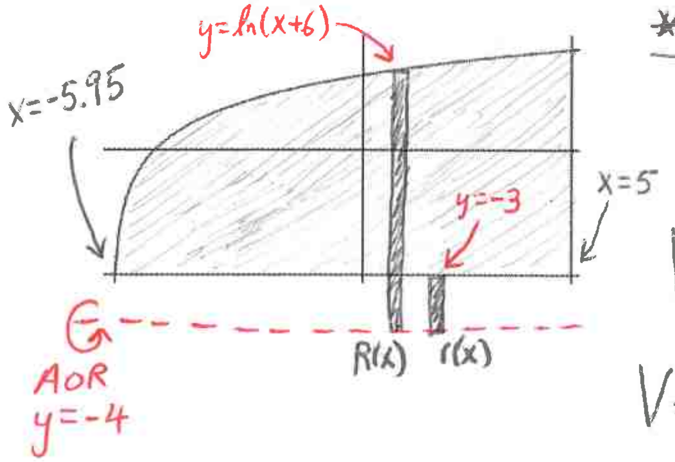
$e^y = x - 3$; $28 - 4y = x$

$3 + e^y = x$

$$Area = \int_{0}^{2.663} 28 - 4y - (3 + e^y)\,dy = \boxed{39.052 \text{ units}^2}$$

2) Given the region below enclosed by $f(x) = \ln(x+6)$, the line $y = -3$, and $x = 5$.

a) Find the Volume of solid generated when the enclosed region is revolved about the line y = –4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Washer Method (Top-Bottom)

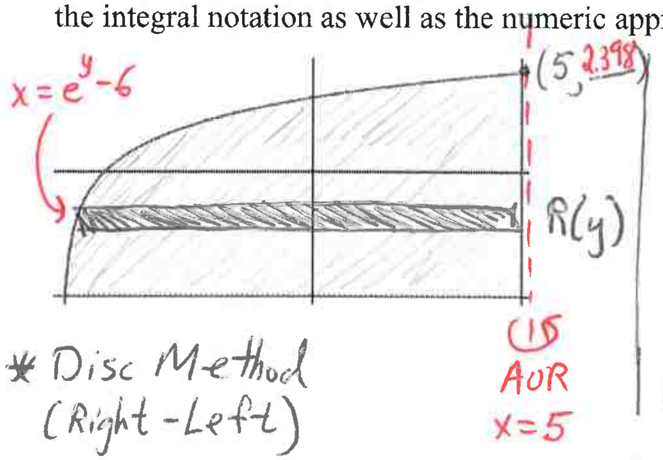
$R(x) = \ln(x+6) - (-4) = \ln(x+6) + 4$

$r(x) = -3 - (-4) = 1$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$V = \pi \int_{-5.95}^{5} [\ln(x+6) + 4]^2 - [1]^2\,dx = \boxed{320.510\pi \text{ units}^3}$$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 5 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Disc Method (Right-Left)

$y = \ln(x+6)$

$e^y = e^{\ln(x+6)}$

$e^y = x + 6$

$e^y - 6 = x$

$R(y) = 5 - (e^y - 6)$

$R(y) = 11 - e^y$

*find upper bound:

$y(x) = \ln(x+6)$

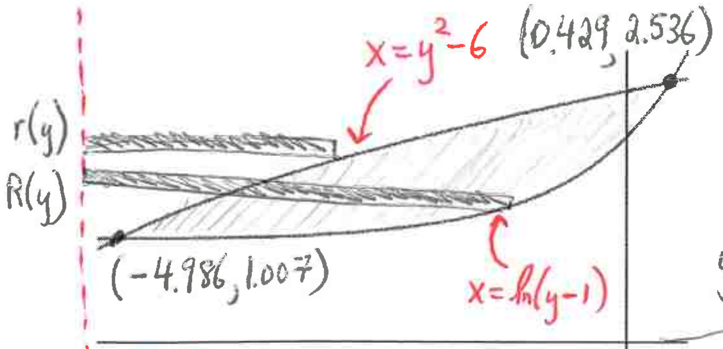
$y(5) = \ln(5+6) = \ln 11 = 2.398$

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2\,dy$$

$$V = \pi \int_{-3}^{2.398} [11 - e^y]^2\,dy = \boxed{472.739\pi \text{ units}^3}$$

3) Given the region below enclosed by $f(x) = \sqrt{x+6}$, the $g(x) = e^x + 1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line x = −6 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



AOR x = −6

* Washer Method (Right − Left)

$y = \sqrt{x+6}$

$(y)^2 = (\sqrt{x+6})^2$

$y^2 = x+6$

$y^2 - 6 = x$

$y = e^x + 1$

$y - 1 = e^x$

$\ln(y-1) = \ln e^x$

$\ln(y-1) = x \ln e$

$\ln(y-1) = x$

$V = \pi \int_{y_1}^{y_2} R(y)^2 - r(y)^2 \, dy$

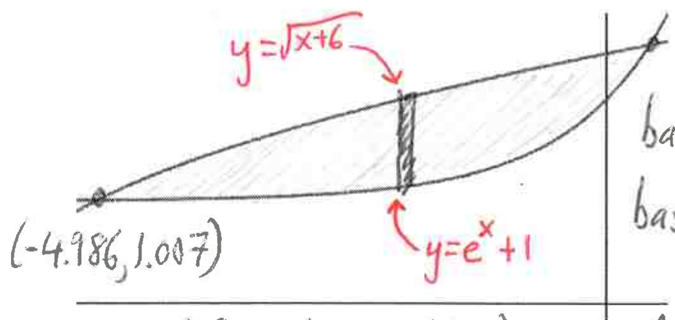
$R(y) = \ln(y-1) - (-6) = \ln(y-1) + 6$

$r(y) = y^2 - 6 - (-6) = y^2$

$V = \pi \int_{1.007}^{2.536} [\ln(y-1) + 6]^2 - [y^2]^2 \, dy$

V = 26.032π units³

b) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with hypotenuse on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Vertical Base (Top − Bottom)

$\text{base} = \sqrt{x+6} - (e^x + 1)$

$\text{base} = \sqrt{x+6} - e^x - 1$

$\text{Area} = \frac{1}{4}(\text{base})^2$

$\text{Area} = \frac{1}{4}(\sqrt{x+6} - e^x - 1)^2$

$V = \int_{x_1}^{x_2} [\text{Area}] \, dx$

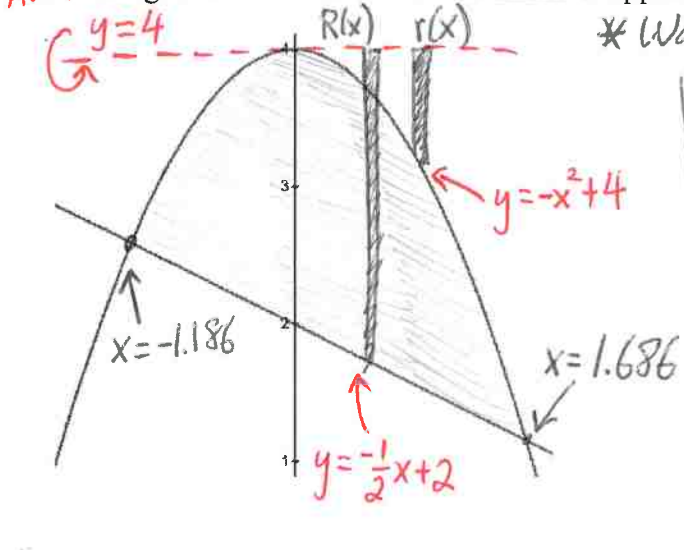
$V = \int_{-4.986}^{0.429} \frac{1}{4}[\sqrt{x+6} - e^x - 1]^2 \, dx$

V = 0.581 units³

4) Given the region below enclosed by $f(x) = -x^2 + 4$ and $g(x) = -\frac{1}{2}x + 2$

AOR y = 4

Find the Volume of solid generated when the enclosed region is revolved about the line y = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Washer Method (Top − Bottom)

$R(x) = 4 - (-\frac{1}{2}x + 2) = 4 + \frac{1}{2}x - 2 = 2 + \frac{1}{2}x$

$r(x) = 4 - (-x^2 + 4) = 4 + x^2 - 4 = x^2$

$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2 \, dx$

$V = \pi \int_{-1.186}^{1.686} [2 + \frac{1}{2}x]^2 - [x^2]^2 \, dx$ = **10.268π units³**