## Review of Antiderivative Word Problems

#1. *. Rate times time passed = Total value

**2002 #2 (calculator question)**

The rate at which people enter an amusement park on a given day is modeled by the function $E$ defined by $E(t) = \frac{15600}{(t^2-24t+160)}$. (people entering per hour)

The rate at which people leave the same amusement park on the same day is modeled by the function $L$ defined by $L(t) = \frac{9890}{(t^2-38t+370)}$. (people leaving per hour)

Both $E(t)$ and $L(t)$ are measured in people per hour and time $t$ is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park. park opens at 9am

a) How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.

b) The price of admission to the park is \$15 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

c) Let $H(t) = \int_9^t (E(x) - L(x))dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park. $H'(t)$ represent rate of people entering or leaving the park.

d) At what time $t$, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

a) $\int_9^{17} E(t)\,dt = \int_9^{17} \frac{15600}{t^2-24t+160}\,dt = 6004.270 = 6004$ people

Total # of people from 9-5pm

b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = \$104,048$

Total # of people through the entrance gate from 5-11pm

c) $H(t) = \int_9^t E(x) - L(x)\,dx$

$H(17) = \int_9^{17} E(x) - L(x)\,dx = 3725$ people in the park at 5pm

$H'(t) = \frac{d}{dx}\int_9^t E(x) - L(x)\,dx$

$H'(t) = E(t) - L(t)$

$H'(17) = E(17) - L(17)$

$= -380.281$

The number of people in park is decreasing at rate of 380 people/hr.

$H'(t) = 0$

d) $H'(t) = E(t) - L(t) = 0$

# of people in the park will keep growing until rate of people leaving is higher than $E(t)$ → rate of people entering.

$E(t) = L(t)$

$\boxed{t = 15.79}$

#2.

**2004 #1 (calculator question)**

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function $F$ defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$, where $F(t)$ is measured in cars per minute and $t$ is measured in minutes.

a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.
d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

a) $\int_0^{30} F(t) = 2474$ cars

b) Find derivative of $F(x)$, then evaluate $F'(7)$

$F'(7) = -1.872$

*calculator Math 8
↳ nDeriv(F(t), x, 7)

Since $F'(7) < 0$, traffic flow is decreasing at $t = 7$.

c) Avg. value theorem

$$\frac{1}{b-a}\int_a^b F(t)\,dt = \frac{1}{15-10}\int_{10}^{15} F(t)\,dt = 81.899 \text{ cars/min}$$

d) Avg. rate of change of traffic flow = $\dfrac{\text{change in traffic flow}}{\text{change in time}}$

$$= \frac{F(15) - F(10)}{15 - 10} = 1.517 \text{ cars/min}^2$$

#3. **2005 #2 (calculator question)**

The tide removes sand from Sandy Point Beach at a rate modeled by the function $R$, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. Removing Sand

A pumping station adds sand to the beach at a rate modeled by the function $S$, given by $S(t) = \frac{15t}{1+3t}$. Adding Sand

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and $t$ is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time $t$.

c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

d) For $0 \le t \le 6$, at what time $t$ is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

Absolute

a) $\int_0^6 R(t)\,dt = 31.815$

b) Total amount = Initial Amount + Sand added − sand removed

$$Y(t) = 2500 + \int_0^t S(x)\,dx - \int_0^t R(x)\,dx$$

c) $Y'(t) = \frac{d}{dx}\int_0^t S(x)\,dx - \frac{d}{dx}\int_0^t R(x)\,dx$

$Y'(t) = S(t) - R(t)$

Is beach growing or shrinking at $t = 4$ hrs?

$Y'(4) = S(4) - R(4) \rightarrow -1.908\ yd^3/hr$

d) Sand is at minimum when $Y'(t) = 0$, so $S(t) - r(t) = 0$

Rel. min at $t = 5.1179$

Find absolute minimum by testing $t = 0$, $t = 6$, $t = 5.117$

$Y(0) = 2500$

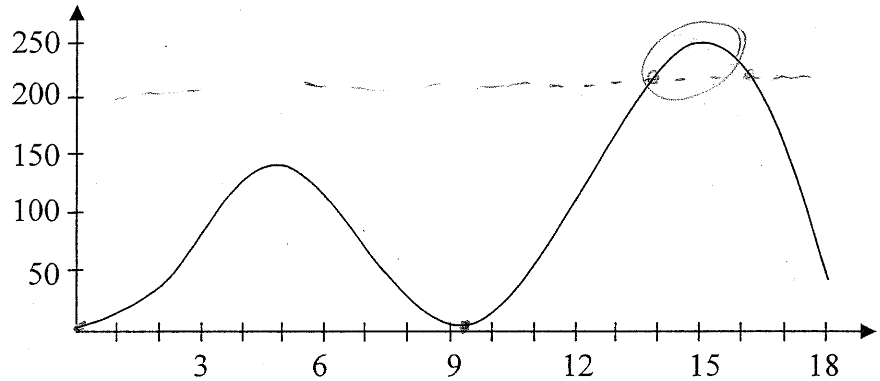
$Y(5.117) = 2500 + \int_0^{5.117} S(x) - R(x)\,dx = 2492.3694$

$Y(6) = 2500 + \int_0^6 S(x) - R(x)\,dx = 2493.2766$

$\boxed{\text{Abs. minimum is } 2492.369\ yd^3 \text{ at } t = 5.117}$

4.

**2006 #2 (calculator question)**




At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.
b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of $t$ for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.
c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

a) $\int_0^{18} L(t)\,dt = 1658$ cars

b) set $L(t) = 150$ or $L(t) - 150 = 0$ $\quad \frac{1}{b-a}\int_a^b L(t)\,dt = \frac{1}{16.122 - 12.42}\int_{12.42}^{16.122} L(t)\,dt$

$t = 12.42,\ 16.122$

$=$

$= 199.426$ cars/hr.

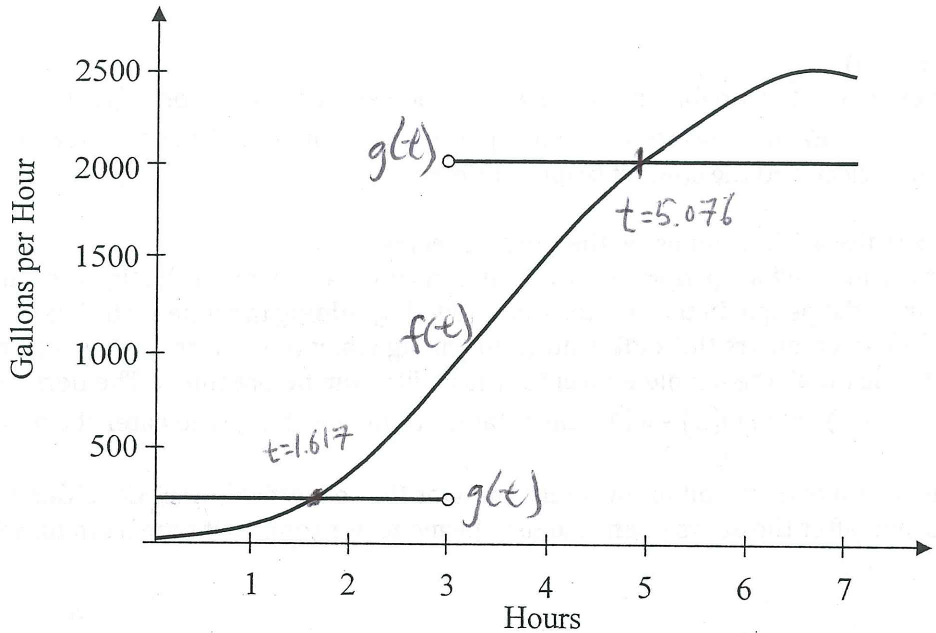
c) Total # of cars turning left $\times$ total # of cars going straight $> 200{,}000$?

$\int_{13}^{15} L(t)\,dt \times 500 = 215{,}965.5$

So a traffic signal is required

5.

**2007 #2 (calculator Question)**




The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where $t$ is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of $f$ and $g$, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

c) For $0 \le t \le 7$, at what time $t$ is the amount of water in the tank the greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

a) $\int_0^7 f(t)\,dt = 8264$ gallons

b) Amt of water in tank is decreasing at $0 < t < 1.617$ and $3 < t < 5.076$ b/c $g(t) > f(t)$

c) Test $t = \cancel{1.617}, t = \cancel{5.076}$ $t = 3, 0, 7$

| $t$ | Amt of water |
|---|---|
| 0 | 5000 |
| 3 | $5000 + \int_0^7 f(t)\,dt - \int_0^3 250\,dt = 5126.591$ |
| 7 | $5126.591 + \int_3^7 f(t)\,dt - \int_3^7 2000\,dt = 4513.807$ |

6.

**2009 #2 (calculator question)**

The rate at which people enter an auditorium for a rock concert is modeled by the function $R$ given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

a) How many people are in the auditorium when the concert begins?
b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function $w$ models the total wait time for all the people who enter the auditorium before time $t$. The derivative of $w$ is given by $w'(t) = (2-t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.
d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

a) $\int_0^2 R(t)\,dt = 980$ people

b) $R'(t) = 0 \quad t = 1.36296$

test endpts and critical pt.

$R(0) = 0$

$R(1.36296) = 854.527$ Max rate is 854.527 people/hr. at $t = 1.362$

$R(2) = 120$

c) Since $w'(t) = (2-t)R(t)$, use FFTC to find $w(2) - w(1)$

$\int_1^2 w'(t) = w(2) - w(1)$

$\int_1^2 (2-t)R(t) = 387.5$ hrs. is total wait time.

d) Avg. wait time $= \frac{1}{980}\int_0^2 (2-t)R(t)\,dt = 0.775$ hrs.