AP Calculus AB Summer Review Packet 2021 Name: _____

Please read <u>all</u> of the following directions carefully (front and back of this page).

- This is a practice and review Pre-Calculus Packet to help students review important Pre-Calculus Concepts to prepare students for AP Calculus. **<u>This is a voluntary practice packet. No grade is given for the packet</u>**. However, this packet could be part of homework assignments once the school year starts. You are welcomed to get a head start and review over summer if you choose to.

- All of the problems in this packet should be solved algebraically **<u>without a calculator</u>**. If you need to leave an answer in logarithmic form such as $x = \ln 5$ that's fine. We would prefer this over a rounded decimal answer anyway.

- All intervals must be written in <u>interval notation</u>. For example, if your answer is $x \geq 2$, you need to write $[2, \infty)$. A great site for learning interval notation is: http://id.mind.net/~zona/mmts/miscellaneousMath/intervalNotation/intervalNotation.html

- All problems should be completed on <u>plain white paper</u>, not notebook paper, and certainly not written on this printout. You may use graph paper for the graphs if you wish but it's not required. Make sure all problems are labeled and easy to find.

- Don't forget that even though you will not have your textbooks over the summer, the internet has some great resources. A few good sites are listed below.
  http://library.thinkquest.org/20991/alg2/index.html http://www.khanacademy.com
  http://www.coolmath.com/algebra http://www.brightstorm.com
  http://www.purplemath.com

- A good site for learning about piecewise functions is:
  http://www.analyzemath.com/Graphing/piecewise_functions.html

- If there is a topic that you just need help with, try googling the topic. For example, if you don't remember how to solve an exponential equation, try googling "solving exponential equations" and a bunch of sites will probably pop right up with help and examples.

- Understand that everything in this packet is expected to be review. If you find that there is a lot in here that you are unable to do and you are leaving whole sections blank, you might want to rethink your decision to enter AP calculus. You can receive help with small questions from past topics, but you will not be re-taught entire units such as logarithms or trigonometry. You need to have some mastery of these topics before entering the class.

- If you have any questions or problems or just need a bit of help over the summer, please feel free to email Mr. Yang at yangd@fultonschools.org.
- If you need additional support and would like to meet for help sessions over Teams**<u>, I will be available weekly Fridays during the summer from 4-5pm.</u>** Please email me if you would like to set up an appointment. (yangd@fultonschools.org.)
- Full worked out solutions for summer packet available top of page of my website: **mryangteacher.weebly.com**

1) Find the x and y-intercepts for each of the following:

a) $y = x^2 + x - 1$

b) $y = (x-1)\sqrt{9-x^2}$

c) $y = \frac{x^2+3x}{(3x+1)^2}$

d) $x^2y - x^2 + 4y = 0$

2) Find all points of intersection of each of the following:

a) $2x - 3y = 13$ and $5x + 3y = 1$

b) $x^2 + y^2 = 25$ and $2x + y = 10$

c) $y = x^3 - 4x$ and $y = -x$

d) $y = x^4 - 2x^2 + 1$ and $y = 1 - x^2$

3) Write the equation of the line with the following characteristics

a) passes through (3, -4) and (5, 2)

b) is a horizontal line with a y-intercept at -4

c) is a vertical line that passes through (7, -8)

d) has an x-intercept at 5 and a y-intercept at -3

e) is parallel to the line $3x + 4y = 7$, passes through the point (-6, 4) and is written in point-slope form

f) is perpendicular to the line $5x - 3y = 0$, passes through the point $\left(\frac{3}{4}, \frac{7}{8}\right)$ and is written in point-slope form

4) For the function $f(x) = x^2 - 4x$, find each of the following:

a) $f(4)$ b) $f\left(\frac{3}{2}\right)$ c) $f(q)$ d) $f(t+4)$

5) Find the value of $\frac{f(x)-f(3)}{x-3}$ for each of the following functions:

a) $f(x) = 3x + 7$ b) $f(x) = 3x^2 - 2x + 1$ c) $f(x) = \frac{6}{x}$

6) Find the value of $\frac{f(x+h)-f(x)}{h}$ for each of the following functions:

a) $f(x) = 3x + 7$ b) $f(x) = 3x^2 - 2x + 1$ c) $f(x) = \frac{6}{x}$

7) For the piecewise function $p(x) = \begin{cases} \sqrt{x+4}, x \le 5 \\ (x-5)^2, x > 5 \end{cases}$ find each of the following:

a) $p(-3)$ b) $p(0)$ c) $p(5)$ d) $p(10)$

8) Write the following as piecewise functions: a) $y = |5x + 2|$ b) $y = |3 - 4x| + 8$

9) If $f(x) = \frac{3}{x}$ and $g(x) = x^2 - 1$, find a) $f(g(x))$ and b) $g(f(x))$ and state the domain of each.

10) Determine the domain for each of the following:

a) $g(x) = \frac{3x^2+2x-8}{2x^2+x-6}$ b) $h(x) = \sqrt{x} + \sqrt{1-x}$ c) $k(x) = \sqrt{x^2 - 3x + 2}$

11) Find the inverse of each of the following:

a) $f(x) = 4x - 3$ b) $g(x) = \frac{2x+3}{x-4}$ c) $h(x) = x^3 + 1$

12) For each of the following, find $f(-x)$ and use it to determine if the function is odd, even, or neither:

a) $f(x) = x^2(4 - x^2)$ b) $f(x) = \sqrt[3]{x}$ c) $f(x) = x \cos x$

13) Graph each of the following:

a) $f(x) = -3x + 2$ b) $f(x) = 1 - x^2$ c) $f(x) = \frac{1}{x}$ d) $f(x) = \begin{cases} 2x - 3, & x \le 3 \\ (x-6)^2 - 2, & x > 3 \end{cases}$

e) $f(x) = |\, 6 - x \,|$ f) $f(x) = \sqrt{9 - x^2}$ g) $f(x) = \ln(x - 1)$ h) $f(x) = e^x + 1$

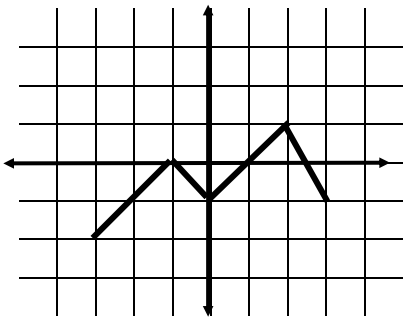
14) a – h) State the domain and range of each function graphed in problem #13

15) a – h) Use end behavior to identify $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for each function graphed in problem #13, with the exception of part g where you should find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to a^+} f(x)$ where *a* is the vertical asymptote for the function. You may skip part f for this problem.

16) a – h) State the intervals of *x* on which each function graphed in problem #13 is increasing or decreasing.

17) If the graph of $f(x)$ is shown below, graph each of the following:

a) $f(-x)$ b) $-f(x)$ c) $|f(x)|$ d) $f(x)+2$ e) $f(x+2)$



18) State the value of all 6 trig functions for each of the following angles:

a) $\frac{5\pi}{4}$ b) $\frac{11\pi}{6}$ c) $\frac{14\pi}{3}$ d) $\frac{\pi}{2}$ e) π

19) State the amplitude, period, phase shift and vertical shift for each of the following:

a) $y = 1 + \cos\left[3\left(x - \frac{\pi}{3}\right)\right]$ b) $y = 2 - 3\sin(4x + \pi)$

20) Simplify the following:

a) $\sin x \cos x \tan x \sec x \csc x$ b) $\frac{\sin x}{1+\cos x} + \frac{\sin x}{1-\cos x}$ c) $\cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x$

21) Solve the following (use identities where necessary) given that $0 \le x < 2\pi$

a) $\cos^2 x - \cos x + 1 = \sin^2 x$ b) $\sin x \tan x = \sin x$ c) $\sin x = \cos 2x - 1$

d) $\sin 4x = \frac{1}{2}$ e) $\cot^2 x - \csc x = 1$ f) $\sec^2 x + 2\sec x = 0$

g) $\sin x = \cos x$ h) $\sin 2x = \cos x$ i) $\cot x \cos^2 x = 2\cot x$

22) Simplify the following given that all angles are between 0 and π:

a) $\sin\left(2\arccos\frac{\sqrt{2}}{2}\right)$ b) $\cos\left(\arccos 0 + \arcsin\frac{1}{2}\right)$

23) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) $\sin(\arccos 2x)$ b) $\cot(\arcsin x)$ c) $\sin(\arctan 3x)$

24) State horizontal asymptote(s), vertical asymptote(s) and hole(s) for each of the following:

a) $y = \frac{2x^2-7x-4}{6x^2+7x+2}$ b) $y = \frac{5x^2+20x}{x^3-3x^2-28x}$

25) Solve the following: a) $\frac{2x^2-7x-4}{6x^2+7x+2}=0$ b) $\frac{5x^2+20x}{x^3-3x^2-28x} = 0$

c) $\frac{2x^2-7x-4}{6x^2+7x+2} < 0$ d) $\frac{5x^2+20x}{x^3-3x^2-28x} \geq 0$ e) $\sqrt{3x+1} \leq 4$ f) $\sqrt{2x^2-13x+6} > 0$

26) Solve the following:

a) $\log_{81}\sqrt{3} = x$ b) $\log_x 64^{\frac{1}{3}} = \frac{1}{2}$ c) $9 = 4 + \log_2(x+3)$ d) $\frac{1}{3}\ln x = \ln 8$

e) $\log_b 8 = \log_b x + \log_b(x-2)$ f) $4\ln(x+3) = 12$ g) $e^{3x} = 6$ h) $7^{(x-4)} = 100$

27) Write each of the following using sigma notation:

a) 4 + 8 + 12 + 16 + 20 b) 1 + 4 + 9 + 16 + 25 + 36 + 49 c) 7 + 4 + 1 + -2 + …

28) "BA" (BAD ALGEBRA) SECTION – The solution to each of the following equations contains at least one step (and possibly more) that involves bad algebra. Your job is to find the bad algebra, explain (very briefly) why it is bad algebra, and re-solve the problem correctly. (All BA's in ap calculus result in full credit lost for any problem, every time they occur.)

a) $10x^2 + 7x = 12$
$x(10x + 7) = 12$
$x = 12$ and $10x + 7 = 12$
$x = 12, x = \frac{1}{2}$

b) $(x - 5)^2 = 16$
$x^2 - 25 = 16$
$x^2 = 41$
$x = \pm\sqrt{41}$

c) $x^2 > 9$
$x > \pm 3$

d) $\sin 2x = \sin x, 0 \leq x < 2\pi$
$2\sin x = \sin x$
$\sin x = 0$
$x = 0$

e) $x^3 = x^2$
divide both sides by x^2
$x = 1$

f) $e^{3\ln x} = 27$
$3x = 27$
$x = 9$

g) $\ln(x - 3) = 2$
$\ln x - \ln 3 = 2$
$\ln x = 2 + \ln 3$
$x = e^{2+\ln 3}$
$x = e^2 \cdot e^{\ln 3}$
$x = 3e^2$

h) $5x + 2x^{-1} = -11$
$\frac{5x+2}{x} = -11$
$5x + 2 = -11x$
$16x = -2$
$x = -\frac{1}{8}$

i) $2x^{-1} = 4$
$\frac{1}{2x} = 4$
$8x = 1$
$x = \frac{1}{8}$

j) $2(x - 1)^2 - x^2 = 14$
$(2x - 2)^2 - x^2 = 14$
$4x^2 - 4 - x^2 = 14$
$3x^2 = 18$
$x^2 = 6$
$x = \sqrt{6}$

k) $\sqrt{x^2 - 16} = 3$
$x - 4 = 3$
$x = 7$

l) $\frac{1}{x} + \frac{1}{2} = \frac{1}{3}$
$x + 2 = 3$
$x = 1$

m) $x = \sqrt{4}$
$x = \pm 2$

n) $\sqrt{x} = 9$
$x = 3$

BONUS (5 points): Find the coordinates of all points, *P*, <u>on the *x*-axis</u> so that the line through *A*(-3, -2) and *P* is perpendicular to the line through *B*(2, 7) and *P*.