6.01: Review of Trigonometry at Any Angle **Date:** ____________

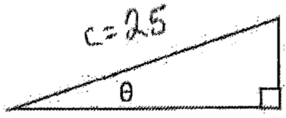
*Degree Mode

Helpful Formulas:

Pythagorean Theorem: $a^2 + b^2 = c^2$ $\quad \sin\theta = \frac{opp}{hyp} \quad \cos\theta = \frac{adj}{hyp} \quad \tan\theta = \frac{opp}{adj}$

Find the length of the hypotenuse and the measure of the angle of elevation for the given right triangle.

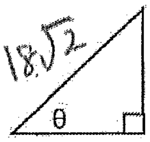
1.



$7^2 + 24^2 = c^2$ $\boxed{c = 25}$

$\tan\theta = \frac{7}{24}$ $\quad \theta = \tan^{-1}\left(\frac{7}{24}\right)$ $\quad \boxed{\theta = 16.26°}$

2.

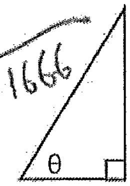


$18^2 + 18^2 = c^2$

$c = 18\sqrt{2}$

$\tan\theta = \frac{18}{18}$ $\quad \theta = \tan^{-1}(1)$ $\quad \boxed{\theta = 45°}$

3



$c = \sqrt{1666}$

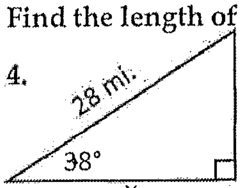
$21^2 + 35^2 = c^2$

$\tan\theta = \frac{35}{21}$ $\quad \boxed{\theta = 59.04°}$

$\theta = \tan^{-1}\left(\frac{35}{21}\right)$

Find the length of the two legs for the given right triangle.

4.

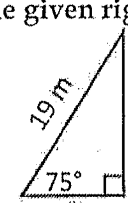


$\cos\theta = \frac{adj}{hyp}$ $\quad \frac{\cos 38}{1} = \frac{x}{28}$

$x = 28\cos 38 = \boxed{22.06 \text{ mi}}$

$\sin\theta = \frac{O}{H}$ $\quad \frac{\sin 38}{1} = \frac{y}{28}$ $\quad y = 28\sin 38$ $\quad \boxed{y = 17.24}$

5.



$\cos 75 = \frac{x}{19}$

$x = 19\cos 75$

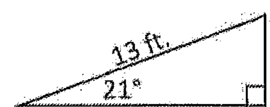
$\boxed{x = 4.92}$

$\sin 75 = \frac{y}{19}$

$y = 19\sin 75$

$\boxed{y = 18.35}$

6.



$\cos 21 = \frac{x}{13}$ $\quad x = 13\cos 21$ $\quad \boxed{x = 12.14}$

$\sin 21 = \frac{y}{13}$

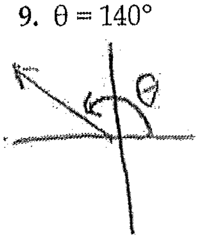
$y = 13\sin 21$

$\boxed{y = 4.66}$

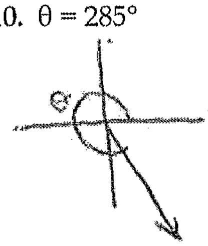
Sketch each angle in standard position and then state the quadrant where each angle has its terminal side.

Q3

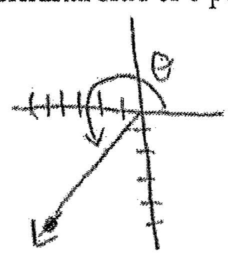
9. θ = 140°



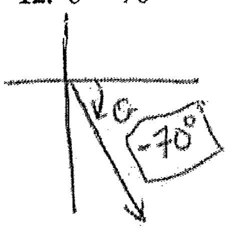
10. θ = 285°



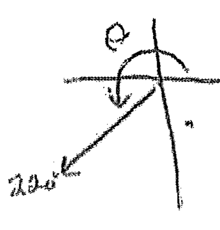
11. The terminal side of θ passes through (-6, -5)



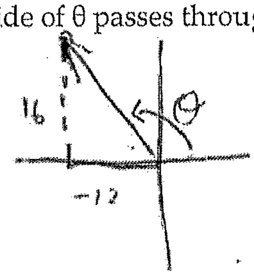
12. θ = -70°



13. θ = 220° Q3



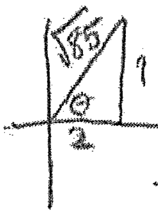
14. The terminal side of θ passes through (-12, 16)



More formulas: Pythagorean Theorem: $a^2 + b^2 = c^2$ $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Sketch each angle described in standard position, where $0 \le \theta < 360°$. Find the distance the point is from the origin and the measure of the angle. (use $\tan\theta$) (hypotenuse)

15. The terminal side of θ passes through (2, 9)

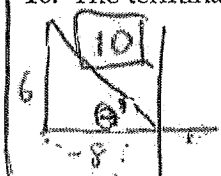


$c^2 = a^2 + b^2$
$c^2 = 2^2 + 9^2$
$c = \sqrt{85}$

$\tan\theta = \frac{9}{2}$ (Reference Angle)

$\theta = \tan^{-1}(9/2) \rightarrow \boxed{\theta = 77.47°}$

16. The terminal side of θ passes through (-8, 6) Q2



$c^2 = 6^2 + 8^2 \rightarrow c = 10$

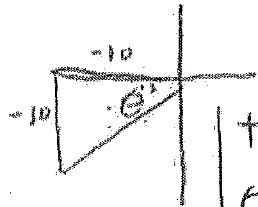
$\tan\theta' = \frac{6}{-8}$ $\theta' = -36.87°$

$\theta = 180 - 36.87°$

$\boxed{\theta = 143.13°}$

Q4

17. The terminal side of θ passes through (-10, -10) Q3



$\tan\theta' = \frac{-10}{-10}$

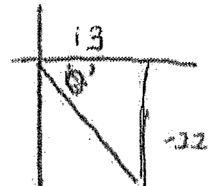
$\theta' = \tan^{-1}(1)$

$\theta' = 45°$

$\theta = 180 + 45 = \boxed{225°}$

$c^2 = a^2 + b^2$
$c^2 = 10^2 + 10^2$
$c = \boxed{10\sqrt{2}}$

18. The terminal side of θ passes through (13, -22)



$\tan\theta' = \frac{-22}{13}$ $\theta' = \tan^{-1}\left(\frac{-22}{13}\right)$

$\theta' = -59.42°$

$\theta = 360 - 59.42°$

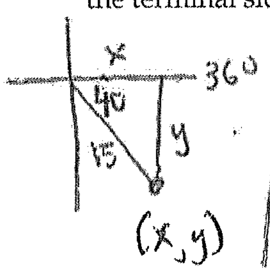
$\boxed{\theta = 300.58°}$

$c^2 = a^2 + b^2$
$c^2 = 13^2 + 22^2$
$\boxed{c = \sqrt{653}}$

Self Check: **Did your measures for the angles in #15 – 18 get larger each time? If not, you did something wrong.**

Sketch each point described with its given distance from the origin and angle measure in standard position. Determine the ordered pair coordinates described.

19. The point lies 15 inches from the origin on the terminal side of θ = 320°. (hypotenuse given) Q4



$\frac{\cos 40}{1} = \frac{x}{15}$

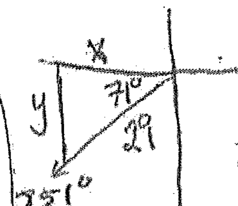
$\frac{\sin 40}{1} = \frac{y}{15}$

$x = 15\cos 40$
$x = 11.49$

$y = 15\sin 40$
$y = 9.64$

$\boxed{(11.49, -9.64)}$

20. The point lies 29 mm from the origin on the terminal side of θ = 251°. Q3



$\cos 71 = \frac{x}{29}$

$x = 29\cos 71$
$x = 9.44$

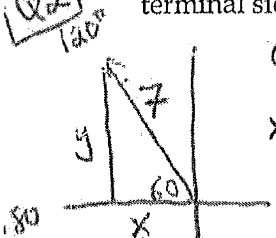
$\sin 71 = \frac{y}{29}$

$y = 29\sin 71$
$y = 27.42$

$\boxed{(-9.44, -27.42)}$

Q2

21. The point lies 7 yds. from the origin on the terminal side of θ = 120°.



$\cos 60 = \frac{x}{7}$

$x = 7\cos 60$
$x = 3.5$

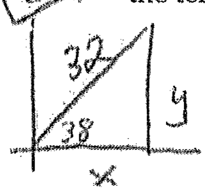
$\sin 60 = \frac{y}{7}$

$y = 7\sin 60$
$y = 6.06$

$\boxed{(-3.5, 6.06)}$

Q1

22. The point lies 32 dm from the origin on the terminal side of θ = 38°.



$\cos 38 = \frac{x}{32}$

$x = 32\cos 38$
$x = 25.22$

$\sin 38 = \frac{y}{32}$

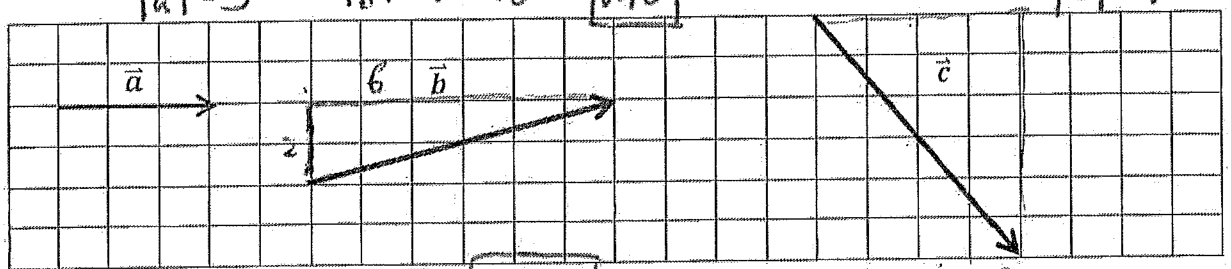
$y = 32\sin 38$
$y = 19.7$

$\boxed{(25.22, 19.7)}$

Vector is a quantity that has both direction and magnitude
4 (velocity) vs. speed

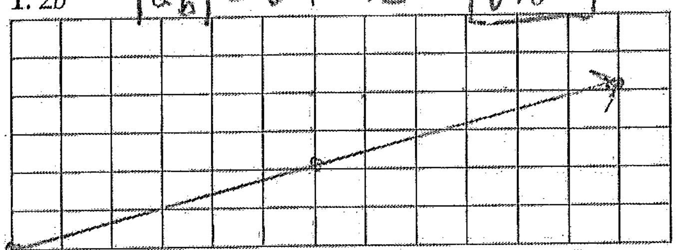
APC 6.02 Geometric Vectors - Notes

Use the following to draw each vector diagram and resultant vector. Find the magnitude of the resultant. $|\vec{a}| = 3$ $|\vec{b}| = \sqrt{2^2+6^2} = \boxed{\sqrt{40}}$ $|\vec{c}| = \sqrt{4^2+6^2} = \boxed{\sqrt{52}}$

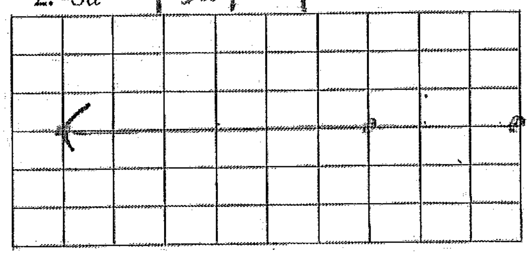


4 →
6 ↓

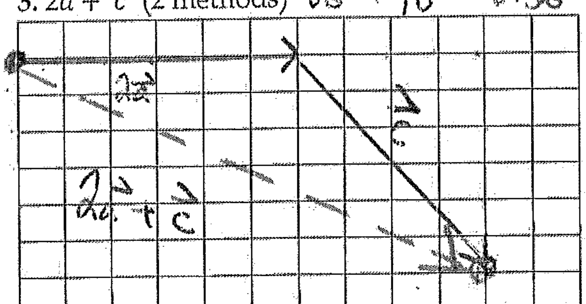
1. $2\vec{b}$ $|2\vec{b}| = \sqrt{4^2+12^2} = \boxed{\sqrt{160}}$



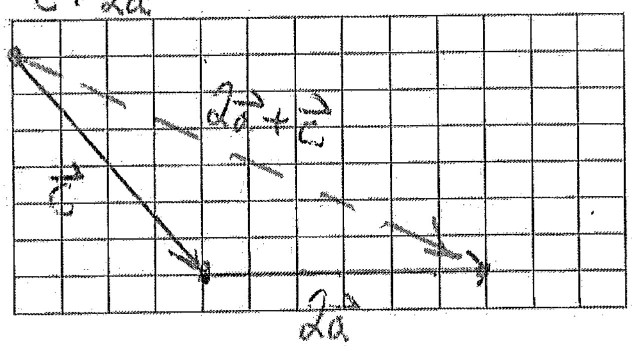
2. $-3\vec{a}$ $|-3\vec{a}| = 9$



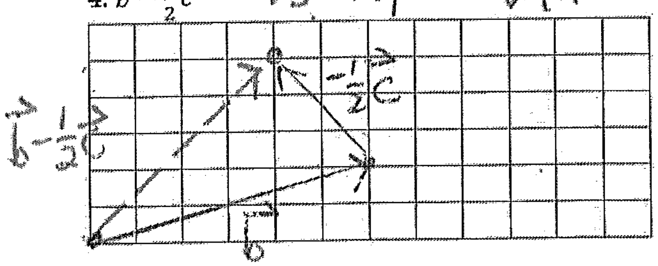
3. $2\vec{a} + \vec{c}$ (2 methods) $\sqrt{6^2+10^2} = \sqrt{136}$

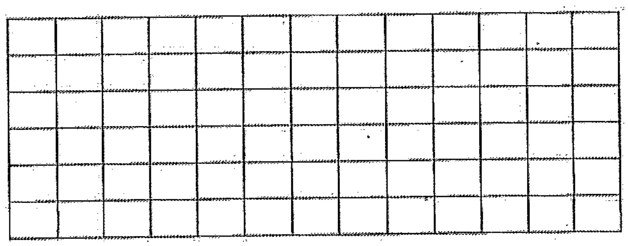


$\vec{c} + 2\vec{a}$



4. $\vec{b} - \frac{1}{2}\vec{c}$ $\sqrt{5^2+4^2} = \sqrt{41}$





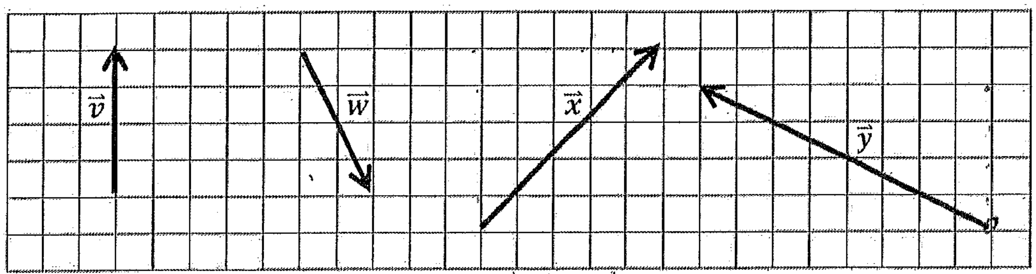
State whether each quantity described is a *vector quantity* or a *scalar quantity*.

1. A box being pushed with a force of 125 newtons Scalar
2. Wind blowing 20 knots Scalar
3. A deer running 15 m/sec due west vector
4. A 15-pound tire hanging from a rope vector

* vector quantity has both direction and magnitude
* scalar quantity only has magnitude

6.02 Practice: Vectors Geometrically

$5\uparrow\ 5\rightarrow$ $\quad$ $4\uparrow\ 8\leftarrow$



1) $\left|\frac{1}{2}\vec{x}\right| = \sqrt{2.5^2+2.5^2} = \sqrt{12.5}$

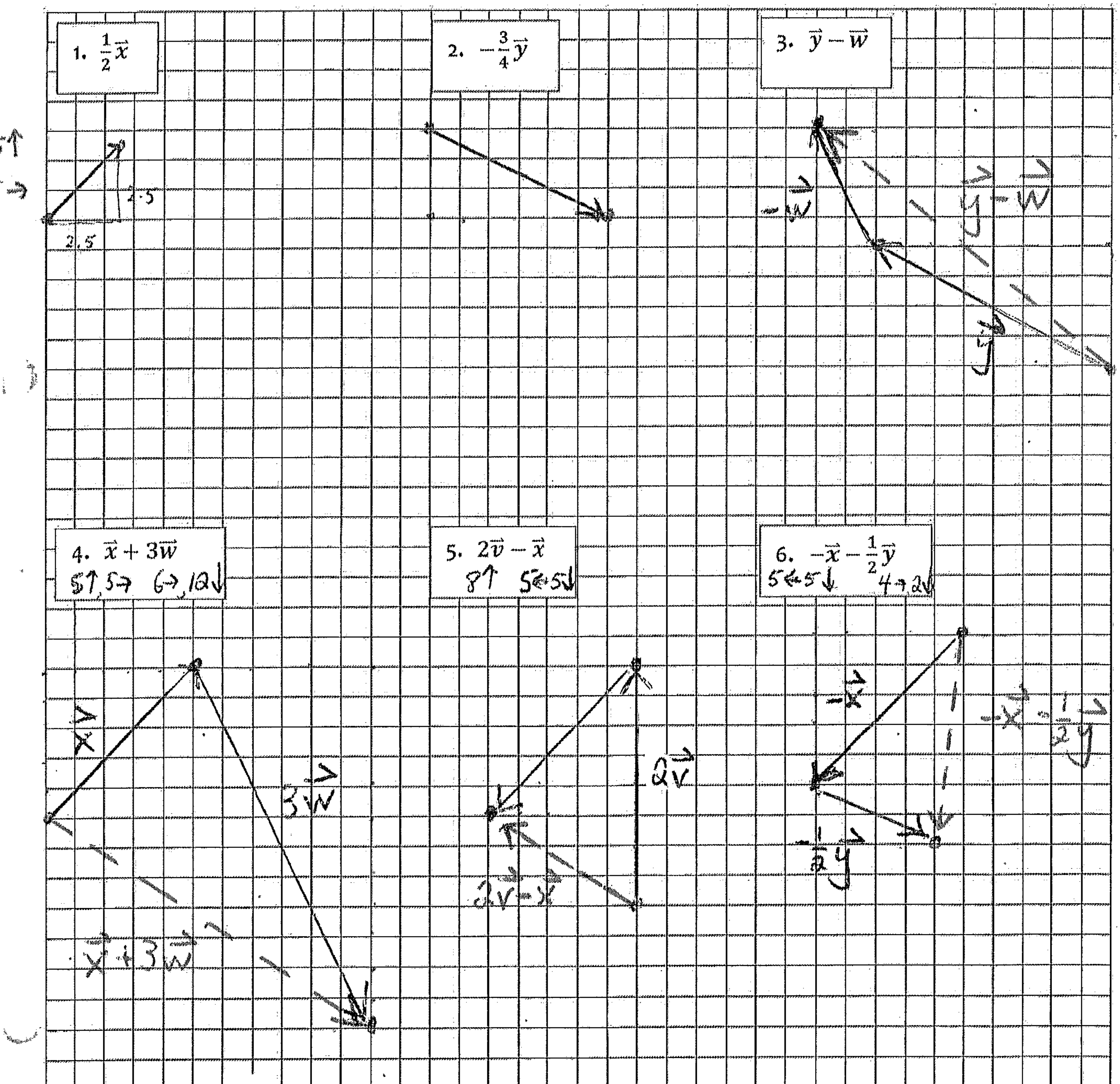
2) $\left|\frac{-3}{4}\vec{y}\right| = \sqrt{3^2+6^2} = \sqrt{45}$

$-3/4\,(4,8) \rightarrow -3\downarrow\ 6\rightarrow$

Draw each and label the resultant. Then, find the magnitude of the resultant vector.

$-\vec{w}$ is $4\uparrow\ 2\leftarrow$

$\sqrt{8^2+10^2} = \sqrt{164}$



$3\vec{w}$ is $6\rightarrow 12\downarrow$

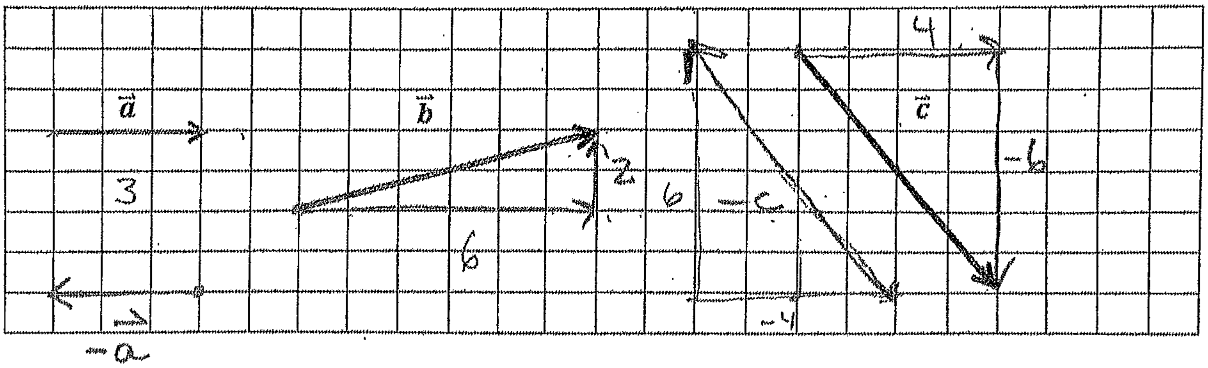
$\sqrt{7^2+11^2} = \sqrt{170}$

$\sqrt{3^2+5^2} = \sqrt{34}$

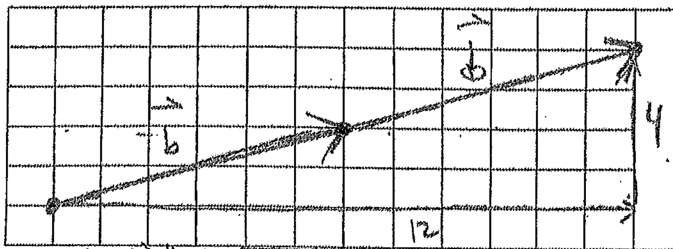
$\sqrt{7^2+1^2} = \sqrt{50}$

APC 5.02 Geometric Vectors - Notes

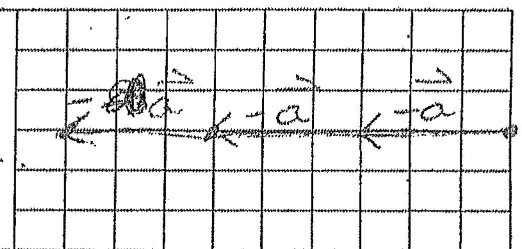
Use the following vectors to draw each. Find the magnitude.



1. $2\vec{b}$

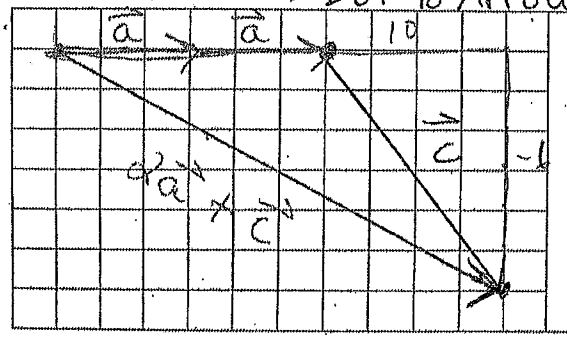


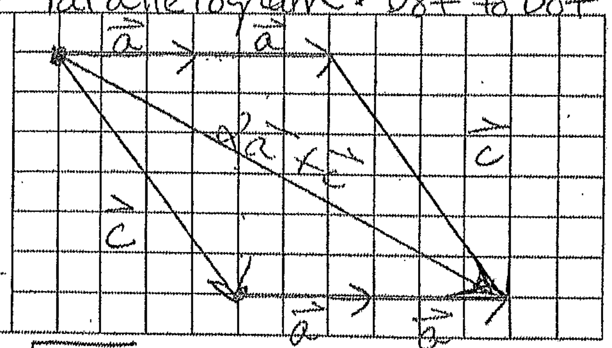
2. $-3\vec{a}$



$|2\vec{b}| = \sqrt{12^2+4^2} = \sqrt{160}$ $\qquad |-3\vec{a}| = 9$ or $\sqrt{(-9)^2+0^2} = 9$

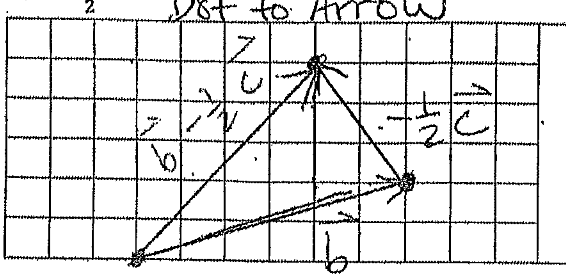
3. $2\vec{a} + \vec{c}$ (2 methods) Dot to Arrow Parallelogram: Dot to Dot

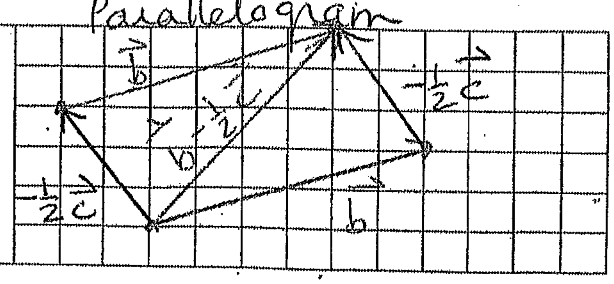




$|2\vec{a}+\vec{c}| = \sqrt{10^2+(-6)^2} = \sqrt{136}$

4. $\vec{b} - \frac{1}{2}\vec{c}$ Dot to Arrow Parallelogram



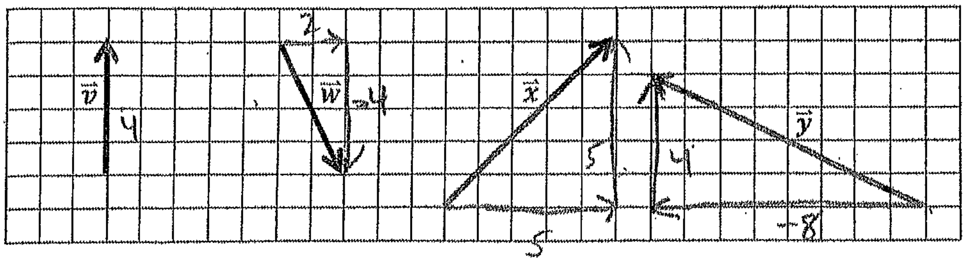


State whether each quantity described is a *vector quantity* or a *scalar* quantity.

1. A box being pushed with a force of 125 newtons Scalar
2. Wind blowing 20 knots Scalar
3. A deer running 15 m/sec due west Vector
4. A 15-pound tire hanging from a rope Vector

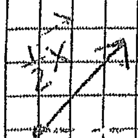
Accelerated Precalculus

5.02 Homework: Vectors Geometrically



Draw each and label the resultant. Then, find the magnitude of the resultant vector.

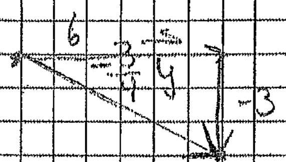
1. $\frac{1}{2}\vec{x}$



$\left\|\frac{1}{2}\vec{x}\right\| = \sqrt{2.5^2+2.5^2}$

$= \sqrt{12.5}$ or 3.536

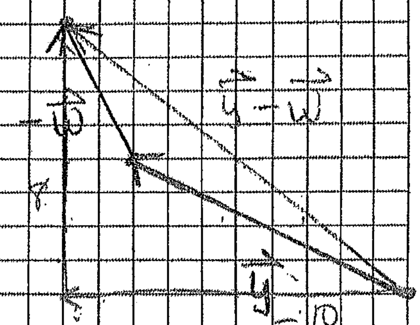
2. $-\frac{3}{4}\vec{y}$



$\left\|-\frac{3}{4}\vec{y}\right\| = \sqrt{6^2+(-3)^2}$

$= \sqrt{45}$ or 6.708

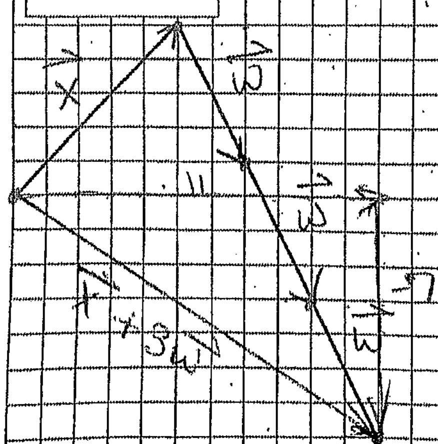
3. $\vec{y} - \vec{w}$



$\|\vec{y}-\vec{w}\| = \sqrt{(-10)^2+(8)^2}$

$= \sqrt{164}$ or 12.806

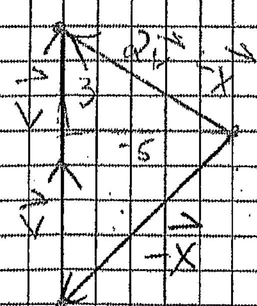
4. $\vec{x} + 3\vec{w}$



$\|\vec{x}+3\vec{w}\| = \sqrt{11^2+(-7)^2}$

$= \sqrt{170}$ or 13.038

5. $2\vec{v} - \vec{x}$

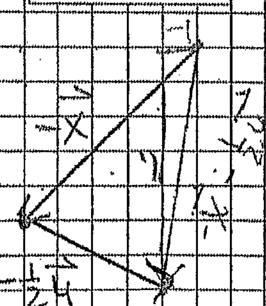


$\|2\vec{v}-\vec{x}\|$

$\sqrt{(-5)^2+3^2}$

$= \sqrt{34}$ or 5.831

6. $-\vec{x} - \frac{1}{2}\vec{y}$



$\left\|-\vec{x}-\frac{1}{2}\vec{y}\right\| = \sqrt{(-1)^2+7^2}$

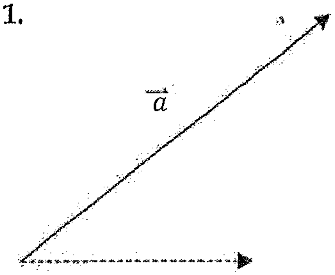
$= \sqrt{50}$ or 7.071

6.03 Notes: More Geometric Vectors

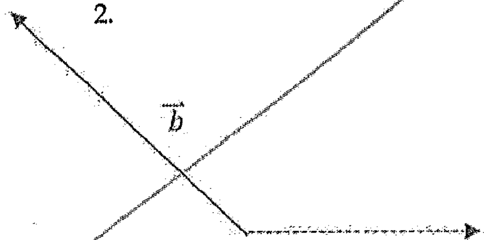
- Use a ______________ to measure the magnitude of a vector.
- Use a ______________ to measure the direction of a vector.

Examples: Measure the magnitude (in cm) and direction (in degrees) of each vector.

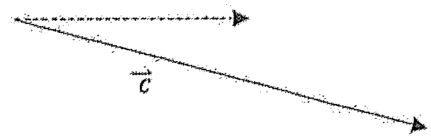
1.



2.



3.



Mag = ______ Dir = ______ Mag = ______ Dir = ______ Mag = ______ Dir = ______

Now, draw each vector diagram and find the magnitude and direction of the resultant vector.

4. $\vec{a} + \vec{b}$

5. $2\vec{c} - \vec{a}$

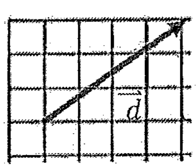
When a grid is used, first find the component form: <horizontal displacement, vertical displacement>. Then:

- Use a $\vec{v} = \sqrt{a^2+b^2}$ to calculate the magnitude of a vector.
- Use a $\theta = \tan^{-1}(b/a)$ to calculate the direction of a vector.

  Caution: The calculator is not always right!

Examples: Find the component form, magnitude, and direction (using standard position) of $\vec{d}$ and $\vec{e}$.

6.

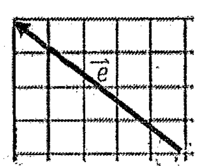


$\langle 4, 3 \rangle$

$|\vec{d}| = \sqrt{4^2+3^2} = 5$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = \boxed{36.87^\circ}$

7.



$\langle -5, 4 \rangle$

$|\vec{e}| = \sqrt{4^2+5^2} = \sqrt{41}$

$\theta = \tan^{-1}\left(\frac{4}{-5}\right) = -38.66^\circ$

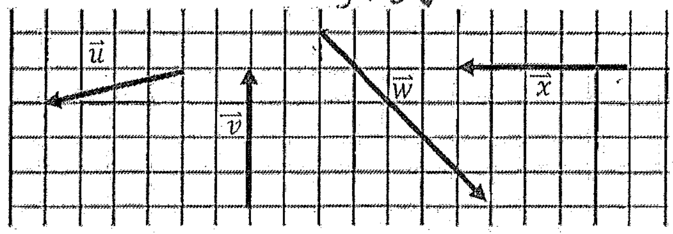
$+180$

$\boxed{\theta = 141.34^\circ}$

6.03 Practice: More Geometric Vectors

Use the vectors to the right to complete problems #1 – 6. Round answers to the nearest thousandth. Use standard position for the direction of a vector.

5 → 5 ↓



1. Find the component form, magnitude, and direction of $\vec{x}$.

$\langle -5, 0 \rangle$

magnitude: $|\vec{x}| = \sqrt{(-5)^2 + 0^2} = \boxed{5}$

direction $\theta = \tan^{-1}\left(\frac{0}{-5}\right) = 0 \rightarrow \boxed{180^\circ}$

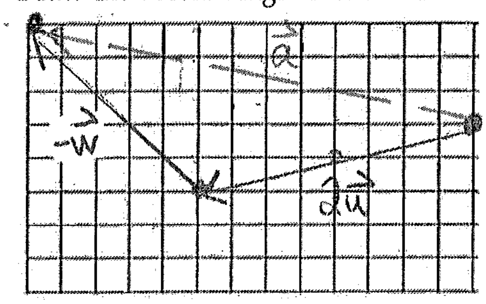
2. Find the component form, magnitude, and direction of $\vec{u}$.

$\langle -4, -1 \rangle$

$|\vec{v}| = \sqrt{(-4)^2 + (-1)^2} = \sqrt{17}$

$\theta = \tan^{-1}\left(\frac{-1}{-4}\right) = 14.036^\circ$
$+180$
$\boxed{194.036^\circ}$

3. Draw the vector diagram of $\vec{a} = 2\vec{u} - \vec{w}$.



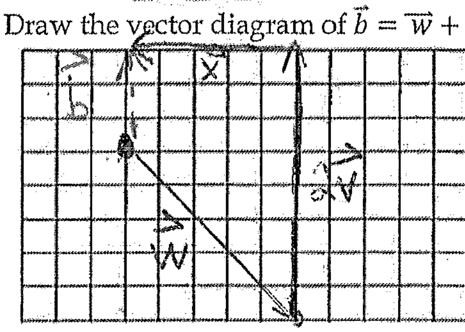
Component form of $\vec{a}$: $\langle -13, 3 \rangle$

Magnitude = $\sqrt{178}$ Direction = 167.005°

$\sqrt{(-13)^2 + (3)^2} =$

$\theta = \tan^{-1}\left(\frac{3}{-13}\right) = -12.995$
$+180$

4. Draw the vector diagram of $\vec{b} = \vec{w} + 2\vec{v} + \vec{x}$.

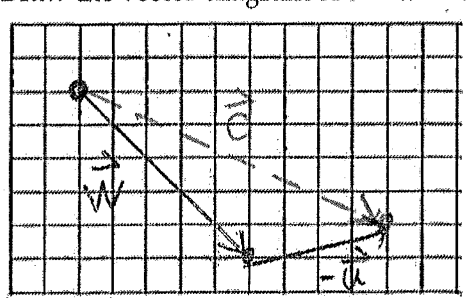


Component form of $\vec{b}$: $\langle 0, 3 \rangle$

Magnitude = 3 Direction = 90°

$\sqrt{0^2 + 3^2} =$

5. Draw the vector diagram of $\vec{c} = \vec{w} - \vec{u}$.



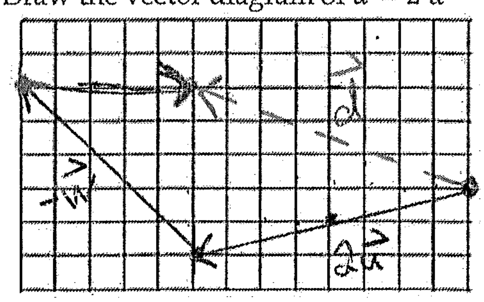
Component form of $\vec{c}$: $\langle 9, -4 \rangle$

Magnitude = $\sqrt{97}$ Direction = 336.038°

$\sqrt{9^2 + (-4)^2}$

$\theta = \tan^{-1}\left(\frac{-4}{9}\right) \rightarrow -23.962$
$+360$

6. Draw the vector diagram of $\vec{d} = 2\vec{u} - \vec{w} - \vec{x}$



Component form of $\vec{d}$: $\langle -8, 3 \rangle$

Magnitude = $\sqrt{73}$ Direction = 159.444°

$\sqrt{(-8)^2 + 3^2}$

$\theta = \tan^{-1}\left(\frac{3}{-8}\right) = -20.556^\circ$
$+180$

Name

Period

Accelerated Pre-Calculus

7.02 TASK: Walking & Flying Around Hogsmeade

Name: ____________

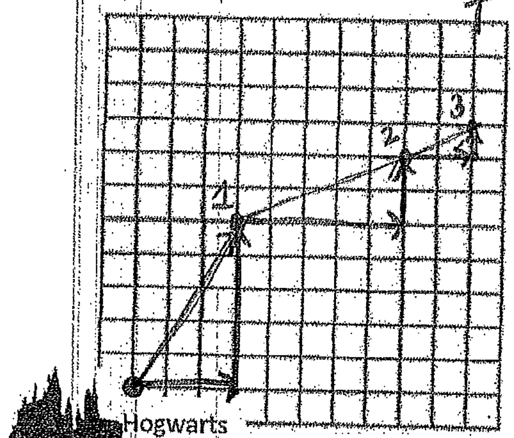
Period: ______

Harry Potter needs to make a few stops around Hogsmeade. Harry's broom is broken, so he must walk between the buildings. The town is laid out in square blocks, which makes it easy to give directions. Here are the directions Harry must follow Monday:

Monday – Start at Hogwarts				
	East/West	North/South		
Stop 1 The 3 Broomsticks	3 blocks East	5 blocks North	8	$\sqrt{34}$
Stop 2 Honeydukes	5 blocks East	2 blocks North	7	$\sqrt{29}$
Stop 3 Gladrags	2 blocks East	1 block North	3	$\sqrt{5}$

T 18 blocks

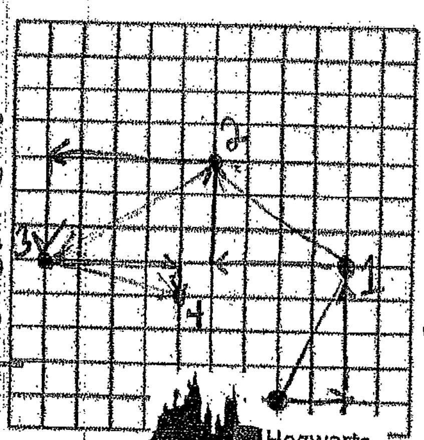
1. Draw the route of Harry's trip on the grid below.
2. Label each of Harry's stops.
3. Complete the chart above calculating Harry's total trek.
4. *How would Harry's trip through Hogsmeade change if he was able to ride his broom to his three stops?* If this would make a different route for Harry, draw this new route in a *different color*.



On Tuesday, Harry has more errands to run. When he is done, he will meet Ron at the Shrieking Shack, found 3 blocks West and 3 blocks North of Hogwarts. His directions for the first 3 stops are listed in the chart below. Draw the route of Harry's trip on the grid. Use the graph to determine his path to the Shrieking Shack. Calculate the totals for Harry's trip.

Tuesday – Start at Hogwarts				
	East/West	North/South		
Stop 1 Zonko's Joke Shop	2 blocks East	4 blocks North	6	$2\sqrt{5}$
Stop 2 Scrivenshaft's	4 blocks West (or -4 blocks East)	3 blocks North	7	5
Stop 3 Dervish and Bangs	5 blocks West (or -5 blocks East)	3 blocks South (or -3 blocks North)	8	$\sqrt{34}$
Stop 4 Shrieking Shack	4 blocks E	1 blocks S	5	$\sqrt{17}$

T 26 blocks

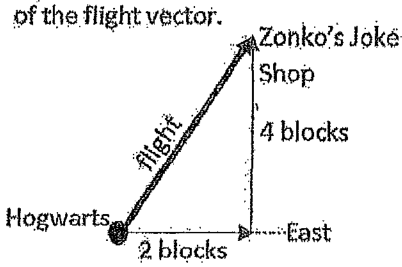


Name______________________________

Period

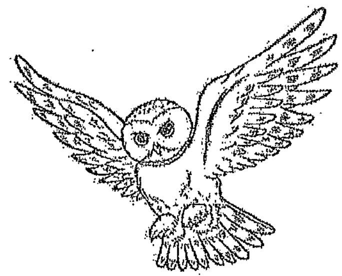
Harry's trusted owl, Hedwig, can fly over buildings, so she travels in a straight line from each stop to the next and waits for Harry to arrive. On Tuesday's graph, use a different color to draw arrows representing Hedwig's path.

How far did Hedwig fly to get to Stop 1 on Tuesday? This is known as the **magnitude** of the flight vector.



$\sqrt{2^2+4^2}$

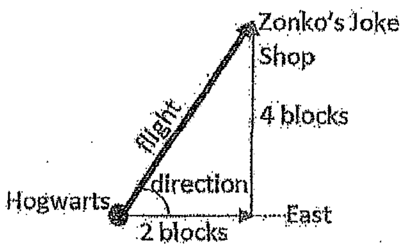
$2\sqrt{5}$ blocks



There are several ways to describe Hedwig's direction during this leg of the trip. We could simply say she traveled "north-east", but this would not be precisely accurate or an exact description. Why not?

NE is considered 45° or b/t 0° & 90°; either way it's not accurate or precise

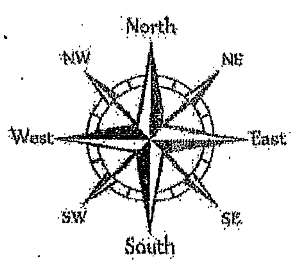
For **vectors**, we include an angle, often measured in **standard position**, to indicate the **direction**. Use **inverse trigonometry** to find the direction of Hedwig's flight from Hogwarts to Tuesday's first stop.



$\theta = \tan^{-1}(2)$

$\theta = 63.4°$

Next stop: Scrivenshaft's. We might say that Hedwig flew north-west, but it would not be an exact description. What is the possible range of values for all angles measured in **standard position** that are generally pointing to the north-west? What is *the* value for an angle pointing precisely to north-west?



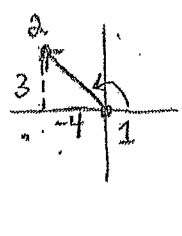
$90° < \theta < 180°$ NW = 135°

Find the **magnitude** (distance) and **direction** of Hedwig's path from Stop 1 to Stop 2 on Tuesday. Show work. CAUTION: The angle must be measured in **standard position**, meaning it *opens counter-clockwise and is measured from the positive x-axis.* **The direction is NOT an acute angle measure!**

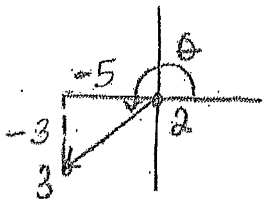
magnitude = 5

$\theta = \tan^{-1}\left(\frac{3}{-4}\right) = -36.870 + 180°$

$\approx 143.130°$



Find the **magnitude** (distance) and **direction** of Hedwig's path from Stop 2 to Stop 3 on Tuesday. Show work.

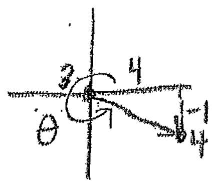


magnitude $= \sqrt{5^2+3^2} = \sqrt{34}$ blocks

$\theta = \tan^{-1}\left(\frac{-3}{-5}\right) = 30.964 + 180$

$\approx 210.964°$

Find the **magnitude** (distance) and **direction** of Hedwig's path from Stop 3 to Stop 4 on Tuesday. Show work.



magnitude $= \sqrt{4^2+(-1)^2} = \sqrt{17}$ blocks

$\theta = \tan^{-1}(-1/4) = -14.036 + 360$

$= 345.964°$

Ron sets out from Hogwarts to the Shrieking Shack to meet Harry. What directions does he take for the shortest path that follows the town's square blocks?

3 West, 3 North

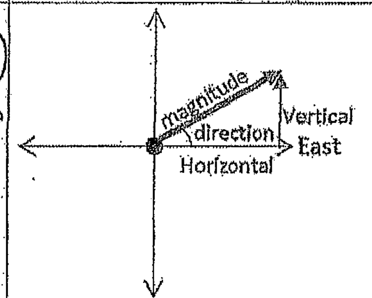
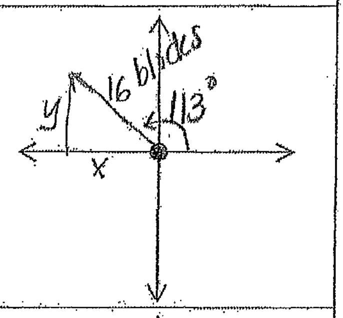
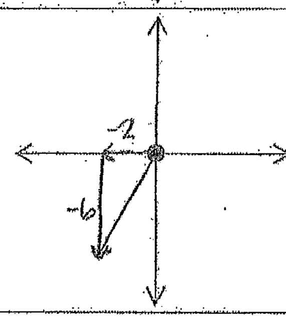
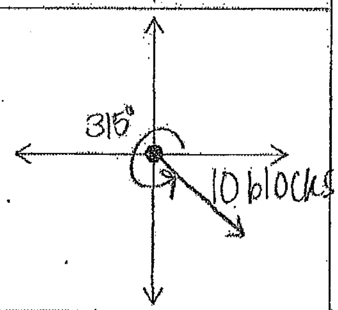
How does this route compare to the overall displacement of Harry during his errands on Tuesday?

They are the same.

The way we have expressed Harry's routes is known as **component form**, since it is made up of two parts, or components, that describe the changes in the horizontal direction and in the vertical direction. The way we have expressed Hedwig's path is known as **magnitude-direction form**, since it gives the distance directly to each stop and the angle measurement for the direction of the flight path.

It is important to be able to convert from one form to another. Practice this skill by filling in the table for Harry's route on Wednesday. Use the Pythagorean Theorem, trigonometry, and inverse trigonometry. Leaving the point (Hogwarts), draw Harry's path as horizontal and vertical components and Hedwig's path as a direct flight to the destination. Remember to include units.

Hint: create reference triangles! Remember, the angle for Hedwig's direction is *measured in standard position.*

Wednesday – Start at Hogwarts					
	Harry's description		Hedwig's description		
	Horizontal	Vertical	Magnitude	Direction	Drawing
a.	6 blocks East	3 blocks North	$\sqrt{6^2+3^2}$ $3\sqrt{5}$ blocks	$\theta=\tan^{-1}\left(\frac{3}{6}\right)$ $\theta=26.565°$	
b.	$\cos 113=\frac{x}{16}$ $16\cos 113$ 6.252 blocks West	$\sin 113=\frac{y}{16}$ $16\sin 113$ 14.728 blocks North	16 blocks	113°	
c.	2 blocks West	6 blocks South	$\sqrt{6^2+2^2}$ $=2\sqrt{10}$ blocks	$\theta=\tan^{-1}\left(\frac{-6}{-2}\right)$ $=71.565$ $+180$ $=251.565°$	
d.	$10\cos 315$ $5\sqrt{2}\approx 7.071$ blocks East	$10\sin 315$ $-5\sqrt{2}=-7.071$ 7.071 blocks south	10 blocks	315°	

Challenge: Thursday, Harry followed these directions through town: 2 blocks East, 5 blocks South, 1 block East, 3 blocks North, 4 blocks West, 2 blocks North, 3 blocks East, 1 block North, 3 blocks West, and 5 blocks South. If Ron wants to walk to Harry's final destination, what route should he take for the shortest trip?

Horizontal $= 2+1-4+3-3=-1$ one block West and 4 blocks south

Vertical $= -5+3+2+1-5=-4$

Hedwig flies from Hogwarts directly to Harry's final destination. What is the **magnitude** of her flight? What **direction**, measured in **standard position**, does she fly?

mag $=\sqrt{(-1)^2+(-4)^2}=\sqrt{17}$ blocks

$\theta=\tan^{-1}\left(\frac{-4}{-1}\right)=75.964+180$

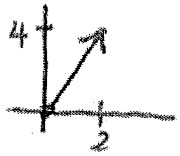
$=255.964°$

6.05 Algebraic Vectors Notes

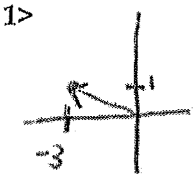
Component form: <a, b> a = horizontal component (displacement) and b = vertical component (displacement)

Ex: Draw the following vectors.

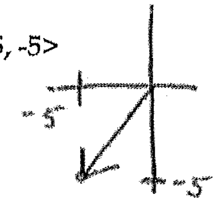
<2, 4>



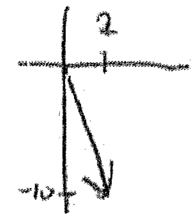
<-3, 1>



<-5, -5>



<2, -10>



Find component form of $\overrightarrow{CD}$ given initial and terminal points: C (7,-3) and D (9,1)

$x_1\ y_1$ $x_2\ y_2$

$\langle a, b\rangle = \langle x_2 - x_1, y_2 - y_1\rangle$

$= \langle 9-7, 1--3\rangle$ | $\overrightarrow{CD} = \langle 2, 4\rangle$

Operations with vectors:

If $\vec{u} = <a_1, b_1>$ and $\vec{v} = <a_2, b_2>$

$\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2\rangle$ $\vec{u} - \vec{v} = \langle a_1 - a_2, b_1 - b_2\rangle$ $k\vec{u} = \langle ka_1, kb_2\rangle$

Ex: Perform the indicated operations given $\vec{u} = <-4, 1>$ and $\vec{v} = <2, 5>$.

$\vec{u} + \vec{v} = \langle -4+2, 1+5\rangle$

$\boxed{\langle -2, 6\rangle}$

$\vec{u} - \vec{v} = \langle -4-2, 1-5\rangle$

$\boxed{\langle -6, -4\rangle}$

$2\vec{u} - \vec{v} =$

$\langle -8, 2\rangle - \langle 2, 5\rangle$

$\boxed{\langle -10, -3\rangle}$

$-3\vec{u} - 4\vec{v} = \underbrace{\langle +12, -3\rangle}_{-3u} - \underbrace{\langle 8, 20\rangle}_{4v}$

$\boxed{\langle 4, -23\rangle}$

Unit Vector: Vector with magnitude of 1

i = unit vector on the positive x-axis $\langle 1, 0\rangle$

j = unit vector on the positive y-axis $\langle 0, 1\rangle$

Linear combination of unit vectors (or "sum of unit vectors"):

$\langle a, b\rangle \rightarrow ai + bj$

Ex: Write <-3, 8> as a sum of unit vectors. $-3i + 8j$

Ex: Write <0, -5> as a linear combination of unit vectors. $0i - 5j = -5j$

Ex: Find the vector with initial point X (5,5) and terminal point Y (-2,6) as a sum of unit vectors.

$x_1\ y_1$ $x_2\ y_2$

$\langle -2-5, 6-5\rangle \rightarrow \langle -7, 1\rangle$ | $\overrightarrow{XY} = \langle -7, 1\rangle = -7i + 1j$

6.05 Practice: ODDS #1 – 17, 29–35

$\langle x_2 - x_1, y_2 - y_1 \rangle$

Find component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1. $A(-3, 1), B(4, 5)$
2. $A(2, -7), B(-6, 9)$
3. $A(10, -2), B(3, -5)$
4. $A(-2, 7), B(-9, -1)$
5. $A(-5, -4), B(8, -2)$
6. $A(-2, 6), B(1, 10)$
7. $A(2.5, -3), B(-4, 1.5)$
8. $A(-4.3, 1.8), B(9.4, -6.2)$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$
10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

1) $\langle 4 - -3, 5 - 1 \rangle \rightarrow \langle 7, 4 \rangle$

3) $\langle 3 - 10, -5 - -2 \rangle \rightarrow \langle -7, -3 \rangle$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$. (Example 3)

11. $4\mathbf{h} - \mathbf{g}$
12. $\mathbf{f} + 2\mathbf{h}$
13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$
14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$
15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$
16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$
17. $4\mathbf{g} - 3\mathbf{f} + \mathbf{h}$
18. $6\mathbf{h} + 5\mathbf{f} - 10\mathbf{g}$

11) $\langle -24, 8 \rangle - \langle -3, -5 \rangle = \langle -21, 13 \rangle$

13) $\langle -9, -15 \rangle - \langle 40, 0 \rangle + \langle -6, 2 \rangle = \langle -55, -13 \rangle$

Let $\overrightarrow{DE}$ be the vector with the given initial and terminal points. Write $\overrightarrow{DE}$ as a linear combination of the vectors $\mathbf{i}$ and $\mathbf{j}$. (Example 5)

28. $D(4, -1), E(5, -7)$
29. $D(9, -6), E(-7, 2)$
30. $D(3, 11), E(-2, -8)$
31. $D(9.5, 1), E(0, -7.3)$
32. $D(-3, -5.7), E(6, -8.1)$
33. $D(-4, -6), E(9, 5)$
34. $D\left(\frac{1}{8}, 3\right), E\left(-4, \frac{2}{7}\right)$
35. $D(-3, 1.5), E(-3, 1.5)$

29) $\langle -7 - 9, 2 - -6 \rangle \rightarrow \langle -16, 8 \rangle$ $\boxed{-16\mathbf{i} + 8\mathbf{j}}$

31) $\langle 0 - 9.5, -7.3 - 1 \rangle \rightarrow \langle -9.5, -8.3 \rangle$ $\boxed{-9.5\mathbf{i} - 8.3\mathbf{j}}$

6.05 Practice: ODDS #1 – 17, 29–35

Find component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1. $A(-3, 1), B(4, 5)$
3. $A(10, -2), B(3, -5)$
5. $A(-5, -4), B(8, -2)$
7. $A(2.5, -3), B(-4, 1.5)$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

① $\langle 7, 4 \rangle$
③ $\langle -7, -3 \rangle$
⑤ $\langle 13, 2 \rangle$
⑦ $\langle -6.5, 4.5 \rangle$
⑨ $\langle \frac{11}{2}, \frac{23}{2} \rangle$ or $\langle 5.5, 11.5 \rangle$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$. (Example 3)

11. $4\mathbf{h} - \mathbf{g}$	$\langle -24, 8 \rangle - \langle -3, -5 \rangle$	$\langle -21, 13 \rangle$
13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$	$\langle -9, -15 \rangle - \langle 40, 0 \rangle + \langle -6, 2 \rangle$	$\langle -55, -13 \rangle$
15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$	$\langle 8, 0 \rangle - \langle -6, -10 \rangle - \langle -12, 4 \rangle$	$\langle 26, 6 \rangle$
17. $4\mathbf{g} - 3\mathbf{f} + \mathbf{h}$	$\langle -12, -20 \rangle - \langle 24, 0 \rangle + \langle -6, 2 \rangle$	$\langle -42, -18 \rangle$

Let $\overrightarrow{DE}$ be the vector with the given initial and terminal points. Write $\overrightarrow{DE}$ as a linear combination of the vectors $\mathbf{i}$ and $\mathbf{j}$. (Example 5)

29. $D(9, -6), E(-7, 2)$	$\langle -16, 8 \rangle$	$-16i + 8j$
31. $D(9.5, 1), E(0, -7.3)$	$\langle -9.5, -8.3 \rangle$	$-9.5i - 8.3j$
33. $D(-4, -6), E(9, 5)$	$\langle 13, 11 \rangle$	$13i + 11j$
35. $D(-3, 1.5), E(-3, 1.5)$	$\langle 0, 0 \rangle$	$0i + 0j$

6.06 Notes: More Algebraic Vectors

Magnitude: Length of a vector ; can represent speed, distance, or force

$|\vec{v}| = \sqrt{a^2+b^2}$

Ex: Find $|\vec{v}|$ if $\vec{v}$ = <-3, 8>. $|\vec{v}| = \sqrt{(-3)^2+8^2} = \sqrt{9+64} = \sqrt{73}$

Ex: Find the magnitude of a vector with initial point A (-2,3) and terminal point B (4,5).

x_1, y_1 $\quad$ $x_2\ y_2$

$\langle 4 - -2, 5-3 \rangle$ | $|\vec{AB}| = \sqrt{6^2+2^2} = \sqrt{40} = \boxed{2\sqrt{10}}$

$\vec{AB} = \langle 6, 2 \rangle$

Direction:

direction that vector is pointing to (in standard form)

$\tan\theta = \frac{b}{a} \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

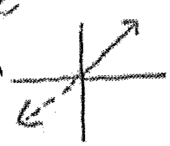
Ex: Find the direction of $\vec{e} = \langle 8, 5 \rangle$ Q1 $\quad \theta = \tan^{-1}\left(\frac{5}{8}\right) = \boxed{32^\circ}$

Ex: Find the direction of $\vec{f} = -6i - 7j$

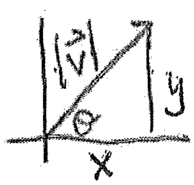
$\langle -6, -7 \rangle$ | $\theta = \tan^{-1}\left(\frac{-7}{-6}\right) = 49.39^\circ + 180 = \boxed{229.39^\circ}$

Q3

Think about it: If the direction of $\vec{x}$ is 50°, how would direction change for each of the following:

- $5\vec{x}$ no direction change
- $-3\vec{x}$ exact opposite direction  $50^\circ + 180 = 230^\circ$
- $\frac{1}{2}\vec{x}$ no direction change

Components from Magnitude and Direction:



$\cos\theta = \frac{x}{|\vec{v}|}$ | $\sin\theta = \frac{y}{|\vec{v}|}$

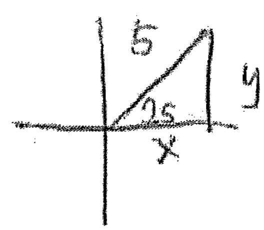
$x = |\vec{v}|\cos\theta$ | $y = |\vec{v}|\sin\theta$

Ex: Find the components of $\vec{g}$ if it has magnitude of 5 and direction of 25°.

$\boxed{\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle}$

$\langle 5\cos 25, 5\sin 25 \rangle$

$\langle 4.532, 2.113 \rangle$



6.06 Practice: ODDS #1–9, 39–51

Find the magnitude of $\overrightarrow{AB}$ with the given initial and terminal points (same 1-9 as yesterday).

1. $A(-3, 1), B(4, 5)$
2. $A(2, -7), B(-6, 9)$
3. $A(10, -2), B(3, -5)$
4. $A(-2, 7), B(-9, -1)$
5. $A(-5, -4), B(8, -2)$
6. $A(-2, 6), B(1, 10)$
7. $A(2.5, -3), B(-4, 1.5)$
8. $A(-4.3, 1.8), B(9.4, -6.2)$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$
10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

1) $\langle 7, 4 \rangle$ $|\overrightarrow{AB}| = \sqrt{7^2+4^2} = \boxed{\sqrt{65}}$

3) $\langle -7, -3 \rangle$ $|\overrightarrow{AB}| = \sqrt{(-7)^2+(-3)^2} = \boxed{\sqrt{58}}$

Find the component form of v with the given magnitude and direction angle. (Example 6)

38. $|v| = 12, \theta = 60°$
39. $|v| = 4, \theta = 135°$
40. $|v| = 6, \theta = 240°$
41. $|v| = 16, \theta = 330°$
42. $|v| = 28, \theta = 273°$
43. $|v| = 15, \theta = 125°$

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

39) $\langle 4\cos 135, 4\sin 135 \rangle = 4\left(\frac{-\sqrt{2}}{2}\right), 4\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\langle -2\sqrt{2}, 2\sqrt{2} \rangle}$

41) $\langle 16\cos 330, 16\sin 330 \rangle$

$16\left(\frac{\sqrt{3}}{2}\right), 16\left(\frac{-1}{2}\right)$

$\boxed{\langle 8\sqrt{3}, -8 \rangle}$

Find the direction angle of each vector to the nearest tenth of a degree. (Example 7)

44. $3\mathbf{i} + 6\mathbf{j}$
45. $-2\mathbf{i} + 5\mathbf{j}$
46. $8\mathbf{i} - 2\mathbf{j}$
47. $-4\mathbf{i} - 3\mathbf{j}$
48. $\langle -5, 9 \rangle$
49. $\langle 7, 7 \rangle$
50. $\langle -6, -4 \rangle$
51. $\langle 3, -8 \rangle$

45) $\langle -2, 5 \rangle$ Q2 $\theta = \tan^{-1}\left(\frac{5}{-2}\right) = -68.19° + 180 = \boxed{111.8°}$

47) $\langle -4, -3 \rangle$ Q3 $\theta = \tan^{-1}\left(\frac{-3}{-4}\right) = 36.87° + 180 = \boxed{216.9°}$

6.06 Practice: ODDS #1–9, 39–51

Find the magnitude of $\overline{AB}$ with the given initial and terminal points (same 1-9 as yesterday).

1. $A(-3, 1), B(4, 5)$ $\overrightarrow{AB} = \langle 7, 4 \rangle$ $|\overrightarrow{AB}| = \sqrt{49+16} = \boxed{\sqrt{65}}$
3. $A(10, -2), B(3, -5)$ $\overrightarrow{AB} = \langle -7, -3 \rangle$ $|\overrightarrow{AB}| = \sqrt{49+9} = \boxed{\sqrt{58}}$
5. $A(-5, -4), B(8, -2)$ $\overrightarrow{AB} = \langle 13, 2 \rangle$ $|\overrightarrow{AB}| = \sqrt{169+4} = \boxed{\sqrt{173}}$
7. $A(2.5, -3), B(-4, 1.5)$ $\overrightarrow{AB} = \langle -6.5, 4.5 \rangle$ $|\overrightarrow{AB}| = \sqrt{42.25+20.25} = \boxed{\sqrt{62.5}}$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$ $\overrightarrow{AB} = \langle 5.5, 11.5 \rangle$ $|\overrightarrow{AB}| = \sqrt{30.25+132.25} = \boxed{\sqrt{162.5}}$

Find the component form of v with the given magnitude and direction angle. (Example 6)

39. $|v| = 4, \theta = 135°$ $\langle 4\cos 135°, 4\sin 135° \rangle = \boxed{\langle -2\sqrt{2}, 2\sqrt{2} \rangle}$
41. $|v| = 16, \theta = 330°$ $\langle 16\cos 330°, 16\sin 330° \rangle = \boxed{\langle 8\sqrt{3}, -8 \rangle}$
43. $|v| = 15, \theta = 125°$ $\langle 15\cos 125°, 15\sin 125° \rangle = \boxed{\langle -8.604, 12.287 \rangle}$

Find the direction angle of each vector to the nearest tenth of a degree. (Example 7)

45. $-2i + 5j$ $\alpha = \tan^{-1}\left(\frac{5}{-2}\right) + 180 = \boxed{111.801°}$
47. $-4i - 3j$ $\alpha = \tan^{-1}\left(\frac{-3}{-4}\right) + 180 = \boxed{216.870°}$
49. $\langle 7, 7 \rangle$ $\alpha = \tan^{-1}\left(\frac{7}{7}\right) = \boxed{45°}$
51. $\langle 3, -8 \rangle$ $\alpha = \tan^{-1}\left(-\frac{8}{3}\right) + 360 = \boxed{290.556°}$

6.07 Angle Between Vectors Notes

Dot Product: if u = <a_1, b_1> and v = <a_2, b_2> then $\vec{u} \cdot \vec{v} =$ $a_1 \cdot a_2 + b_1 \cdot b_2$

Ex: Find the dot product between the following pairs of vectors.

1. u = <3, 6>, v = <-4, 2>

$\vec{u} \cdot \vec{v} = 3(-4) + 6(2) = 0$

2. a = 2i + 5j, b = 8i – 4j

$\vec{a} \cdot \vec{b} = 2(8) + 5(-4) = 16 - 20 = \boxed{-4}$

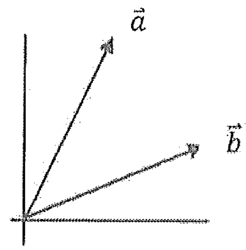
Angle Between 2 Vectors:

$\vec{a} =$ $\langle |\vec{a}| \cos$

$\vec{b} =$

$\vec{a} \cdot \vec{b} =$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$



$\cos\theta = \dfrac{\vec{c} \cdot \vec{d}}{|\vec{c}| \cdot |\vec{d}|} \rightarrow \dfrac{(-5)(4) + 8(-2)}{\sqrt{5^2 + 2^2} \cdot \sqrt{4^2 + 8^2}}$

$\cos\theta = \dfrac{-36}{\sqrt{29} \cdot \sqrt{80}} \rightarrow \cos\theta = \dfrac{-36}{\sqrt{2320}} = -0.7474$

~~$\theta = \cos^{-1}(-0.7474)$~~

$\boxed{\theta = 138.366°}$

Ex: Find the angle between vectors $\vec{c} = \langle -5, -2 \rangle$ and $\vec{d} = \langle 4, 8 \rangle$.

Orthogonal Vectors: Two vectors are orthogonal (perpendicular) if and only if their dot product is zero.

Ex: Create a non-zero vector $\vec{n}$ that is orthogonal to $\vec{m} = \langle -4, 2 \rangle$.

$-4 \cdot (\_?\_) + 2(\_?\_) = 0$ $\quad$ <2, 4> or <1, 2>, or <-3, -6>

(infinitely many possibilities)

Parallel Vectors: Vectors are parallel if the dot product is equal to the product of their magnitudes ($\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}|$)

Ex: Determine the angle between the following pairs of vectors.

$< -2, 5 > and < -6, 15 >$

$\cos\theta = \dfrac{-2(-6) + 5(15)}{\sqrt{4+25} \cdot \sqrt{36+225}}$

$< 8, 12 > and < 6, 9 >$

$\cos\theta = \dfrac{8(6) + 12(9)}{\sqrt{64+144} \cdot \sqrt{36+81}}$

$< 10, 11 > and < -5, -5.5 >$

$\cos\theta = \dfrac{10(-5) + 11(-5.5)}{\sqrt{10^2+11^2} \cdot \sqrt{25+5.5^2}}$

6.07 Practice: #1-9, 19-24

Determine if the following vectors are parallel, orthogonal, or neither.

1. $u = < 3, -5 >, v = < 6, 2 >$

2. $u = < -10, -16 >, v = < -8, 5 >$

3. $u = < 9, -3 >, v = < 1, 3 >$

4. $u = < 4, -4 >, v = < 7, 5 >$

5. $u = < 1, -4 >, v = < 2, -8 >$

6. $u = 11i + 7j;\ v = -7i + 11j$

7. $u = < -4, 6 >, v = < -5, -2 >$

8. $u = 8i + 6j; v = -i + 2j$

9. **SPORTING GOODS** The vector $\mathbf{u} = \langle 406, 297 \rangle$ gives the numbers of men's basketballs and women's basketballs, respectively, in stock at a sporting goods store. The vector $\mathbf{v} = \langle 27.5, 15 \rangle$ gives the prices in dollars of the two types of basketballs, respectively. (Example 1)

   a. Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

   b. Interpret the result in the context of the problem.

1) $u \cdot v = 3(6) + 2(-5) = 18 - 10 = \boxed{8}$ (not orthogonal)

3) $u \cdot v = 9(1) + 3(-3) = \boxed{0}$ (orthogonal)

5) $u \cdot v = 1(2) + 4(8) = \boxed{34}$ (not orthogonal)

7) $u \cdot v = (-4)(-5) + 6(-2) = 20 - 12 = \boxed{8}$ (not orthogonal)

9) $u \cdot v = 406(27.5) + 297(15) = \$15,620$ (total cost of all basketballs in stock)

Find the angle θ between u and v to the nearest tenth of a degree. (Example 3)

16. $\mathbf{u} = \langle 0, -5\rangle$, $\mathbf{v} = \langle 1, -4\rangle$
17. $\mathbf{u} = \langle 7, 10\rangle$, $\mathbf{v} = \langle 4, -4\rangle$
18. $\mathbf{u} = \langle -2, 4\rangle$, $\mathbf{v} = \langle 2, -10\rangle$
19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$
20. $\mathbf{u} = \langle -9, 0\rangle$, $\mathbf{v} = \langle -1, -1\rangle$
21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$
22. $\mathbf{u} = \langle 6, 0\rangle$, $\mathbf{v} = \langle -10, 8\rangle$
23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$
24. **CAMPING** Regina and Luis set off from their campsite to search for firewood. The path that Regina takes can be represented by $\mathbf{u} = \langle 3, -5\rangle$. The path that Luis takes can be represented by $\mathbf{v} = \langle -7, 6\rangle$. Find the angle between the pair of vectors. (Example 3)

24) $u = \langle 3, -5 \rangle \quad v = \langle -7, 6 \rangle$

$$\cos\theta = \frac{3(-7) + 6(-5)}{\sqrt{9+25} \cdot \sqrt{49+36}} = \frac{-51}{\sqrt{34} \cdot \sqrt{85}}$$

$$\cos\theta = \frac{-3}{\sqrt{10}} \quad \theta = \cos^{-1}\left(\frac{-3}{\sqrt{10}}\right)$$

$$\boxed{\theta = 161.565^\circ}$$

20) $u = \langle -9, 0 \rangle \quad v = \langle -1, -1 \rangle$

$$\cos\theta = \frac{-9(-1) + 0(-1)}{\sqrt{81+0} \cdot \sqrt{1+1}} = \frac{+9}{9\sqrt{2}}$$

$$\cos\theta = \frac{+1}{\sqrt{2}}$$

$$\boxed{\theta = 45^\circ}$$

21) $u = \langle -1, -3 \rangle \quad v = \langle -7, -3 \rangle$

$$\cos\theta = \frac{(1)(7) + (-3)(-3)}{\sqrt{1^2+3^2} \cdot \sqrt{7^2+3^2}} = \frac{16}{\sqrt{10} \cdot \sqrt{58}}$$

$$\cos\theta = \frac{16}{\sqrt{580}} \quad \theta = \cos^{-1}\left(\frac{16}{\sqrt{580}}\right)$$

$$\boxed{\theta = 48.366^\circ}$$

22) $u = \langle 6, 0 \rangle \quad v = \langle -10, 8 \rangle$

$$\cos\theta = \frac{6(-10) + 0(8)}{\sqrt{36+0} \cdot \sqrt{100+64}}$$

$$\cos\theta = \frac{-60}{(6)(\sqrt{164})}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{41}}\right)$$

$$\boxed{\theta = 141.340^\circ}$$

23) $u = \langle -10, 1 \rangle \quad v = \langle 10, -5 \rangle$

$$\cos\theta = \frac{-10(10) + 1(-5)}{\sqrt{100+1} \cdot \sqrt{100+25}}$$

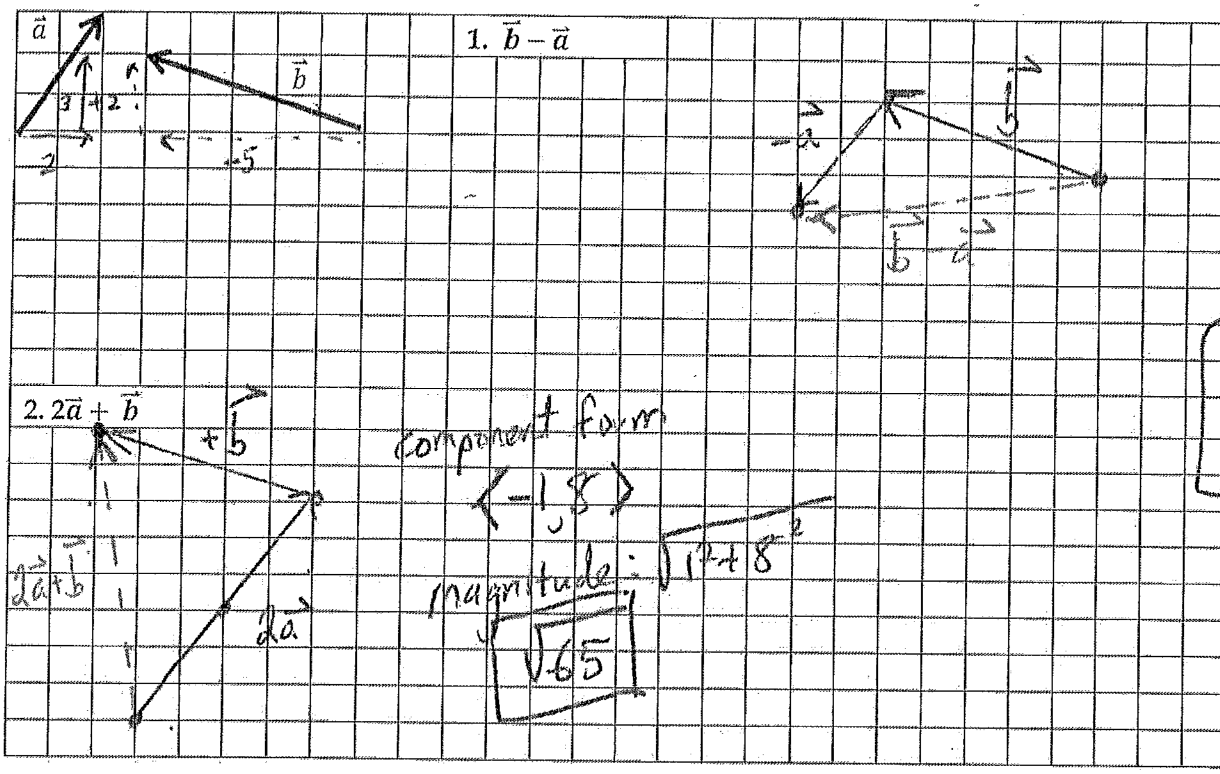
$$\cos\theta = \frac{-105}{\sqrt{101} \cdot \sqrt{125}}$$

$$\theta = \cos^{-1}\left(\frac{-21}{\sqrt{505}}\right)$$

$$\boxed{\theta = 159.146^\circ}$$

6.08 2D Vector Quiz Review

Draw each and label the resultant. Then find the component form and magnitude of each resultant.



component form

$\langle -7, -1 \rangle$

magnitude

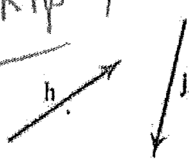
$\sqrt{7^2+1^2}$

$\boxed{= \sqrt{50} = 5\sqrt{2}}$

Draw the vector diagram and the resultant vector. Use either the triangle or parallelogram method.

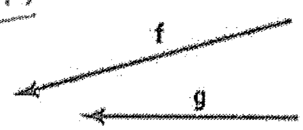
Skip

3. $\vec{h} + \vec{j}$



Skip

4. $\vec{f} - 2\vec{g}$



$\langle x_2 - x_1, y_2 - y_1 \rangle$

Use each set of points to create a vector with initial point & terminal point in *alphabetical order*. For each, find the component form, write as a sum of unit vectors, and find the magnitude and direction.

5. A (−3, 5) and B (5, −1)

$\langle 5 - -3, -1 - 5 \rangle$

Q4 $\langle 8, -6 \rangle \rightarrow 8i - 6j$

$|\vec{AB}| = \sqrt{8^2+6^2} = \sqrt{100} = \boxed{10}$

$\theta = \tan^{-1}\left(\frac{-6}{8}\right) = -36.87 + 360°$

$\boxed{\theta = 323.13°}$

6. C(10, −6) and D(−8, 2)

$\langle -8 - 10, 2 - -6 \rangle$

$\langle -18, 8 \rangle \rightarrow -18i + 8j$

$|\vec{CD}| = \sqrt{18^2+8^2} = \boxed{2\sqrt{97}}$

$\theta = \tan^{-1}\left(\frac{8}{-18}\right) = -23.96$

Q2 $\rightarrow -23.96 + 180$

$\theta = \boxed{156.04°}$

7. S(1, 12) and T(−2, −9)

$\langle -2 - 1, -9 - 12 \rangle$

$\langle -3, -21 \rangle \rightarrow -3i - 21j$

$|\vec{ST}| = \sqrt{3^2+21^2} = \boxed{15\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{-21}{-3}\right) = 81.87°$

Q3 $\rightarrow 81.87 + 180$

$\boxed{\theta = 261.87°}$

$\vec{v} = \langle 1, -2 \rangle$

Given: $\vec{u} = < 5, -12 >$, $\vec{v} = i - 2j$ and $\vec{w} = < 9, 3 >$

8. a) Find: $-\vec{u} - \frac{1}{3}\vec{w}$

$\langle -5, 12 \rangle - \langle 3, 1 \rangle = \boxed{\langle -8, 11 \rangle}$

b) $2\vec{v} - \vec{u}$

$\langle 2, -4 \rangle - \langle 5, -12 \rangle$

$\boxed{\langle -3, 8 \rangle}$

c) Are any of the vectors orthogonal? Show work.

$u \cdot v = 5(1) + -2(-12) = 5 + 24 = 29 \neq 0$, not orthogonal

$u \cdot w = 5(9) + 3(-12) = 45 - 36 = 9 \neq 0$, not orthogonal

$\vec{v} \cdot \vec{w} = 1(9) + 3(-2) = 9 - 6 = 3 \neq 0$, not orthogonal

d) If the vectors are not orthogonal, use $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$ to find the angle between each pair of vectors.

$\cos\theta = \dfrac{5(1) + -2(-12)}{\sqrt{5^2 + 12^2} \cdot \sqrt{1^2 + 2^2}}$

$\cos\theta = \dfrac{29}{13 \cdot \sqrt{5}}$

$\cos\theta = 0.9976$

$\theta = \cos^{-1}(0.9976)$

$\boxed{\theta = 3.945^\circ}$

Find the component form of $\vec{v}$ given the following. $\langle |\vec{v}| \cos\theta, |\vec{v}| \sin\theta \rangle$

9. $|\vec{v}| = 4, \theta = 135°$

$\langle 4\cos 135, 4\sin 135 \rangle$

$\langle -2\sqrt{2}, 2\sqrt{2} \rangle$

10. $|\vec{v}| = 6, \theta = 240°$

$\langle 6\cos 240, 6\sin 240 \rangle$

$\langle -3, -3\sqrt{3} \rangle$

11. $|\vec{v}| = 15, \theta = 330°$

$\langle 15\cos 330, 15\sin 330 \rangle$

$\langle \frac{15\sqrt{3}}{2}, -\frac{15}{2} \rangle$

12. A boat is traveling west at 25 mph. The current is moving south at 3 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

$\vec{b} = \langle -25, 0 \rangle$

$\vec{c} = \langle 0, -3 \rangle$

$\vec{r} = \vec{b} + \vec{c}$

$\vec{r} = \langle -25, -3 \rangle$

Speed (magnitude) $\rightarrow |\vec{r}| = \sqrt{25^2 + 3^2} = 25.18$ mph

$\tan\theta = \frac{b}{a}$

$\theta = \tan^{-1}\left(\frac{-3}{-25}\right)$

$\theta = 6.843° + 180$

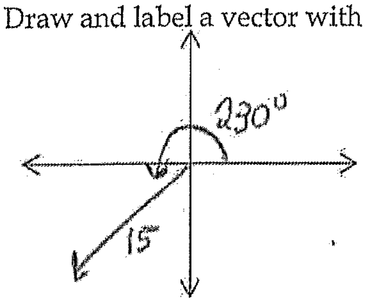
$\boxed{\theta = 186.84°}$

13. Alvin pulls a sled through the snow with a force of 50 newtons at an angle of 35° with the horizontal. Find the ~~magnitude of the force.~~ horizontal and vertical components

$\langle 50\cos 35, 50\sin 35 \rangle$

$\boxed{\langle 40.958, 28.679 \rangle}$

14. Draw and label a vector with magnitude of 15 meters per second at a direction of 230°. (horizontal and vertical components)



$\langle 15\cos 230, 15\sin 230 \rangle$

$\boxed{\langle -9.642, -11.491 \rangle}$

15) Kaya is swimming due west at a rate of 1.5 meters/sec. A strong current flowing due north at rate of 1 meter per second. Find Kaya's resulting speed and direction.

$\vec{k} = \langle -1.5, 0 \rangle$ | resultant vector $\vec{r} = \vec{k} + \vec{c}$

$\vec{c} = \langle 0, 1 \rangle$ | $\vec{r} = \langle -1.5, 1 \rangle$

speed (magnitude) $= \sqrt{1.5^2 + 1^2} = 1.803$ meters/sec.

direction: $\theta = \tan^{-1}\left(\frac{1}{-1.5}\right) = -33.69°$

Q2 $\rightarrow -33.69 + 180 = \boxed{146.31°}$

16) vector. 1 is 15 meters per second squared at 60° angle to the horizontal and vector 2 is 9.8 meters per second squared downward. Determine the magnitude and direction of resultant of vector sum.

$\vec{v1} = \langle 15\cos 60, 15\sin 60 \rangle$

$\vec{v1} = \langle \frac{15}{2}, \frac{15\sqrt{3}}{2} \rangle$

$\vec{v2} = \langle 0, -9.8 \rangle$

resultant vector $\vec{r} = \langle \frac{15}{2}, 3.19 \rangle$ or $\langle 7.5, 3.19 \rangle$

$|\vec{r}| = \sqrt{7.5^2 + 3.19^2} = \boxed{8.15\ m/s^2}$

direction: $\theta = \tan^{-1}\left(\frac{3.19}{7.5}\right) = \boxed{23.04°}$

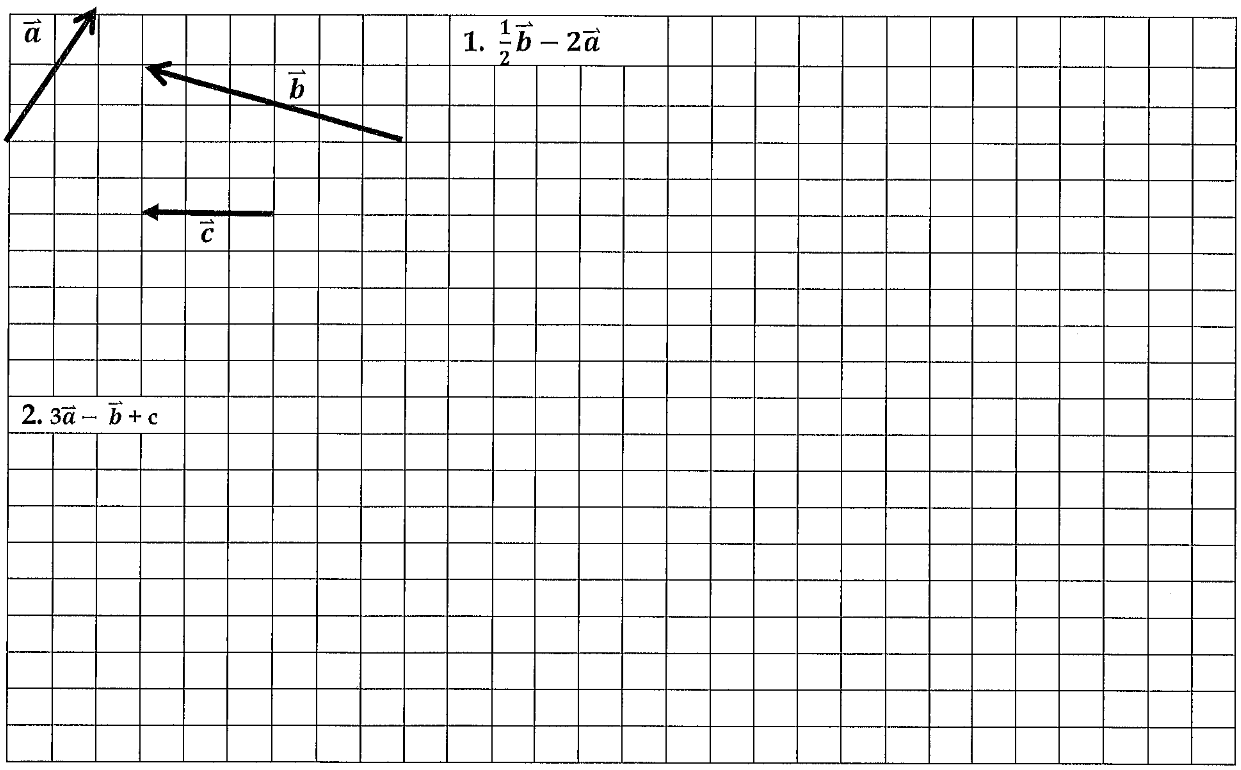
Q1

Accelerated Precalculus

6.08 Vector Quiz Review WS 2

The angle between 2 vectors: $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|}$

Draw each and label the resultant. Then find the component form and magnitude of each resultant.



3. Given points A(-3, 6) and B(5, -4) with the initial& terminal point in alphabetical order, find the resulting vector $\vec{v}$ in component form. Find the magnitude and direction of vector.

4. The terminal point of ***vector*** $\vec{k}$ is B(-3, 4). If k = <1, -6>, find the initial point A.

5. Determine the measure of the angle made between ***a*** = <-1, -3> and ***b*** = <5, -6>.

6. Find $\boldsymbol{u} \cdot \boldsymbol{v}$ if $|\boldsymbol{u}| = 3$, $|\boldsymbol{v}| = 5$, and the angle between the vectors is $\theta = \frac{\pi}{6}$

Given: $\vec{u} = < 4, -8 >$, $\vec{v} = 2i - 3j$ and $\vec{w} = < 16,4 >$

7. a) Find: $-\overrightarrow{2u} - \frac{1}{4}\vec{w}$

b) $3\vec{v} - \frac{1}{2}\vec{u}$

Find the component form of $\vec{v}$ given the following.

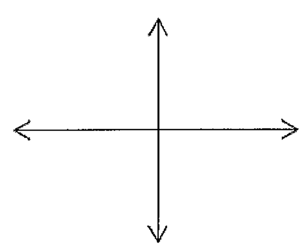
8. $|\vec{v}| = 2, \theta = 120°$

9. $|\vec{v}| = 2, \theta = 60°$

10. A boat is traveling west at 35 mph. The current is moving 60 degrees at 2 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

11. Draw and label a vector with magnitude of 12 meters per second

At the direction of 330 degrees. Represent vector in component form.

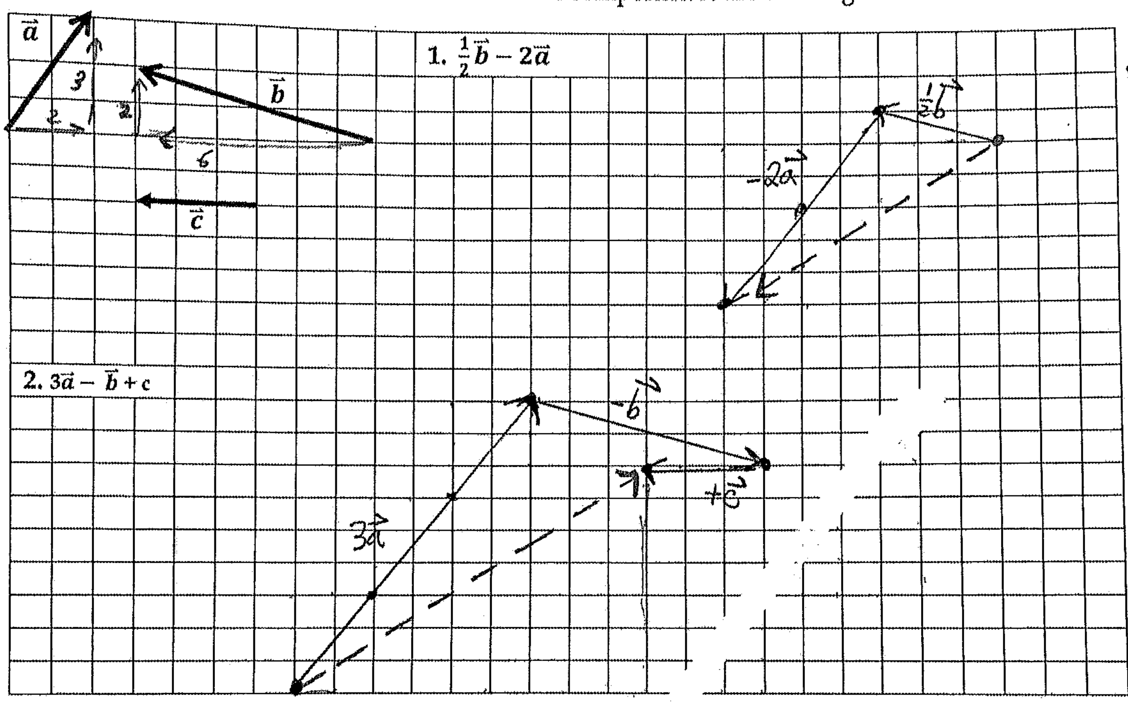


Accelerated Precalculus
6.08 Vector Quiz Review WS 2

Key

The angle between 2 vectors: $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|}$

Draw each and label the resultant. Then find the component form and magnitude of each resultant.



$\langle -7, -5 \rangle$
magnitude
$\sqrt{7^2+5^2}$
$\boxed{\sqrt{74}}$

2) $\langle 9, 7 \rangle$
magnitude
$\sqrt{9^2+7^2}$
$\boxed{=\sqrt{130}}$

3. Given points A(-3, 6) and B(5, -4) with the initial& terminal point in alphabetical order, find the resulting vector $\vec{v}$ in component form. Find the magnitude and direction of vector.

$\vec{V} = \langle 5--3, -4-6 \rangle$

$\vec{V} = \langle 8, -10 \rangle \rightarrow 8i - 10j$

$|\vec{V}| = \sqrt{8^2+10^2} = \sqrt{164} = \boxed{2\sqrt{41}}$

$\theta = \tan^{-1}\left(\frac{-10}{8}\right) = -51.34°$

Q4 → $360 - 51.34°$

$\boxed{\theta = 308.66°}$

4. The terminal point of *vector* $\vec{k}$ is B(-3, 4). If $k = <1, -6>$, find the initial point A.

$A(x_1, y_1)$ $B(-3, 4)$

$k = \langle -3-x_1, 4-y_1 \rangle$

$\langle 1, -6 \rangle = \langle -3-x_1, 4-y_1 \rangle$

$-3 - x_1 = 1$

$-4 = x_1$

$4 - y_1 = -6$

$10 = y_1$

$\boxed{\text{point } A \text{ is } (-4, 10)}$

5. Determine the measure of the angle made between $\boldsymbol{a} = \langle -1, -3 \rangle$ and $\boldsymbol{b} = \langle 5, -6 \rangle$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{-1(5)+-3(-6)}{\sqrt{1^2+3^2}\cdot\sqrt{5^2+6^2}} \rightarrow \cos\theta = \frac{13}{\sqrt{10}\cdot\sqrt{61}}$$

$$\theta = \cos^{-1}\left(\frac{13}{\sqrt{610}}\right) \quad \boxed{\theta = 58.241^\circ}$$

6. Find $\boldsymbol{u}\cdot\boldsymbol{v}$ if $|\boldsymbol{u}| = 3$, $|\boldsymbol{v}| = 5$, and the angle between the vectors is $\theta = \frac{\pi}{6}$

$$\frac{\cos\theta}{1} = \frac{u\cdot v}{|u||v|} \quad \frac{\cos(\pi/6)}{1} = \frac{u\cdot v}{3\cdot 5} \quad \frac{\sqrt{3}}{2} = \frac{u\cdot v}{15} \quad 15\sqrt{3} = 2(u\cdot v) \quad \boxed{\frac{15\sqrt{3}}{2} = u\cdot v}$$

Given: $\vec{u} = \langle 4, -8 \rangle$, $\vec{v} = 2i - 3j$ and $\vec{w} = \langle 16, 4 \rangle$

$\vec{v} = \langle 2, -3 \rangle$

7. a) Find: $-2\vec{u} - \frac{1}{4}\vec{w}$

$$-2\langle 4,-8\rangle - \frac{1}{4}\langle 16,4\rangle$$
$$\langle -8, 16\rangle - \langle 4, 1\rangle$$
$$= \boxed{\langle -12, 15\rangle}$$

b) Are vectors $\vec{u}$ *and* $\vec{v}$ *orthogonal*? Justify with reason.

$$u\cdot v = 4(2) + -8(-3) = 8 + 24 = 32 \neq 0$$

Since $u\cdot v \neq 0$, vectors u and v are not orthogonal to each other.

Find the component form of $\vec{v}$ given the following.

8. $|\vec{v}| = 2, \theta = 120°$

$$\langle 2\cos 120, 2\sin 120\rangle$$
$$\boxed{\langle -1, \sqrt{3}\rangle}$$

9. $|\vec{v}| = 2, \theta = 60°$

$$\langle 2\cos 60, 2\sin 60\rangle$$
$$\boxed{\langle 1, \sqrt{3}\rangle}$$

10. A boat is traveling west at 35 mph. The current is moving 60 degrees at 2 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

$$\vec{b} = \langle -35, 0\rangle$$
$$\vec{c} = \langle 2\cos 60, 2\sin 60\rangle$$
$$\vec{c} = \langle 1, \sqrt{3}\rangle$$

resultant $\vec{r} = \langle -34, \sqrt{3}\rangle$

speed $|\vec{r}| = \sqrt{34^2 + \sqrt{3}^2} = \boxed{34.04 \text{ mph}}$

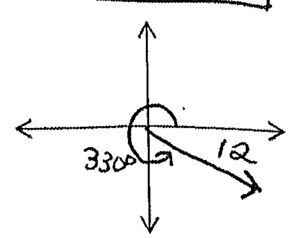
$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-34}\right) = -2.916^\circ + 180 = \boxed{177.08^\circ}$$

Q2

11. Draw and label a vector with magnitude of 12 meters per second

At the direction of 330 degrees. Represent vector in component form.

$$\langle 12\cos 330, 12\sin 330\rangle = \boxed{\langle 6\sqrt{3}, -6\rangle}$$

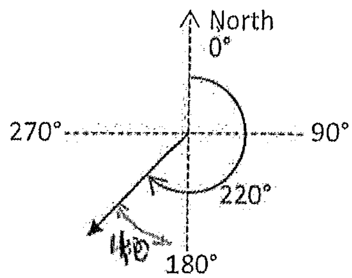


5th

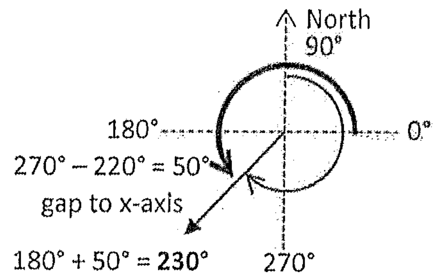
6.10 Bearing and Directional Bearing

<u>Bearing</u> directions are angles given as rotations measured clockwise from North.

Example: bearing of 220°



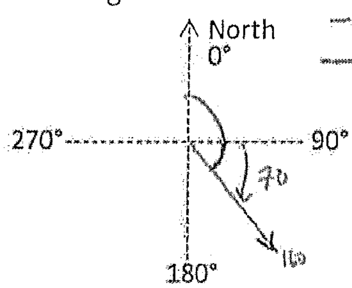
To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)



Directions: Sketch the angle given in bearing. Then, determine the angle measured in standard position that is coterminal with the given angle.

* standard position = 450° - bearing

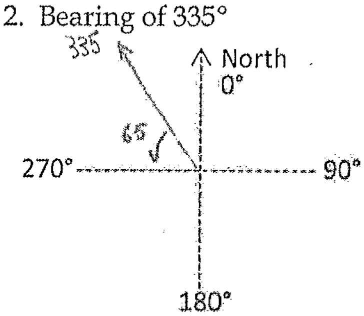
1. bearing of 160°



450
-160
290°

360-70 =
Standard Position: 290°

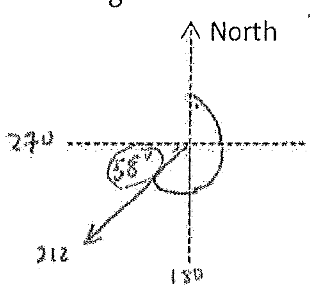
2. Bearing of 335°



450
-335
115°

180-65=
Standard Position: 115°

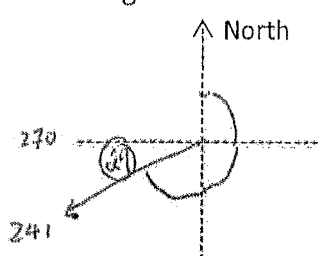
3. bearing of 212°



450
-212
238°

180+58=
Standard Position: 238°

4. Bearing of 241°

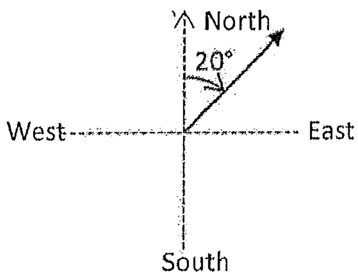


450
-241
209°

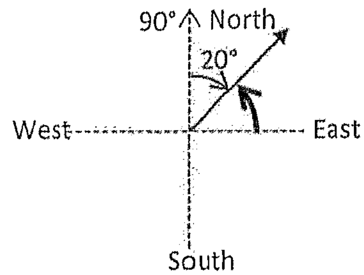
180+29
Standard Position: 209°

Directional bearing (also called Quadrant Bearing) directions are angles given as a starting direction (either North or South) and then an acute angle of rotation measured toward the second direction (either East or West).

Example: bearing of N 20° E

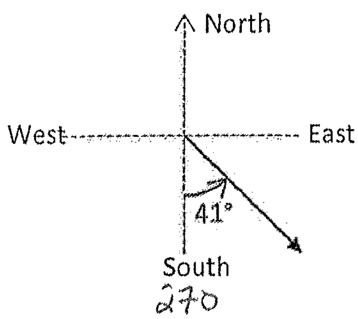


To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)

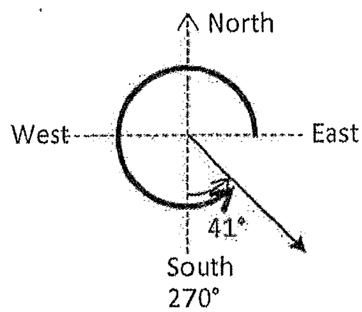


Direction occurs 20° *before* 90° quadrantal
90° – 20° = **70°**

Example: bearing of S 41° E



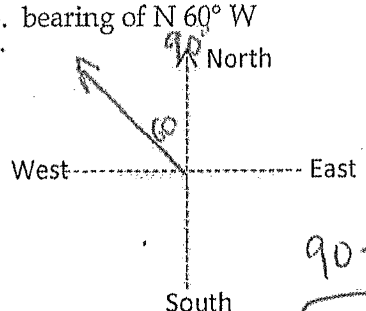
To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)



Direction occurs 41° *after* 270° quadrantal
270° + 41° = **311°**

Directions: Sketch the angle given in bearing. Then, determine the angle measured in standard position that is coterminal with the given angle.

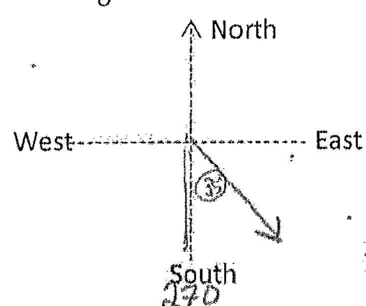
5. bearing of N 60° W



90+60° =

Standard Position: 150°

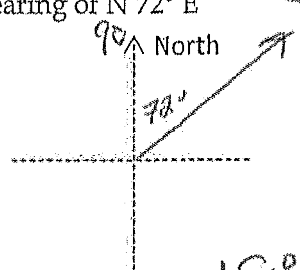
6. bearing of S 35° E



270+35 =

Standard Position: 305°

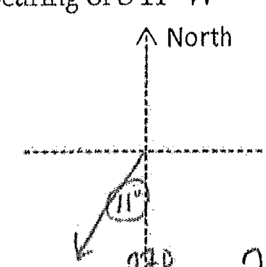
7. bearing of N 72° E



Standard Position: 18°

90-72 =

8. bearing of S 11° W



Standard Position: 259°

270-11 =

6.11 Applications of Vectors: Notes Date: ____________

Formulas: $|\vec{v}| = \sqrt{a^2+b^2}$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ $\cos\theta = \frac{u \cdot v}{|u|\cdot|v|}$

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

Ex 1: Train A and Train B depart from the same station. The path that train A takes can be represented by $\langle 33, 12\rangle$. If the path that train B takes can be represented by $\langle 55, 4\rangle$, find the angle between the pair of vectors.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$\cos\theta = \frac{33(55)+12(4)}{\sqrt{33^2+12^2}\cdot\sqrt{55^2+4^2}}$$

$$\cos\theta = \frac{1863}{\sqrt{1233}\cdot\sqrt{3041}}$$

$$\theta = \cos^{-1}\left(\frac{1863}{\sqrt{1233}\cdot\sqrt{3041}}\right)$$

$$\boxed{\theta = 15.823^\circ}$$

Ex 2: An airplane is flying at a direction of 115° at 530 mph. Find the component form of the velocity of the airplane.

$\theta = 115^\circ$

$|\vec{v}| = 530$

$\langle 530\cos 115, 530\sin 115\rangle$

$\langle -223.988, 480.343\rangle$

Ex 3: A captain sails a boat for 200 kilometers at a bearing of 150°. Find the component form of the velocity of the boat.

$450 - 150^\circ$

$\theta = 300^\circ$

$\langle 200\cos 300, 200\sin 300\rangle$

$\langle 100, -173.205\rangle$

Ex 4: Jordan is riding the bus to school. The bus travels north for 4.5 miles, east for 2 miles, and then N60°E for 1.5 miles. Find the component form of the resultant.

$\vec{v}_1 \langle 0, 4.5\rangle$

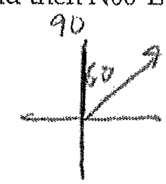
$\vec{v}_2 \langle 2, 0\rangle$

$v_3 \langle 1.299, 0.75\rangle$

$\vec{r} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$

$\boxed{\vec{r} = \langle 3.299, 5.25\rangle}$

$90 - 60 = 30^\circ$



$\vec{V_3} \langle 1.5\cos 30, 1.5\sin 30\rangle$

$\langle 1.299, 0.75\rangle$

Ex 5: An airplane is flying with an airspeed of 500 miles per hour on a heading due north. If a 50-mile per hour wind is blowing at a bearing of 280°, determine the velocity and direction of the plane relative to the ground.

$\theta = 450 - 280 = 170^\circ \rightarrow \vec{w} = \langle 50\cos 170, 50\sin 170\rangle$

$\vec{w} = \langle -49.240, 8.682\rangle$

$\vec{a} = \langle 0, 500\rangle$

$\vec{w} = \langle -49.240, 8.682\rangle$

$\vec{r} = \vec{a} + \vec{w}$

$\vec{r} = \langle -49.240, 508.682\rangle$

velocity (magnitude) $\rightarrow |\vec{r}| =$

$= \sqrt{49.240^2 + 508.682^2}$

$\boxed{|\vec{r}| = 511.060 \text{ mph}}$

Direction: $\theta = \tan^{-1}\left(\frac{508.682}{-49.240}\right)$

$\theta = -84.529 + 180$

Q2 $\rightarrow$ $\boxed{\theta = 95.529^\circ}$

6.11 Applications of Vectors: Notes Date: ___________

Formulas: $|\vec{v}| = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$ $\quad \cos\theta = \frac{u \cdot v}{|u| \cdot |v|}$

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

Ex 1: Train A and Train B depart from the same station. The path that train A takes can be represented by $\langle 33, 12\rangle$. If the path that train B takes can be represented by $\langle 55, 4\rangle$, find the angle between the pair of vectors.

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ $\quad \cos\theta = \frac{33(55)+12(4)}{\sqrt{33^2+12^2} \cdot \sqrt{55^2+4^2}}$ $\quad \cos\theta = \frac{1863}{\sqrt{1233} \cdot \sqrt{3041}}$

$\theta = \cos^{-1}\left(\frac{1863}{\sqrt{1233} \cdot \sqrt{3041}}\right)$ $\quad \boxed{\theta = 15.823^\circ}$

Ex 2: An airplane is flying at a direction of 115° at 530 mph. Find the component form of the velocity of the airplane.

$\theta = 115^\circ$ $\quad \langle 530\cos 115, 530\sin 115 \rangle$

$|\vec{v}| = 530$ $\quad \langle -223.988, 480.343 \rangle$

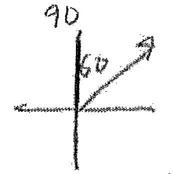
Ex 3: A captain sails a boat for 200 kilometers at a bearing of 150°. Find the component form of the velocity of the boat.

$450 - 150^\circ$ $\quad \langle 200\cos 300, 200\sin 300 \rangle$

$\theta = 300^\circ$ $\quad \langle 100, -173.205 \rangle$

Ex 4: Jordan is riding the bus to school. The bus travels north for 4.5 miles, east for 2 miles, and then N60°E for 1.5 miles. Find the component form of the resultant.

$90 \quad 90 - 60 = 30^\circ$



$\vec{v1}\ \langle 0, 4.5 \rangle$ $\quad \vec{r} = \vec{v1} + \vec{v2} + \vec{v3}$

$\vec{v_2}\ \langle 2, 0 \rangle$ $\quad \boxed{\vec{r} = \langle 3.299, 5.25 \rangle}$

$v_3\ \langle 1.299, 0.75 \rangle$ $\quad \vec{v_3}\ \langle 1.5\cos 30, 1.5\sin 30 \rangle$ $\quad \langle 1.299, 0.75 \rangle$

Ex 5: An airplane is flying with an airspeed of 500 miles per hour on a heading due north. If a 50-mile per hour wind is blowing at a bearing of 280°, determine the velocity and direction of the plane relative to the ground.

$\theta = 450 - 280 = 170^\circ \rightarrow \vec{w} = \langle 50\cos 170, 50\sin 170 \rangle$

$\vec{a} = \langle 0, 500 \rangle$ $\quad \vec{w} = \langle -49.240, 8.682 \rangle$

$\vec{w} = \langle -49.240, 8.682 \rangle \leftarrow$

$\vec{r} = \vec{a} + \vec{w}$

$\vec{r} = \langle -49.240, 508.682 \rangle$

velocity (magnitude) $\rightarrow |\vec{r}| =$

$= \sqrt{49.240^2 + 508.682^2}$

$\boxed{|\vec{r}| = 511.060 \text{ mph}}$

Direction: $\theta = \tan^{-1}\left(\frac{508.682}{-49.240}\right)$

$\theta = -84.529 + 180$

Q2 $\rightarrow$ $\boxed{\theta = 95.529^\circ}$

$* \cos\theta = \frac{\vec{p}\cdot\vec{c}}{|\vec{p}|\cdot|\vec{c}|}$

6.11 Applications of Vectors Practice Day 1

1. Two clay pigeons are thrown at the same time. If the path of the clay pigeons can be represented by the vectors $\mathbf{p} = \langle 42, 58\rangle$ and $\mathbf{c} = \langle 59, 73\rangle$, what is the measure of the angle between the clay pigeons?

$\cos\theta = \frac{42(59)+58(73)}{\sqrt{42^2+58^2}\cdot\sqrt{59^2+73^2}}$ | $\cos\theta = \frac{6712}{\sqrt{5128}\cdot\sqrt{8810}}$ | $\theta = \cos^{-1}\left(\frac{6712}{\sqrt{45177680}}\right)$

$\boxed{\theta = 3.036^\circ}$

2. A hiker is walking a trail at 2.5 miles per hour at a bearing of N50°W. Find the component form of the velocity of the hiker.

90, 50

$\theta = 90+50$

$\theta = 140^\circ$

$\langle 2.5\cos 140, 2.5\sin 140\rangle$

$\boxed{\langle -1.915, 1.607\rangle}$

3. An airplane is traveling 300 kilometers per hour due east. A wind is blowing 35 kilometers per hour at an angle of 255°. A) What is the resulting speed of the airplane? B) What is the direction of the plane?

$* \vec{a} + \vec{w} = \vec{r}$

$\vec{w} = \langle 35\cos 255, 35\sin 255\rangle$

$\vec{a} = \langle 300, 0\rangle$

$+\ \vec{w} = \langle -9.059, -33.807\rangle$

$\vec{r} = \langle 290.941, -33.807\rangle$

A) $|\vec{r}| = \sqrt{290.941^2 + 33.807^2}$

$|\vec{r}| = 292.899$ Kph

B) $\theta = \tan^{-1}\left(\frac{-33.807}{290.941}\right)$

Q4

$\theta = -6.628 + 360$

$\boxed{\theta = 353.372^\circ}$

4. A helicopter is moving at a bearing of 105° with a velocity of 52 km/h. If a 30-kilometer per hour wind is blowing at S25°E, find the helicopter's resulting velocity and direction.

$* \vec{h} + \vec{w} = \vec{r}$

helicopter $\theta = 450 - 105 = 345^\circ$

25, 270

wind $\theta = 270 + 25$

$\theta = 295^\circ$

$\vec{h} = \langle 52\cos 345, 52\sin 345\rangle \rightarrow \vec{h} = \langle 50.228, -13.459\rangle$

$\vec{w} = \langle 30\cos 295, 30\sin 295\rangle \rightarrow +\ \vec{w} = \langle 12.679, -27.189\rangle$

$\vec{r} = \langle 62.907, -40.648\rangle$

velocity $|\vec{r}| = \sqrt{62.907^2 + 40.648^2}$

$|\vec{r}| = 74.967$ Km/hr

$\theta = \tan^{-1}\left(\frac{-40.648}{62.907}\right)$

$\theta = -32.869 + 360$

$\boxed{\theta = 327.131^\circ}$

5. Meredith is skateboarding along a path at a bearing of 70° for 35 meters. She then changes paths and travels for 45 meters along path at a bearing of 60°. A) Find the resulting distance, and B) the direction (bearing) of her path.

$\theta_1 = 20^\circ \quad \theta_2 = 30^\circ$

$* \vec{p1} + \vec{p2} = \vec{r}$

$\vec{p1} = \langle 35\cos 20, 35\sin 20\rangle \rightarrow \langle 32.889, 11.971\rangle$

$+\ \vec{p2} = \langle 45\cos 30, 45\sin 30\rangle \rightarrow \langle 38.971, 22.5\rangle$

$\vec{r} = \langle 71.860, 34.471\rangle$

A) magnitude

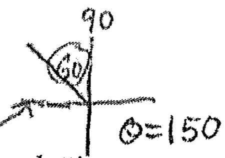
$|\vec{r}| = \sqrt{71.860^2 + 34.471^2} = \boxed{79.700\text{m}}$

B) $\theta = \tan^{-1}\left(\frac{34.471}{71.860}\right)$

$\theta = 25.627$

bearing $= 90 - 25.627$

$= \boxed{64.373^\circ}$



6.12 Apps of Vectors Day 2: Notes

Date: ______________

Ex 1: To reach a destination, a pilot is plotting a course that will result in a velocity of 450 miles per hour at an angle of N60°W. The wind is blowing 50 miles per hour to the north. Find the direction and speed the pilot should set to achieve the desired resultant.

$\vec{p} + \vec{w} = \vec{r}$

$\vec{p} = \vec{r} - \vec{w}$

$\vec{r} = \langle 450\cos 150, 450\sin 150 \rangle$

$-\vec{w} = \langle 0, 50 \rangle$

$\vec{p} = \langle -389.711, 175 \rangle$

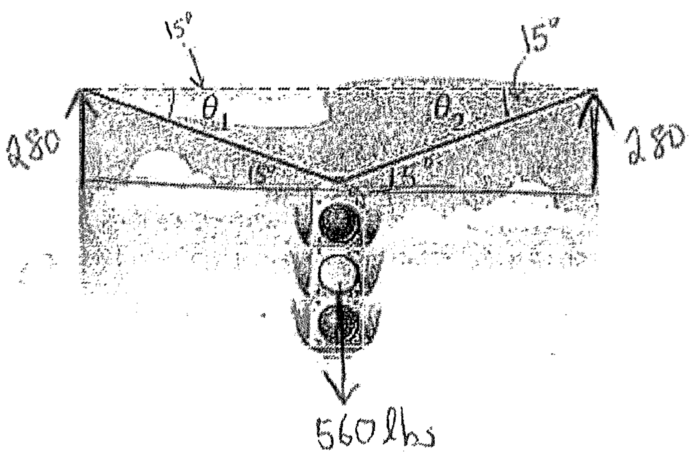
speed $|\vec{p}| = \sqrt{389.711^2 + 175^2}$

$= \boxed{427.200\text{ mph}}$

$\theta = \tan^{-1}\left(\frac{175}{-389.711}\right)$

Q2 $-24.182 + 180 = \boxed{155.818°}$

Ex 2: A traffic light at an intersection is hanging from two wires of equal length at 15° below the horizontal as shown. If the traffic light weighs 560 pounds, what is the tension in each wire keeping the light at equilibrium?



*use $\sin\theta = \frac{opp}{hyp}$

$\sin 15 = \frac{280}{|\vec{t}|}$

$\frac{\sin 15}{1} = \frac{280}{|\vec{t}|}$

$|\vec{t}|\sin 15 = 280$

$|\vec{t}| = \frac{280}{\sin 15}$

$\boxed{|\vec{t}| = 1081.837\text{ lbs}}$

6.12 Applications of Vectors Practice Day 2

1. Anne and Mike are lifting a stone statue and moving it to a new location in their garden. Anne is pushing the statue with a force of 120 newtons at a 60° angle while ~~Henry~~ Mike is pulling the statue with a force of 180 newtons at a 40° angle. What is the magnitude of the combined force they exert on the statue?

$\vec{A} + \vec{M} = \vec{r}$

$\vec{A} = \langle 120\cos 60, 120\sin 60 \rangle$

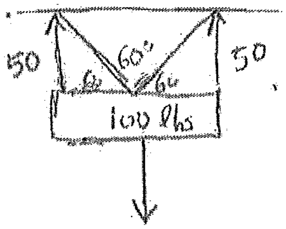
$+\ \vec{M} = \langle 180\cos 40, 180\sin 40 \rangle$

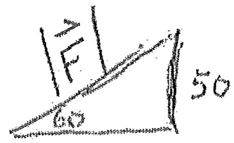
$\vec{r} = \langle 197.888, 219.625 \rangle$

$|\vec{r}| = \sqrt{197.888^2 + 219.625^2}$

$= \boxed{295.626\text{ N}}$

2. Dr. Smith is hanging a sign for her medical practice that will be held by two support bars. If the bars make a 60° angle with each other and the sign weighs 100 pounds, what are the magnitudes of the forces exerted by the sign of each support bar?





$* \sin\theta = \frac{opp}{hyp}$

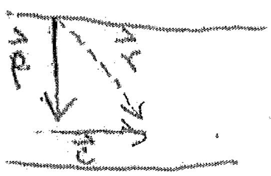
$\frac{\sin 60}{1} = \frac{50}{|\vec{F}|}$

$|\vec{F}|\sin 60 = 50$

$|\vec{F}| = \frac{50}{\sin 60}$

$\boxed{|\vec{F}| = 57.735 \text{ lbs}}$

3. A person in a canoe wants to cross a 65-foot-wide river. He begins to paddle straight across the river at 1.2 m/s while a current is flowing perpendicular to the canoe. If the resulting velocity of the canoe is 3.2 m/s, what is the speed of the current to the nearest tenth?



$\vec{p} + \vec{c} = \vec{r}$

$\cancel{\vec{p} = \langle 0, 1.2 \rangle}$

$\cancel{\vec{c} = \langle \quad \rangle}$

$\cancel{\vec{r} =}$

$p^2 + c^2 = r^2$

$c^2 = r^2 - p^2$

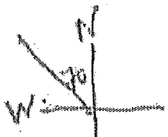
$c^2 = 3.2^2 - 1.2^2$

$c^2 = 8.8$

$\boxed{c = 2.966 \text{ m/s}}$

4. Kristin walks N70°W for 200 meters. She then walks due east for 90 meters. How far and at what bearing is Kristin from her starting point?

$\vec{p1} = \langle 200\cos 160, 200\sin 160 \rangle \qquad \vec{p2} = \langle 90, 0 \rangle \qquad * \vec{p1} + \vec{p2} = \vec{r}$



$\theta_1 = 160°$

$\vec{r} = \langle -97.939, 68.404 \rangle$

$|\vec{r}| = \sqrt{97.939^2 + 68.404^2} = \boxed{119.462 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{68.404}{-97.939}\right) = -34.932$ Q2 $+180$

$\theta = 145.068$

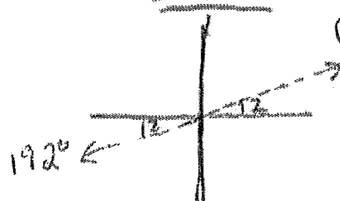
$\boxed{\text{bearing is } 304.932°}$

5. A pilot needs to plot a course that will result in a velocity of 500 miles per hour in a direction of due west. If the wind is blowing 100 miles per hour from 192°, find the direction and the speed the pilot should set to achieve this resultant.

$\vec{p} + \vec{w} = \vec{r}$

$\vec{p} = \vec{r} - \vec{w}$

$\theta = 12°$



$\vec{r} = \langle -500, 0 \rangle$

$-\vec{w} = \langle 100\cos 12, 100\sin 12 \rangle$

$\vec{p} = \langle -597.815, -20.791 \rangle$

$|\vec{p}| = \sqrt{597.815^2 + 20.791^2} = \boxed{598.176 \text{ mph}}$

$\theta = \tan^{-1}\left(\frac{-20.791}{-597.815}\right) = 1.992°$ Q3 $+180$

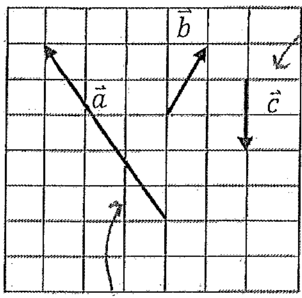
$\boxed{\theta = 181.992°}$

6.13 Test Review - Vectors Date: ______________

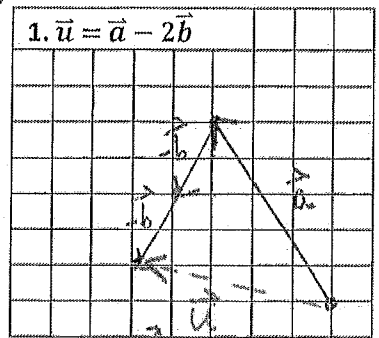
$\langle 1,2 \rangle$

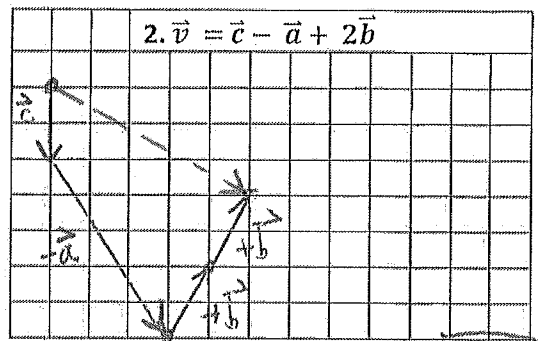
Given the vectors below, draw and label the given resultant vectors. $\vec{v} = \langle 5,-3 \rangle$

$\langle 0,-2 \rangle$



$\langle -3,5 \rangle$





3. a) Give the <u>component</u> form of $\vec{u}$: $\langle -5, 1 \rangle$

b) Find $|\vec{v}|$: $\sqrt{5^2 + 3^2} = \boxed{\sqrt{34}}$

magnitude

4. Given points A(4,-5) and B(1,-3):
a) write $\overrightarrow{AB}$ in component form
b) calculate $|\overrightarrow{AB}|$ → magnitude $|\overrightarrow{AB}| = \sqrt{3^2 + 2^2} = \sqrt{13}$
c) write as the sum of unit vectors

a) $\langle -3, 2 \rangle$
b) $\sqrt{13}$
c) $-3i + 2j$

a) $\langle x_2 - x_1, y_2 - y_1 \rangle$ | $\langle -3, 2 \rangle$
$\langle 1-4, -3--5 \rangle$

5. Give an example of 2 vectors, < 5 , 2 > and < 1 , 7 > that are *not* perpendicular. Show why they are no

*dot product ≠ 0

$5(1) + 2(7) = 19 \neq 0$, so these 2 vectors are not perpendicular

6. Given: $\vec{a} = \langle -3, 6 \rangle, \vec{b} = 5\vec{\imath} - 2\vec{\jmath}, \vec{c} = \langle 2, 3 \rangle$, write $\vec{v} = 2\vec{b} - \vec{c} + 3\vec{a}$ as the sum of unit vectors.

$\vec{v} = 2\langle 5,-2 \rangle - \langle 2,3 \rangle + 3\langle -3,6 \rangle$

$\vec{v} = \langle 10,-4 \rangle - \langle 2,3 \rangle + \langle -9,18 \rangle$

$\vec{v} = \langle -1, 11 \rangle$

6. $-1i + 11j$

7. A ship leaves port and sails for 40 miles in a direction N55°E. Find the component form (ordered pair) of the distance the ship sails.

N 55

$\theta = 90 - 55 = 35$ | $\langle 40\cos 35, 40\sin 35 \rangle$
$\theta = 35°$

7. $\langle 32.766, 22.943 \rangle$

8. Two hot air balloons take off at a spring festival. After about twenty minutes the path of the first balloon can be represented by $\langle 55, 81\rangle$. If the path of the second balloon can be represented by $\langle 62, 77\rangle$, find the angle between the vectors. Use: $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$

$$\cos\theta = \frac{55(62)+81(77)}{\sqrt{55^2+81^2}\cdot\sqrt{62^2+77^2}}$$

$$\cos\theta = \frac{9647}{\sqrt{9586}\cdot\sqrt{9773}}$$

$$\theta = \cos^{-1}\left(\frac{9647}{\sqrt{9586}\cdot\sqrt{9773}}\right)$$

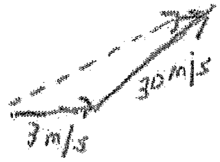
$$\boxed{\theta = 4.664^\circ}$$

9. A batter on the opposing softball team hits a ground ball that rolls out to left field. The left fielder runs toward the ball at a velocity of 3 meters per second, scoops it, and proceeds to throw it to the catcher at a speed of 30 meters per second and at an angle of 25° with the horizontal in an effort to throw out the runner. What is the resultant speed and direction of the throw?

$$\vec{v1} + \vec{v2} = \vec{r}$$

$$\vec{v1} = \langle 3, 0\rangle$$

$$\vec{v2} = \langle 30\cos 25, 30\sin 25\rangle$$



$$\vec{r} = \langle 30.189, 12.679\rangle$$

$$\text{speed} = \sqrt{30.189^2+12.679^2} \approx \boxed{32.743 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{12.679}{30.189}\right) \approx \boxed{22.781^\circ}$$

Q1

~~10. A corporate jet is flying at a bearing of 320° at 425 mph. If a 40 mph wind blows at a bearing of 290°,~~ find the bearing and speed that the pilot must use to maintain the jet's former course.

$$\vec{p} + \vec{w} = \vec{r}$$

$$\vec{p} = \vec{r} - \vec{w}$$

* Entire problem in terms of bearing, so leave in bearing!

$$\vec{r} = \langle 425\cos 320, 425\sin 320\rangle$$

$$-\ \vec{w} = \langle 40\cos 290, 40\sin 290\rangle$$

$$\vec{p} = \langle 311.888, -235.597\rangle$$

$$\text{speed} = \sqrt{311.888^2 + 235.597^2} = \boxed{390.871 \text{ mph}}$$

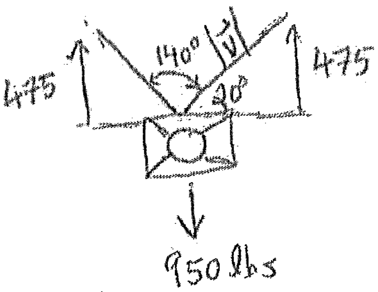
$|\vec{p}| \rightarrow$

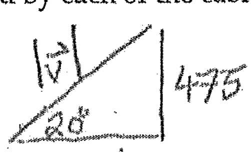
$$\text{direction: } \theta = \tan^{-1}\left(\frac{-235.597}{311.888}\right) = -37.067$$

Q4 $\quad +360$

$$\boxed{\theta = \text{bearing of } 322.933^\circ}$$

11. The lighting system for Milton theater is supported equally by two cables suspended from the ceiling of the auditorium. The cables form a 140° angle with each other. If the lighting system weighs 950 pounds, what is the force exerted by each of the cables on the lighting system?





$$\sin\theta = \frac{O}{H}$$

$$\frac{\sin 20}{1} = \frac{475}{|\vec{v}|}$$

$$|\vec{v}|\sin 20 = 475$$

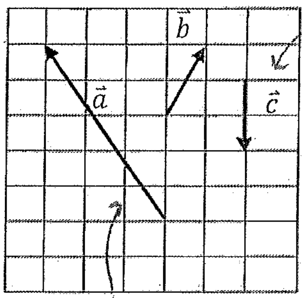
$$|\vec{v}| = \frac{475}{\sin 20}$$

$$\boxed{|\vec{v}| = 1388.807 \text{ lbs}}$$

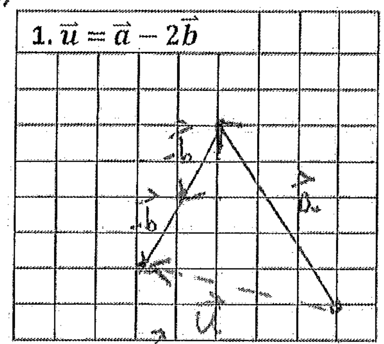
6.13 Test Review - Vectors Date: ____________

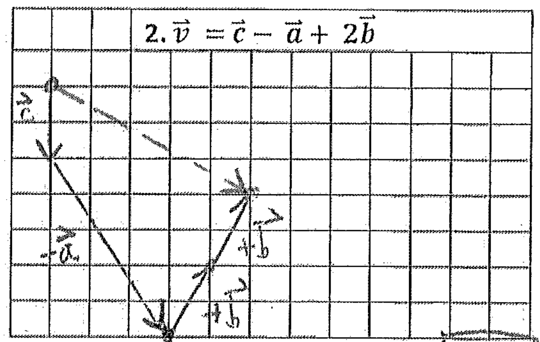
Given the vectors below, draw and label the given resultant vectors. $\vec{v} = \langle 5, -3 \rangle$

$\langle 1, 2 \rangle$ $\langle 0, -2 \rangle$



$\langle -3, 5 \rangle$





3. a) Give the <u>component</u> form of $\vec{u}$: $\langle -5, 1 \rangle$

b) Find $|\vec{v}|$: $\sqrt{5^2 + 3^2} = \boxed{\sqrt{34}}$

magnitude

4. Given points A(4,-5) and B(1,-3):
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c) write as the sum of unit vectors

a) $\langle -3, 2 \rangle$
b) $\sqrt{13}$
c) $-3i + 2j$

a) $\langle x_2 - x_1, y_2 - y_1 \rangle$ | $\langle -3, 2 \rangle$
$\langle 1 - 4, -3 - -5 \rangle$

5. Give an example of 2 vectors, < 5 , 2 > and < 1 , 7 > that are *not* perpendicular. Show why they are no

* dot product ≠ 0

$5(1) + 2(7) = 19 \neq 0$, so these 2 vectors are not perpendicular

6. Given: $\vec{a} = \langle -3, 6 \rangle, \vec{b} = 5\vec{\imath} - 2\vec{\jmath}, \vec{c} = \langle 2, 3 \rangle$, write $\vec{v} = 2\vec{b} - \vec{c} + 3\vec{a}$ as the sum of unit vectors.

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$\theta = 90 - 55 = 35$
$\theta = 35°$

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$$\cos\theta = \frac{9647}{\sqrt{9586}\cdot\sqrt{9773}}$$

$$\theta = \cos^{-1}\left(\frac{9647}{\sqrt{9586}\cdot\sqrt{9773}}\right)$$

$$\boxed{\theta = 4.664^\circ}$$

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$$\vec{v1} = \langle 3, 0\rangle$$

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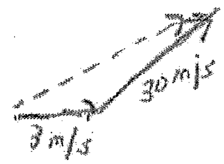

$$\vec{r} = \langle 30.189, 12.679\rangle$$

$$\text{speed} = \sqrt{30.189^2+12.679^2} \approx \boxed{32.743\ m/s}$$

$$\theta = \tan^{-1}\left(\frac{12.679}{30.189}\right) \approx \boxed{22.781^\circ}$$

Q1

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$$\vec{p} = \vec{r} - \vec{w}$$

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$$\text{Speed} = \sqrt{311.888^2+235.597^2} = \boxed{390.871\ mph}$$

$|\vec{p}| \rightarrow$

$$\text{direction: } \theta = \tan^{-1}\left(\frac{-235.597}{311.888}\right) = -37.067$$

Q4 $\quad +360$

$$\boxed{\theta = \text{bearing of } 322.933^\circ}$$

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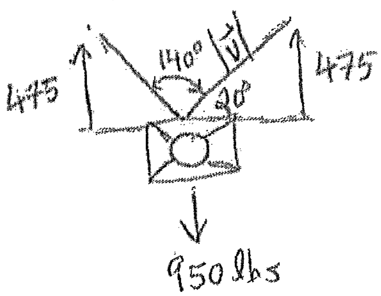
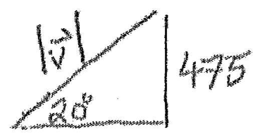




$$\sin\theta = \frac{O}{H}$$

$$\frac{\sin 20}{1} = \frac{475}{|\vec{v}|}$$

$$|\vec{v}|\sin 20 = 475$$

$$|\vec{v}| = \frac{475}{\sin 20}$$

$$\boxed{|\vec{v}| = 1388.807\ lbs}$$

6.14 More Vector Review

use $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ Date: ______________

Given: X (-2, 8) and Y (-5, 12)

1. Find the component form of $\overrightarrow{XY}$.

$\langle x_2 - x_1, y_2 - y_1 \rangle$

$\overrightarrow{XY} = \langle -5 - -2, 12 - 8 \rangle = \boxed{\langle -3, 4 \rangle}$

2. Find the direction, in standard position, of $\overrightarrow{XY}$.

$\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.130 + 180$ $\boxed{\theta = 126.870^\circ}$

Q2

3. Write $\overrightarrow{YX}$ as the sum of unit vectors.

$Y(-5, 12)$ $X(-2, 8)$ $\boxed{3i - 4j}$

$x_1 \; y_1 \quad x_2 \; y_2$

$\overrightarrow{YX} = \langle -2 - -5, 8 - 12 \rangle \to \langle 3, -4 \rangle$

4. Find the magnitude of $\overrightarrow{YX}$.

$|\overrightarrow{YX}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$

$|\overrightarrow{YX}| = \boxed{5}$

Given: $\vec{u} = \langle -5, -1 \rangle$, $\vec{v} = \langle 4, -2 \rangle$

5. Find the angle between $\vec{u}$ and $\vec{v}$.

$* \cos\theta = \frac{u \cdot v}{|u||v|} = \frac{-5(4) + (-1)(-2)}{\sqrt{5^2+1^2} \cdot \sqrt{4^2+2^2}}$

$\cos\theta = \frac{-18}{\sqrt{26} \cdot \sqrt{20}}$ $\theta = \cos^{-1}\left(\frac{-18}{\sqrt{26} \cdot \sqrt{20}}\right)$

$\boxed{\theta = 142.125^\circ}$

6. Find the magnitude and direction of $\vec{u}$.

$|\vec{u}| = \sqrt{5^2 + 1^2} = \boxed{\sqrt{26}}$

$\theta = \tan^{-1}\left(\frac{-1}{-5}\right) = 11.310^\circ + 180$

Q3 $\boxed{\theta = 191.310^\circ}$

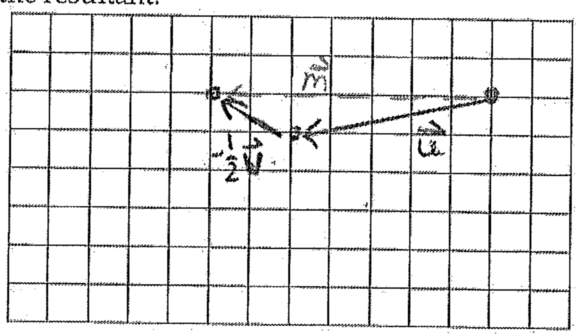
7. Find $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$. Show work algebraically.

$\vec{m} = \langle -5, -1 \rangle - \frac{1}{2}\langle 4, -2 \rangle$

$= \langle -5, -1 \rangle - \langle 2, -1 \rangle$

$\boxed{\vec{m} = \langle -7, 0 \rangle}$

8. Draw the vector diagram for $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$ and label the resultant.



$\vec{m} = \langle -7, 0 \rangle$

9. Find the magnitude and direction of $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$

$\vec{m} = \sqrt{7^2 + 0^2} = \sqrt{49} = \boxed{7}$

$\theta = 180^\circ$

10. Write $\vec{m}$ as the sum of unit vectors.

$-7i + 0j = \boxed{-7i}$

$\theta = 450 - 210$

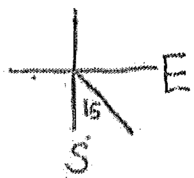
$\theta = 240°$ 30

11. a) A pilot directs his plane to fly on a bearing of 210° at 460 mph. State the measurement of the angle for the direction in standard position. Find the component form of the velocity of the airplane.

$\langle 460\cos 240, 460\sin 240\rangle = \boxed{\langle -230, -398.372\rangle}$

b) An 80 mph wind blowing in the direction of S15°E is pushing the plane off course. Find the ground speed and direction of the airplane (resultant path).

$\vec{p} + \vec{w} = \vec{r}$



$\theta = 270 + 15$

$\theta = 285°$

$\vec{w} = \langle 80\cos 285, 80\sin 285\rangle$

$\vec{p}\ \langle -230, -398.372\rangle$

$+\vec{w}\ \langle 20.706, -77.274\rangle$

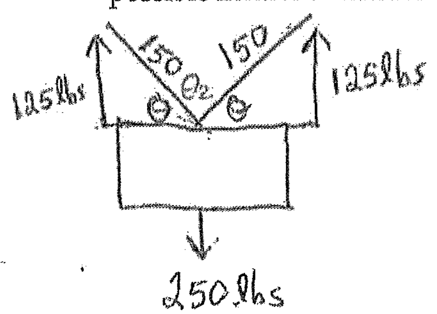
$\vec{r} = \langle -209.294, -475.646\rangle$

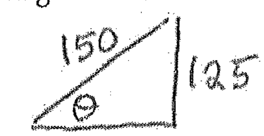
$\text{Speed} = |\vec{r}| = \sqrt{209.294^2 + 475.646^2} = \boxed{519.657\text{ mph}}$

$\theta = \tan^{-1}\left(\frac{-475.646}{-209.294}\right) = 66.249$

$Q3 \rightarrow \theta = 66.249 + 180 = \boxed{246.249°}$

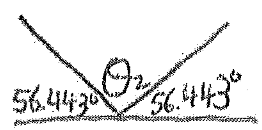
12. A new score board is being place in the gym. It will be supported from the ceiling by 2 cables. Each cable can withstand 150 pounds of tension. If the score board weighs 250 pounds, what is the largest possible measurement for the angle that the two cables make with each other?





$\sin\theta = \frac{125}{150} \rightarrow \theta = \sin^{-1}\left(\frac{125}{150}\right)$

$\theta = 56.443°$



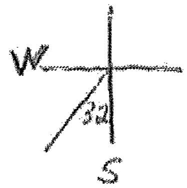
$\theta_2 = 180 - 56.443 - 56.443$

$\boxed{\theta_2 = 67.115°}$

13. A pilot needs to plot a course that will result in a velocity of 520 miles per hour in a direction of S32°W. If the wind is blowing 65 miles per hour in the direction of 12°, find the direction and the speed the pilot should set to achieve this resultant.

$\vec{p} + \vec{w} = \vec{r}$

$\vec{p} = \vec{r} - \vec{w}$



$\theta = 270 - 32$

$\theta = 238°$

$\vec{r} = \langle 520\cos 238, 520\sin 238\rangle$

$-\vec{w} = \langle 65\cos 12, 65\sin 12\rangle$

$\vec{p} = \langle -339.138, -454.499\rangle$

$\text{Speed} = |\vec{p}| = \sqrt{339.138^2 + 454.499^2} = \boxed{567.084\text{ mph}}$

direction: $\theta = \tan^{-1}\left(\frac{-454.499}{-339.138}\right) = 53.270$

$+180$

Q3

$\boxed{\theta = 233.270°}$

Vectors

2D Vectors: $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

1. **Component form** shows the vector from the *initial point* to the *terminal point* based on the displacement of its dimensional values:
   - 2D vector, from (x_1, y_1) to (x_2, y_2): $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

2. **Unit vector** is a vector of length 1. The standard unit vectors are $\vec{\imath} = \langle 1,0 \rangle$ and $\vec{\jmath} = \langle 0,1 \rangle$. A vector can be written as the *sum of unit vectors* by using its components as scalars of standard unit vectors:
   - 2D vector: $\vec{v} = a\vec{\imath} + b\vec{\jmath}$

3. **Magnitude** (length) of a vector:
   - 2D vector: $|\vec{v}| = \sqrt{a^2 + b^2}$

4. **Direction of a vector:**
   - 2D vector: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, add 180° if in quadrants 2 or 3.

5. Given the **magnitude** and the **direction** of a vector, it is possible to determine its components:
   - 2D vector with magnitude $|\vec{v}|$ and direction θ, $\vec{v} = |\vec{v}|\langle \cos\theta, \sin\theta \rangle = \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

6. **Resultant vector** is the sum of two or more vectors.
   - Geometrically, this is shown with the *tip-to-tail* method, also known as the *triangle* method. The *parallelogram* method also can determine the resultant vector.
   - Algebraically, this is calculated by finding the sum of the corresponding components.
   - 2D vectors: $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$

7. **Scalar multiplication:**
   - 2D vector: $k\vec{v} = \langle ka, kb \rangle$

8. **Dot product** (inner product) is used to determine if two vectors are perpendicular:
   - 2D vectors: $\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$
   - For magnitude: $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$
   - 2 vectors are orthogonal (perpendicular) if their dot product equals 0.

9. **Angle between two vectors** can be found with a dot product:
   - 2D vectors: $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$ (will be provided on test)

10. **Angles** have different ways of being measured:
    - **Standard Position** is measured from the positive x-axis, with positive angles opening counter-clockwise.
    - **True Bearing** or **Compass Bearing** is measured from North, with positive angles opening clockwise.
      True bearing measurement = 450° − Standard position measurement
      Standard position measurement = 450° − True bearing measurement
    - **Quadrant Bearing** is measured either from North or from South, opening toward East or toward West in such a way that the angle value is always acute.

Vectors Help Session Test Review WS 3

Given: X (-1, 9) and Y (-3,7)

1. Find the component form of $\overrightarrow{XY}$.

2. Find the direction, in standard position, of $\overrightarrow{XY}$.

3. Write $\overrightarrow{YX}$ as the sum of unit vectors.

4. Find the magnitude of $\overrightarrow{YX}$.

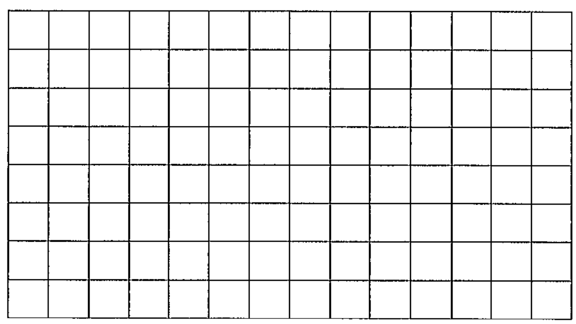
Given: $\vec{u}$ = <-2, -3>, $\vec{v}$ = <1, -3>

5. Find the angle between $\vec{u}$ and $\vec{v}$.

6. Find the magnitude and direction of $\vec{u}$.

7. Find $\vec{m} = \vec{u} - 2\vec{v}$. Show work algebraically.

8. Draw the vector diagram for $\vec{m} = \vec{u} - 2\vec{v}$ and label the resultant.



9. Find the magnitude and direction of $\vec{m} = \vec{u} - 2\vec{v}$.

10. Create 2 vectors that are parallel to $\vec{v}$. Justify your answer.

11. Determine whether $\vec{a}$ and $\vec{b}$ are orthogonal. $\vec{a} = \langle 2, -3 \rangle$ and $\vec{b} = \langle 4, 5 \rangle$

12. Convert bearing of 314° to standard position.

13. Convert S 12° W to standard position.

14. Convert the standard position of 197° into bearing.

15. A plane is flying at a speed of 320 mph on a bearing N 70° E. Its resultant speed is 370 mph and resultant direction is 60°. Find the speed and direction of the wind.

16. A ship is sailing through the water in the English Channel with a velocity of 22 knots along a bearing of 157°. The current has a velocity of 5 knots along a bearing of 213°. The actual velocity of the ship is the vector sum of the ship's velocity and the water's velocity. Find the actual velocity.

17. A bear travels 70 miles in a northeasterly direction from his den. It then travels 150 miles 60 degrees north of west. Determine how far and in what direction the bear is from his den.

18. A motorboat with a speed of 9 mph in still water must aim upstream at an angle of 25.5 degrees in order to travel directly across the stream. What is the speed of the current? What is the resultant speed of the boat?

Vectors Help Session Test Review WS 3

Key

Given: X (-1, 9) and Y (-3,7)

1. Find the component form of $\overrightarrow{XY}$.

$\langle -3--1, 7-9 \rangle$

$\langle -2, -2 \rangle$

2. Find the direction, in standard position, of $\overrightarrow{XY}$.

$\theta = \tan^{-1}\left(\frac{-2}{-2}\right)$

$\theta = 45° + 180 = \boxed{225°}$

3. Write $\overrightarrow{YX}$ as the sum of unit vectors.

$\langle -1--3, 9-7 \rangle = \langle 2, 2 \rangle$

$\boxed{2i + 2j}$

4. Find the magnitude of $\overrightarrow{YX}$.

$|\overrightarrow{YX}| = \sqrt{2^2 + 2^2} = \sqrt{8} = \boxed{2\sqrt{2}}$

Given: $\vec{u}$ = <-2, -3>, $\vec{v}$ = <1, -3>

5. Find the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \frac{-2(1)+3(3)}{\sqrt{2^2+3^2} \cdot \sqrt{1^2+3^2}} = \frac{7}{\sqrt{13} \cdot \sqrt{10}}$

$\theta = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \rightarrow \boxed{\theta = 52.125°}$

6. Find the magnitude and direction of $\vec{u}$.

$|\vec{u}| = \sqrt{2^2 + 3^2} = \boxed{\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{-3}{-2}\right) = 56.309 + 180 = \boxed{236.309°}$

Q3

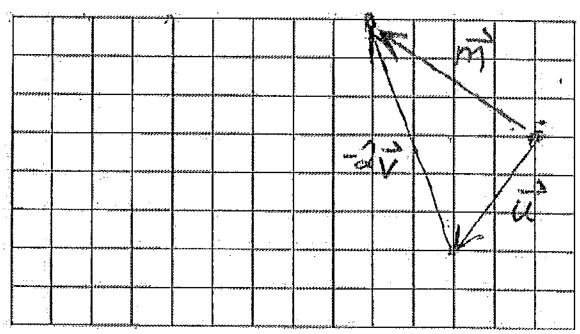
7. Find $\vec{m} = \vec{u} - 2\vec{v}$. Show work algebraically.

$\langle -2, -3 \rangle - 2\langle 1, -3 \rangle$

$\langle -2, -3 \rangle - \langle 2, -6 \rangle$

$\langle -4, 3 \rangle = \vec{m}$

8. Draw the vector diagram for $\vec{m} = \vec{u} - 2\vec{v}$ and label the resultant.



9. Find the magnitude and direction of $\vec{m} = \vec{u} - 2\vec{v}$.

$|\vec{m}| = \sqrt{(-4)^2 + 3^2} = \boxed{5}$ $\quad \theta = \tan^{-1}\left(\frac{3}{-4}\right) = -36.869 + 180 = \boxed{143.130°}$

10. Create 2 vectors that are parallel to $\vec{v}$. Justify your answer.

* multiples of $\vec{v}\langle 1, -3 \rangle$

$\vec{a}\langle 2, -6 \rangle$

$\vec{b}\langle 3, -9 \rangle$

$\cos\theta = \frac{u \cdot v}{|u||v|}$

vectors are parallel if numerator is equal to denominator

$\vec{v}$ and $\vec{a} \rightarrow \frac{1(2)+3(6)}{\sqrt{1+3^2} \cdot \sqrt{2^2+6^2}} \rightarrow \frac{20}{\sqrt{400}} \rightarrow \frac{20}{20}$ ✓

$\vec{v}$ and $\vec{b}$

$\frac{1(3)+3(9)}{\sqrt{1+3^2} \cdot \sqrt{3^2+9^2}}$

$\downarrow$

$\frac{30}{\sqrt{10} \cdot \sqrt{90}} \rightarrow \frac{30}{30}$

11. Determine whether $\vec{a}$ and $\vec{b}$ are orthogonal. $\vec{a} = \langle 2, -3 \rangle$ and $b = \langle 4, 5 \rangle$

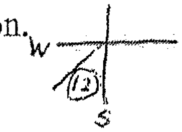
* $\vec{a}$ and $\vec{b}$ are orthogonal (perpendicular) if dot product = 0.

$\vec{a} \cdot \vec{v} = 2(4) + (-3)(5) = 8 - 15 = -7 \neq 0$ so vectors $\vec{a}$ and $\vec{b}$ are not orthogonal.

12. Convert bearing of 314° to standard position.

$450 - 314 = \boxed{136°}$

13. Convert S 12° W to standard position.



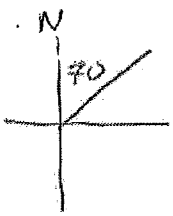
$270 - 12 = \boxed{258°}$

14. Convert the standard position of 197° into bearing.

$450 - 197 = 253$

$\boxed{\text{bearing of } 253°}$

15. A plane is flying at a speed of 320 mph on a bearing N 70° E. Its resultant speed is 370 mph and resultant direction is 60°. Find the speed and direction of the wind.



$90 - 70$

$\theta = 20°$

$\vec{p} + \vec{w} = \vec{r}$

$\vec{w} = \vec{r} - \vec{p}$

$\vec{r} = \langle 370\cos 60, 370\sin 60 \rangle$

$-\vec{p} = \langle 320\cos 20, 320\sin 20 \rangle$

$\vec{w} = \langle -115.702, 210.983 \rangle$

$\text{speed} = \sqrt{115.7^2 + 210.98^2}$

$= \boxed{|\vec{w}| = 240.626 \text{ mph}}$

$\theta = \tan^{-1}\left(\frac{210.983}{-115.702}\right)$

$-61.259 + 180 = \boxed{118.74°}$

16. A ship is sailing through the water in the English Channel with a velocity of 22 knots along a bearing of 157°. The current has a velocity of 5 knots along a bearing of 213°. The actual velocity of the ship is the vector sum of the ship's velocity and the water's velocity. Find the actual velocity.

$\theta_v = 450 - 157 = 293°$

$\theta_c = 450 - 213 = 237°$

$\vec{v} + \vec{c} = \vec{r}$

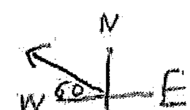
$\vec{v}\ \langle 22\cos 293, 22\sin 293 \rangle$

$+\vec{c}\ \langle 5\cos 237, 5\sin 237 \rangle$

$\vec{r} = \langle 5.873, -24.444 \rangle$

$|\vec{r}| = \sqrt{5.873^2 + 24.444^2}$

$\boxed{|\vec{r}| = 25.139 \text{ knots}}$



$\theta = 45°$

17. A bear travels 70 miles in a northeasterly direction from his den. It then travels 150 miles 60 degrees north of west. Determine how far and in what direction the bear is from his den.

$\theta = 180 - 60$

$\theta = 120°$

$\vec{v1} + \vec{v2} = \vec{r}$

$\vec{v1} = \langle 70\cos 45, 70\sin 45 \rangle$

$+\ \vec{v2} = \langle 150\cos 120, 150\sin 120 \rangle$

$\vec{r} = \langle -25.503, 179.401 \rangle$

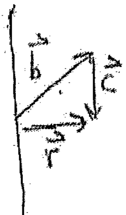
$|\vec{r}| = \sqrt{25.503^2 + 179.401^2}$

$|\vec{r}| = 181.205 \text{ mi}$

$\theta = \tan^{-1}\left(\frac{179.401}{-25.503}\right) = -81.909 + 180$

$\boxed{\theta = 98.091°}$

18. A motorboat with a speed of 9 mph in still water must aim upstream at an angle of 25.5 degrees in order to travel directly across the stream. What is the speed of the current? What is the resultant speed of the boat?



$\vec{b} + \vec{c} = \vec{r}$

$\vec{c} = \vec{r} - \vec{b}$

$\vec{r} = \langle 9, 0 \rangle$

$-\vec{b} = \langle 9\cos 25.5, 9\sin 25.5 \rangle$

$\vec{c} = \langle 0.877, -3.875 \rangle$

$|\vec{c}| = \sqrt{0.877^2 + 3.875^2}$

$= 3.973 \text{ mph}$

$|\vec{b}| = \sqrt{8.123^2 + 3.875^2}$

$= 8.999 \text{ mph}$