

Calculus - SUMMER PACKET

NAME: _____

NO CALCULATOR!!!

* Not for a grade *

* Voluntary work *

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x + 2) =$

3. $f(x + h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

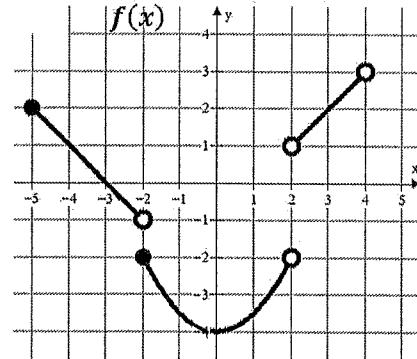
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2 \text{ when } x = ?$

$f(x) = -3 \text{ when } x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.

$$y - y_1 = m(x - x_1)$$

5. slope = 3 and $(4, -2)$

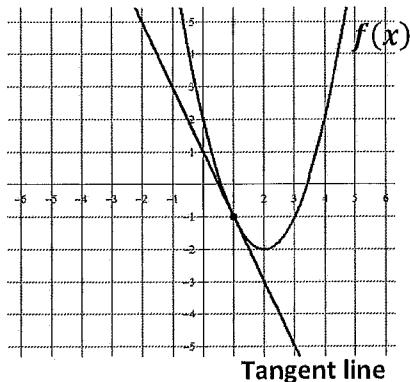
6. $m = -\frac{3}{2}$ and $f(-5) = 7$

7. $f(4) = -8$ and $f(-3) = 12$

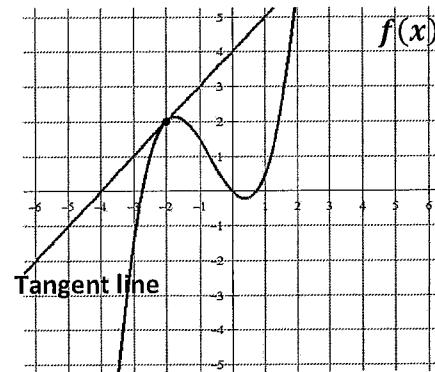
2

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



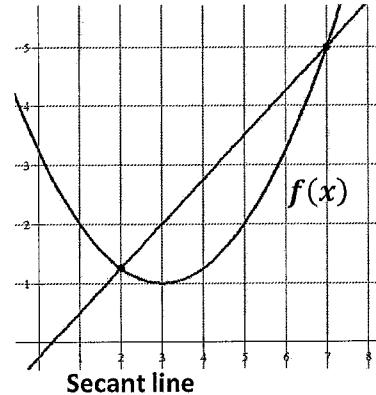
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

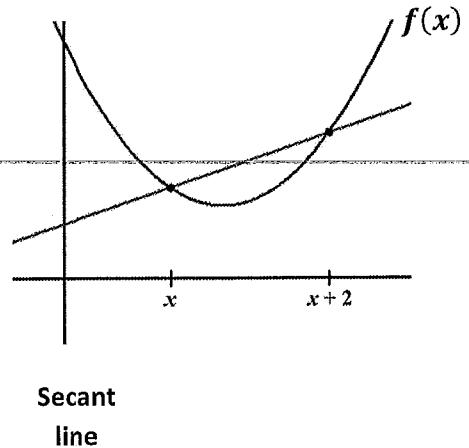
10. Which choice represents the slope of the secant line shown?

A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$
 D) $\frac{x+2-x}{f(x)-f(x+2)}$



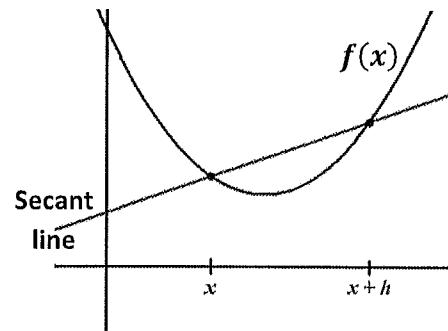
12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

C) $\frac{f(x+h)-f(x)}{x+h-x}$

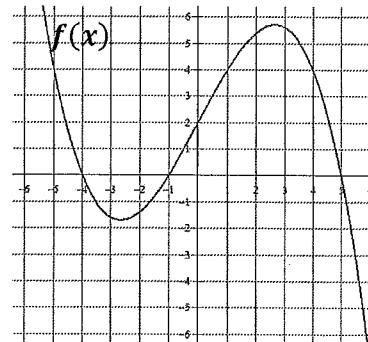
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

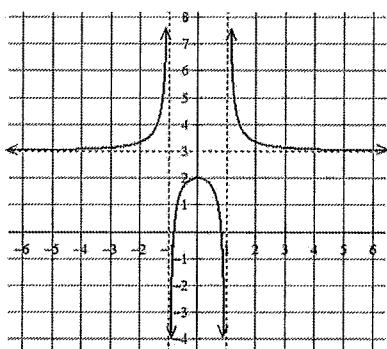
- I. $f(2) = 0$
- II. $(x + 4)$ is a factor of $f(x)$
- III. $f(5) = f(-1)$

- (A) I only
- (B) II only
- (C) III only
- (D) I and III only
- (E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



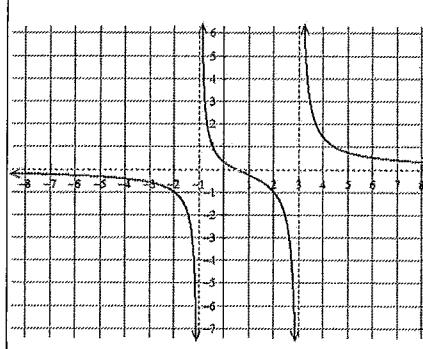
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



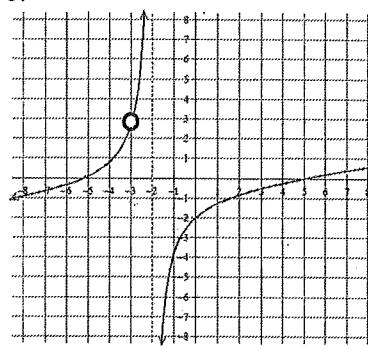
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

(4)

MULTIPLE CHOICE!17. Which of the following functions has a vertical asymptote at $x = 4$?

- (A) $\frac{x+5}{x^2-4}$
 (B) $\frac{x^2-16}{x-4}$
 (C) $\frac{4x}{x+1}$
 (D) $\frac{x+6}{x^2-7x+12}$
 (E) None of the above

18. Consider the function: $f(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

- I. $f(x)$ has a vertical asymptote of $x = 2$
 II. $f(x)$ has a vertical asymptote of $x = -2$
 III. $f(x)$ has a horizontal asymptote of $y = 1$
- (A) I only
 (B) II only
 (C) I and III only
 (D) II and III only
 (E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$

20. $\sqrt{x+1}$

21. $\frac{1}{\sqrt{x+1}}$

22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$

23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

27. $3x^{-\frac{1}{2}}$

28. $(x+4)^{-\frac{1}{2}}$

29. $x^{-2} + x^{\frac{1}{2}}$

30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin \frac{\pi}{6}$

32. $\cos \frac{\pi}{4}$

33. $\sin 2\pi$

34. $\tan \pi$

35. $\sec \frac{\pi}{2}$

36. $\cos \frac{\pi}{6}$

37. $\sin \frac{\pi}{3}$

38. $\sin \frac{3\pi}{2}$

39. $\tan \frac{\pi}{4}$

40. $\csc \frac{\pi}{2}$

41. $\sin \pi$

42. $\cos \frac{\pi}{3}$

43. Find x where $0 \leq x \leq 2\pi$,

$$\sin x = \frac{1}{2}$$

44. Find x where $0 \leq x \leq 2\pi$,

$$\tan x = 0$$

45. Find x where $0 \leq x \leq 2\pi$,

$$\cos x = -1$$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$

47. $3e^x + 5 = 8$

48. $e^{2x} = 1$

49. $\ln x = 0$

50. $3 - \ln x = 3$

51. $\ln(3x) = 0$

52. $x^2 - 3x = 0$

53. $e^x + xe^x = 0$

54. $e^{2x} - e^x = 0$

6

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$

56. $\cos x = -1$

57. $\cos x = \frac{\sqrt{3}}{2}$

58. $2\sin x = -1$

59. $\cos x = \frac{\sqrt{2}}{2}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

61. $\tan x = 0$

62. $\sin(2x) = 1$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

<u>Function</u>	<u>Domain</u>	<u>Range</u>
64. $y = \sqrt{x - 4}$		
65. $y = (x - 3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4 - x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$

70. $e^{\ln x}$

71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $(5a^{2/3})(4a^{3/2})$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $(4a^{5/3})^{\frac{3}{2}}$

If $f(x) = \{(3,5), (2,4), (1,7)\}$,
 $g(x) = \sqrt{x-3}$,
 $h(x) = \{(3,2), (4,3), (1,6)\}$,
 $k(x) = x^2 + 5$, then determine each of the following.

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

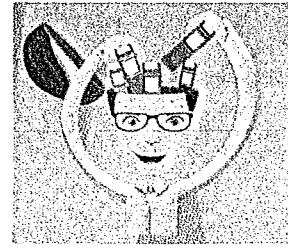
87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

The following formulas and identities will help you complete this packet



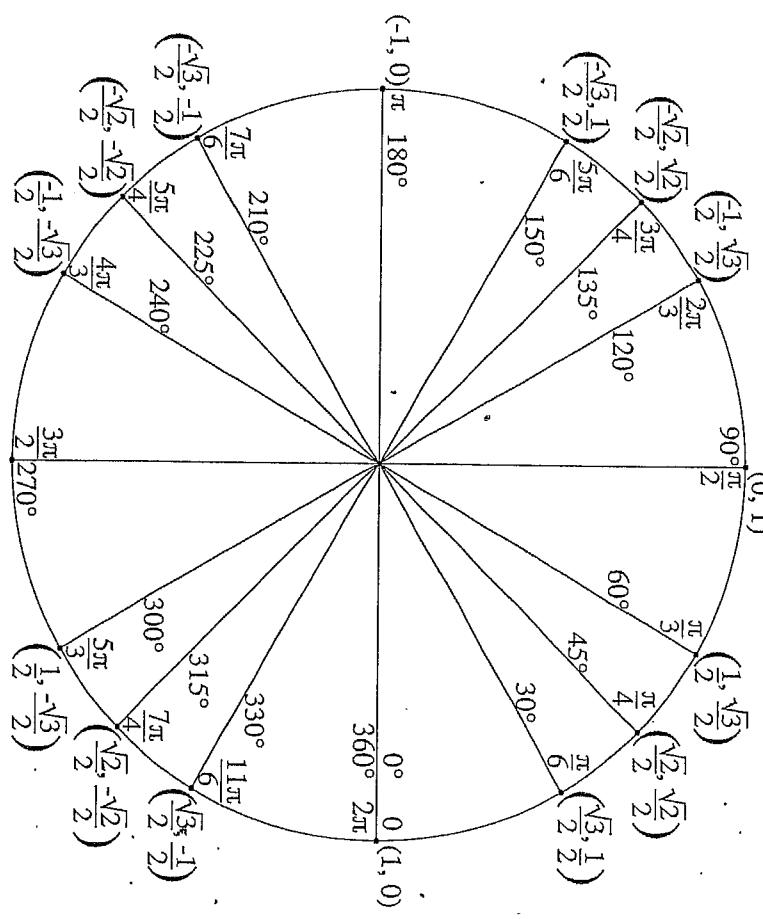
LINES	QUADRATICS
Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$ Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0) Vertical line: $x = a$ (slope = undefined) Parallel \rightarrow same slope Perpendicular \rightarrow opposite reciprocal slopes	Standard: $y = ax^2 + bx + c$ Vertex: $y = a(x - h)^2 + k$ Intercept: $y = a(x - p)(x - q)$ Parabola opens: up if $a > 0$ down if $a < 0$ Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
EXPONENTIAL PROPERTIES	LOGARITHMS
$x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $\frac{x^a}{x^b} = x^{a-b}$ $\sqrt[n]{x^m} = x^{m/n}$ $x^0 = 1$ ($x \neq 0$) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ $x^{-n} = \frac{1}{x^n}$ <div style="border: 1px solid black; padding: 2px; margin-left: 10px;"> In general, it is fine to have negative exponents in your answers! </div>	$y = \log_a x$ is equivalent to $a^y = x$ $\log_b(mn) = \log_b m + \log_b n$ $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ $\log_b(m^p) = p \log_b m$
TRIGONOMETRIC IDENTITIES	
$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$ $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$ $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$ $\sin(2x) = 2 \sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x$ or $1 - 2 \sin^2 x$ or $2 \cos^2 x - 1$	



The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

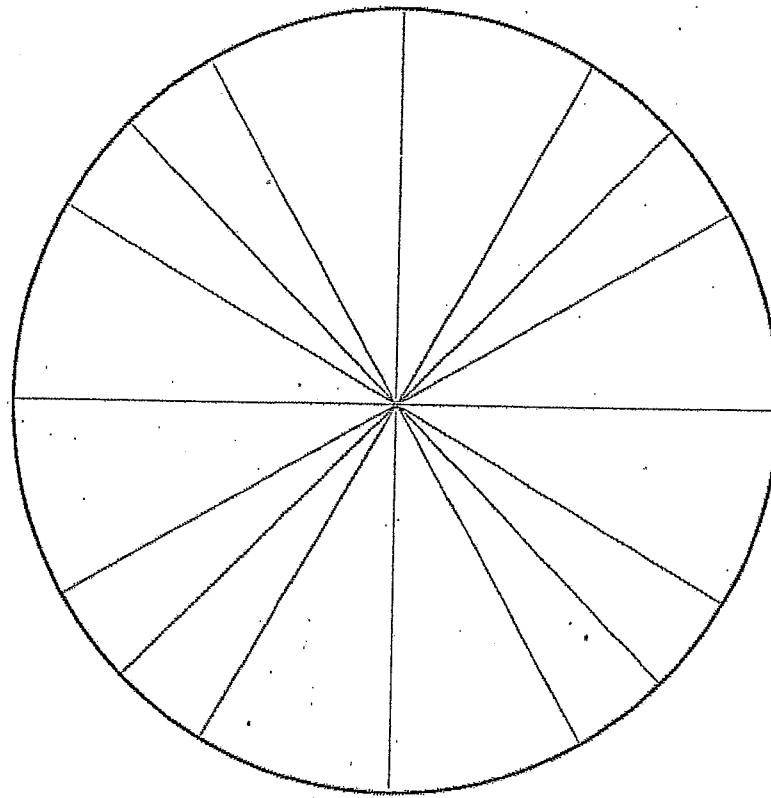
Positive: sin, cos, tan, sec, csc, cot
Negative: none



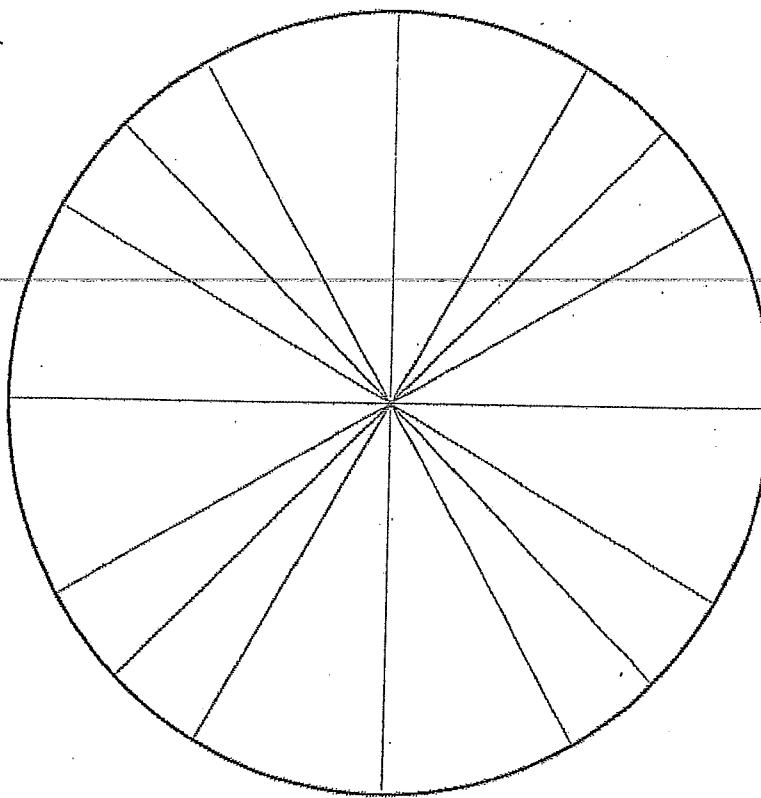
Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

(Blank) Unit Circle for Practice



(Blank) Unit Circle for Practice

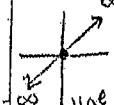
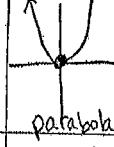
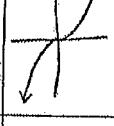


10

Properties of Families of Functions

*any
breaks
or gaps*

∅

Name	Graph	Algebraic Equation	Continuity	Extrema max or min	End Behavior	Symmetry Even, Odd	Asymptotes	Domain and Range
Identity Function Linear		$y = x^1$	No breaks continuous	none	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow -\infty$	odd can rotate 180° about origin and it's self	none	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
Quadratic Function		$y = x^2$	continuous	no max 1 min at $(0, 0)$	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow \infty$	Even can flip over y-axis and it's self	none	D: $(-\infty, \infty)$ R: $[0, \infty)$
Cubic Function		$y = x^3$	continuous	parent has none * can be 1 max & 1 min infinitely	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow -\infty$	ODD	none	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
Rational Function Reciprocal Function		$y = \frac{1}{x}$ $y = 1/x$	Infinite discontinuity at $x=0$	none	As $x \rightarrow \infty$, $y \rightarrow 0$ As $x \rightarrow -\infty$, $y \rightarrow 0$	ODD	VA: $x=0$ (y-axis) HA: $y=0$ (x-axis)	D: $(-\infty, 0) \cup (0, \infty)$ R: $(-\infty, 0) \cup (0, \infty)$
Square Root Function		$y = \sqrt{x}$	continuous	abs min at $(0, 0)$	As $x \rightarrow \infty$, $y \rightarrow \infty$ as $x \rightarrow \infty$, $y \rightarrow \infty$	none	none	D: $[0, \infty)$ R: $[0, \infty)$

1/2 of parabola

Properties of Families of Functions

Name	Graph	Algebraic Equation	Continuity	Extrema	End Behavior	Symmetry	Asymptotes	Domain and Range
Absolute Value Function		$y = x $	continuous	abs min at $(0, 0)$	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow \infty$	even	none	D: $[-\infty, \infty)$ R: $[0, \infty)$
Exponential Function		$y = e^x$ Or $y = a^x$	continuous	none	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow 0$	none	HA $y=0$	D: $(-\infty, \infty)$ R: $(0, \infty)$
Logarithmic Function		$y = \log_b x$ Or $y = \ln x$	continuous	none	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow 0^+$, $y \rightarrow -\infty$	none	VA $x=0$	D: $(0, \infty)$ R: $(-\infty, \infty)$
Sine Function		$y = \sin x$	discontinuous	abs min @ $y=-1$ abs max @ $y=1$	Does not exist oscillates and cannot converge to a value	odd	none	D: $(-\infty, \infty)$ R: $[-1, 1]$
Greatest Integer Function/Step Function		$y = [x]$	jump (non-removable) dis @ $x =$ every integer	none	As $x \rightarrow \infty$, $y \rightarrow \infty$ As $x \rightarrow -\infty$, $y \rightarrow -\infty$	none	none	D: $(-\infty, \infty)$ R: {all integers} \mathbb{Z}

The output of $[x]$ is the greatest integer that is less than or equal to x .

$$[1.2] = 1 \quad [1.75] = 1 \quad [1.99] = 1 \quad [2] = 2$$

Calculus - SUMMER PACKET

NAME: SolutionsSummer + Math = (Best Summer Ever)²**NO CALCULATOR!!!**Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$$f(-2) = (-2)^2 - 2(-2) + 5$$

$$f(-2) = 4 + 4 + 5$$

$$\boxed{f(-2) = 13}$$

2. $f(x+2) =$

$$\boxed{x^2 + 2x + 5}$$

3. $f(x+h) =$

$$(x+h)^2 - 2(x+h) + 5$$

$$(x+h)(x+h) - 2x - 2h + 5$$

$$x^2 + xh + xh + h^2 - 2x - 2h + 5$$

$$\boxed{x^2 + 2xh + h^2 - 2x - 2h + 5}$$

Use the graph $f(x)$ to answer the following.

4. $f(0) = -4$

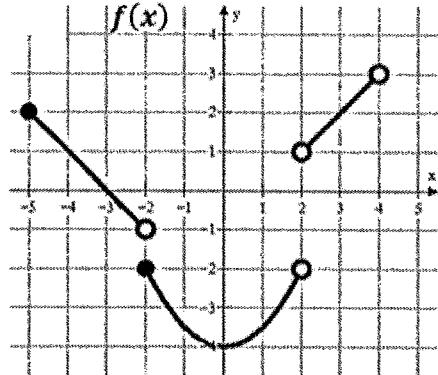
 $f(4) = \text{DNE (Does not exist)}$
 or undefined

$f(-1) = -3.5$

$f(-2) = -2$

 $f(2) = \text{DNE (Does not exist)}$
 or undefined

$f(3) = 2$

 $f(x) = 2$ when $x = ?$
 -5 and 3
 $f(x) = -3$ when $x = ?$
 -1.5 and 1.5


Write the equation of the line meets the following conditions. Use point-slope form.

$y - y_1 = m(x - x_1)$

5. slope = 3 and $(4, -2)$

$$y - 2 = 3(x - 4)$$

$$\boxed{y + 2 = 3(x - 4)}$$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

$$\boxed{y - 7 = -\frac{3}{2}(x + 5)}$$

7. $f(4) = -8$ and $f(-3) = 12$

$$m = \frac{12 - -8}{-3 - 4} = \frac{20}{-7}$$

$$\boxed{y - 12 = -\frac{20}{7}(x + 3)}$$

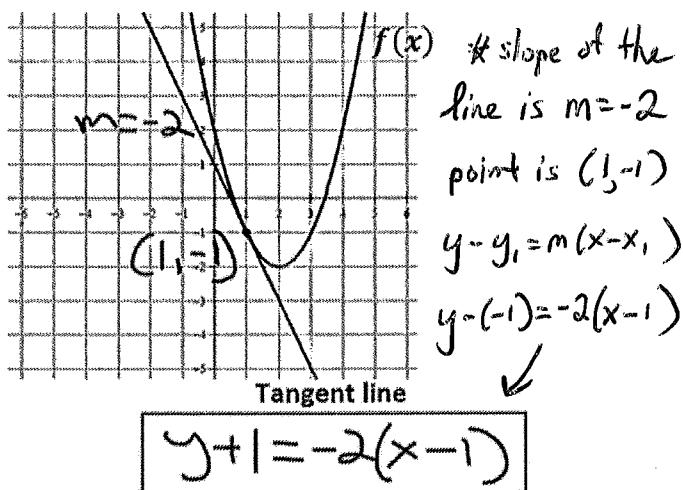
or

$$\boxed{y + 8 = -\frac{20}{7}(x - 4)}$$

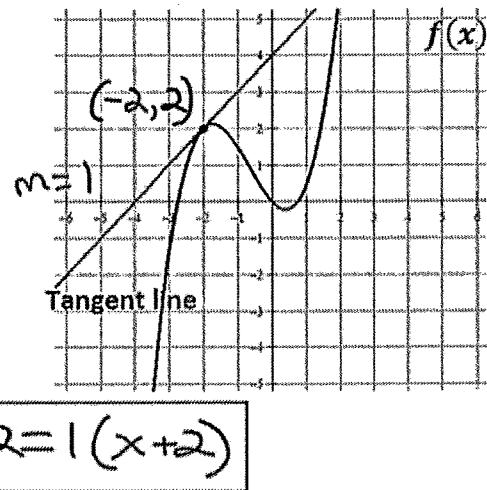
14

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



9. The line tangent to $f(x)$ at $x = -2$



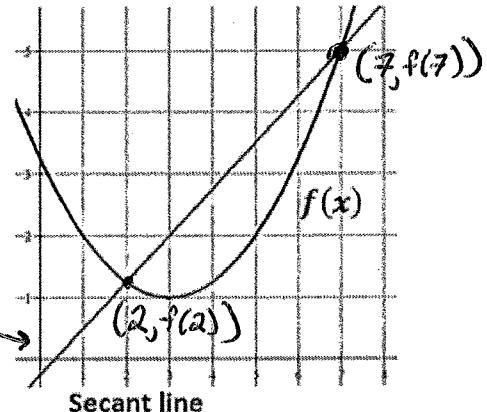
MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

*secant line is the line that intersects the curve at 2 points

Slope: $\frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{f(7) - f(2)}{7 - 2}$

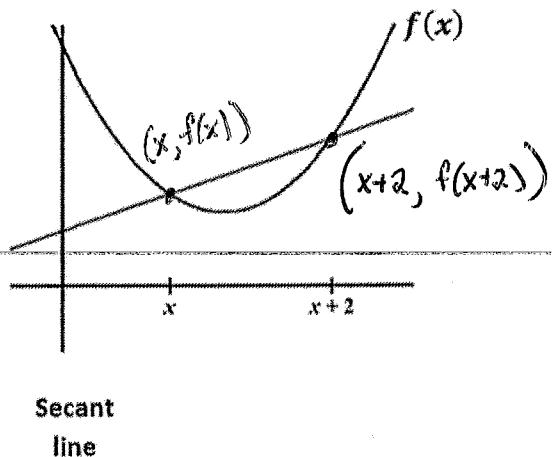


11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

Slope: $\frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{f(x+2) - f(x)}{x+2 - x}$



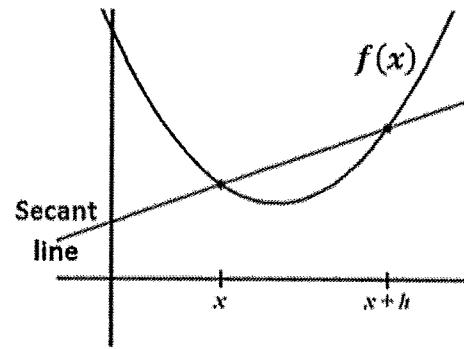
12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

C) $\frac{f(x+h)-f(x)}{x+h-x}$
()

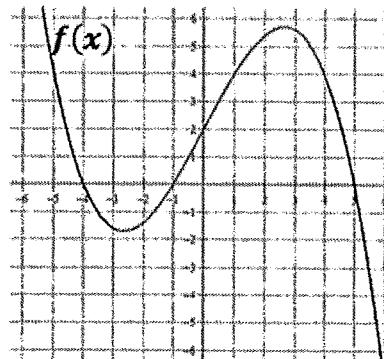
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

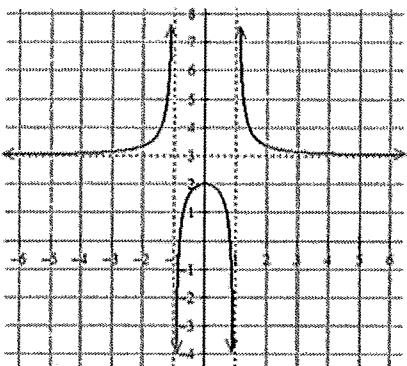
- I. $f(2) = 0$
- II. $(x + 4)$ is a factor of $f(x)$
- III. $f(5) = f(-1)$

- (A) I only
- (B) II only
- (C) III only
- (D) I and III only
- (E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range:

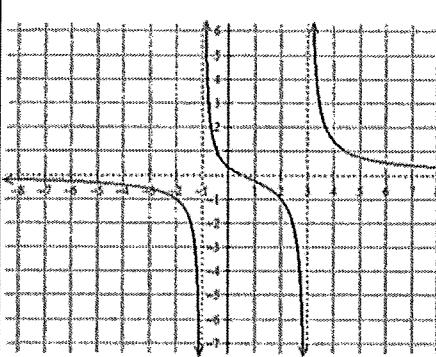
$$(-\infty, 2] \cup (3, \infty)$$

Horizontal Asymptote(s):

$$y = 3$$

Vertical Asymptotes(s): $x = 1$
 $x = -1$

15.



Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

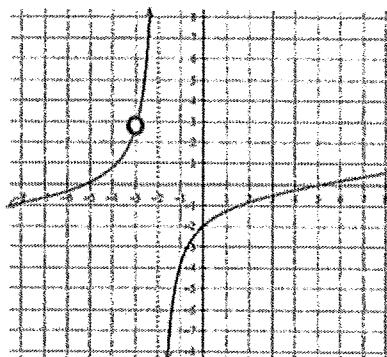
Range:

$$(-\infty, \infty)$$

Horizontal Asymptote(s): $y = 0$

Vertical Asymptotes(s): $x = -1$
 $x = 3$

16.



Domain: $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

Range:

$$(-\infty, \infty)$$

Horizontal Asymptote(s):

none

Vertical Asymptotes(s): $x = -2$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

* Find vertical asymptotes by setting denominator = 0

$$\frac{x+6}{(x-4)(x-3)}$$

$$x-4=0 \rightarrow x=4$$

$$x-3=0 \rightarrow x=3$$

Vertical at $x=3$ and $x=4$
Asymptotes

18. Consider the function: $f(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

- degrees match
so the ratio of
coefficients is 1
so horizontal asymptote at $y=1$
- I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$
- (A) I only
(B) II only
(C) I and III only
(D) II and III only

x -intercept
at $x=3$

$\frac{(x-2)(x-3)}{(x-2)(x+2)}$ ← VA at
 $x=-2$

hole at $x=2$

- (E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$
 $x^{\frac{3}{5}} + (2x)^{\frac{1}{5}}$

20. $\sqrt{x+1}$
 $(x+1)^{\frac{1}{2}}$

21. $\frac{1}{\sqrt{x+1}}$
 $(x+1)^{-\frac{1}{2}}$

22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$
 $x^{-\frac{1}{2}} - 2x^{-1}$

23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$
 $\frac{1}{4}x^{-3} + \frac{1}{2}x^{\frac{3}{4}}$

24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$
 $\frac{1}{4}x^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$
 $\frac{1}{\sqrt{x}} - \sqrt{x^3}$

26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$
 $\frac{1}{2\sqrt{x}} + \frac{1}{x}$

27. $3x^{-\frac{1}{2}}$
 $\frac{3}{\sqrt{x}}$

28. $(x+4)^{-\frac{1}{2}}$
 $\frac{1}{\sqrt{x+4}}$

29. $x^{-2} + x^{\frac{1}{2}}$
 $\frac{1}{x^2} + \sqrt{x}$

30. $2x^{-2} + \frac{3}{2}x^{-1}$
 $\frac{2}{x^2} + \frac{3}{2x}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin \frac{\pi}{6}$ $\frac{1}{2}$

32. $\cos \frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$

33. $\sin 2\pi$ 0

34. $\tan \pi$ 0

35. $\sec \frac{\pi}{2}$ undefined

36. $\cos \frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$

37. $\sin \frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$

38. $\sin \frac{3\pi}{2}$ -1

39. $\tan \frac{\pi}{4}$ 1

40. $\csc \frac{\pi}{2}$ 1

41. $\sin \pi$ 0

42. $\cos \frac{\pi}{3}$ $\frac{1}{2}$

 43. Find x where $0 \leq x \leq 2\pi$,

$$\sin x = \frac{1}{2}$$
 $\frac{\pi}{6} \text{ and } \frac{5\pi}{6}$

 44. Find x where $0 \leq x \leq 2\pi$,

$$\tan x = 0$$
 $0, \pi, \text{ and } 2\pi$

 45. Find x where $0 \leq x \leq 2\pi$,

$$\cos x = -1$$
 π

 Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$

$e^x = 1$

$\ln(e^x) = \ln(1)$

$x = 0$

47. $3e^x + 5 = 8$

$3e^x = 3$

$e^x = 1$

$\ln e^x = \ln 1$

$x = 0$

48. $e^{2x} = 1$

$\ln e^{2x} = \ln(1)$

$2x = 0$

$x = 0$

49. $\ln x = 0$

$e^x = e^0$

$x = 1$

50. $3 - \ln x = 3$

$-\ln x = 0$

$\ln x = 0$

$e^{\ln x} = e^0$

$x = 1$

51. $\ln(3x) = 0$

$e^{\ln(3x)} = e^0$

$3x = 1$

$x = \frac{1}{3}$

52. $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$

53. $e^x + xe^x = 0$

$e^x(1+x) = 0$

$e^x = 0 \quad 1+x = 0$

$\text{not possible} \quad x = -1$

54. $e^{2x} - e^x = 0$

$e^x(e^x - 1) = 0$

$e^x = 0 \quad e^x - 1 = 0$

$\text{not possible} \quad e^x = 1$

$x = 0$

18

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}$

56. $\cos x = -1$

$x = \pi$

57. $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} \text{ and } \frac{11\pi}{6}$

58. $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}$

59. $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} \text{ and } \frac{7\pi}{4}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{6} \quad \frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{\pi}{3}$

$x = \frac{11\pi}{3}$
Not in the domain interval

61. $\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$

$x = 0, \pi, 2\pi$

62. $\sin(2x) = 1$

$2x = \frac{\pi}{2} \text{ and } 2x = \frac{5\pi}{2}$

$x = \frac{\pi}{4} \text{ and } x = \frac{5\pi}{4}$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{4} = \frac{\pi}{3} \quad \frac{x}{4} = \frac{2\pi}{3}$

$x = \frac{4\pi}{3} \text{ and } x = \frac{8\pi}{3}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x - 4}$	$x \geq 4$	$y \geq 0$
65. $y = (x - 3)^2$	\mathbb{R} <small>real numbers</small>	$y \geq 0$
66. $y = \ln x$	$x > 0$	\mathbb{R}
67. $y = e^x$	\mathbb{R}	$y > 0$
68. $y = \sqrt{4 - x^2}$	$-2 \leq x \leq 2$	$0 \leq y \leq 2$

Simplify.

69. $\frac{\sqrt{x}}{x} \cdot \frac{x^{\frac{1}{2}-1}}{x^{-\frac{1}{2}}} = \frac{1}{\sqrt{x}}$

$$\boxed{\frac{1}{\sqrt{x}}}$$

70. $e^{\ln x}$

$$\boxed{x}$$

71. $e^{1+\ln x} = e^1 \cdot e^{\ln x}$

$$\boxed{ex}$$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

$$(\log_3 3)^{-1}$$

75. $\log_{1/2} 8$
 $\log_{1/2} (1/2)^3$

76. $\ln \frac{1}{2}$ Calculator needed

$$\approx -0.693$$

77. $27^{\frac{2}{3}}$
$$\sqrt[3]{27}^2$$

$$3^2$$

78. $(5a^{2/3})(4a^{3/2})$
 $20a^{\frac{3}{2} + \frac{3}{2}}$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-\frac{5}{3}}} = x^{\frac{1}{3}}y^{\frac{1}{3}}y^{-2} = x^{\frac{1}{3}}y^{-\frac{5}{3}}$

$$\frac{1}{3}x^{\frac{1}{3}}y^{\frac{1}{3}}$$

80. $(4a^{\frac{5}{3}})^{\frac{3}{2}}$

$$\sqrt[3]{4} a^{\frac{5}{3} \cdot \frac{3}{2}}$$

If $f(x) = \{(3, 5), (2, 4), (1, 7)\}$
 $h(x) = \{(3, 2), (4, 3), (1, 6)\}$

$g(x) = \sqrt{x-3}$, then determine each of the following.
 $k(x) = x^2 + 5$

81. $(f+h)(1)$

$$f(1) + h(1)$$

$$7 + 6$$

82. $(k-g)(5)$

$$k(5) - g(5)$$

$$(25+5) - (\sqrt{2})$$

$$30 - \sqrt{2}$$

83. $f(h(3))$

$$f(2)$$

84. $g(k(7))$

$$g(7^2 + 5)$$

$$g(54)$$

$$\sqrt{54-3} = \sqrt{51}$$

85. $h(3)$

86. $g(g(9))$

$$g(\sqrt{6})$$

$$\sqrt{\sqrt{6}-3}$$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

$$x = y^2 + 5$$

$$x - 5 = y^2$$

$$y = \sqrt{x-5}$$

89. $k(g(x)) = (\sqrt{x-3})^2 + 5$

$$x-3+5$$

$$x+2$$

90. $g(f(2))$

$$g(4) = \sqrt{4-3}$$

20

20

Precalculus Cheat Sheet

Basic Algebraic Properties

Arithmetic Operations

$$ab + ac = a(b + c)$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{ab + ac}{a} = b + c, \quad a \neq 0$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\begin{aligned} \left(\frac{a}{b}\right) &= \frac{ad}{bc} \\ \left(\frac{c}{d}\right) &= \frac{ad}{bc} \end{aligned}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$$

$$\sqrt[n]{a^n} = a, \text{ if } a \text{ is odd}$$

$$\sqrt[n]{a^n} = |a|, \quad \text{if } a \text{ is even}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Exponent Properties

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$a^0 = 1, a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0$$

$$|ab| = |a||b|$$

$$|a + b| \leq |a| + |b|$$

$$|-a| = |a|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

Distance Formula

If (x_1, y_1) & (x_2, y_2) are two points, the distance between them is:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1}, \quad i^2 = -1$$

$$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 - b^2$$

Complex Conjugate:

$$\overline{(a + bi)} = a - bi$$

Properties of Inequalities

If $a < b$, then $a + c < b + c$
and $a - c < b - c$.

If $a < b$ and $c > 0$, then

$$ac < bc \text{ and } \frac{a}{c} < \frac{b}{c}$$

If $a < b$ and $c < 0$, then

$$ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}$$

Properties of Logarithms

$$\log_a(a) = 1, \quad \ln(e) = 1$$

$$\log_a(a^x) = x, \quad \ln(e^x) = x$$

$$\log_a(1) = 0, \quad \ln(1) = 0$$

$$a^{\log_a(x)} = x, \quad e^{\ln(x)} = x$$

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\log_a(x^y) = y \log_a(x)$$

$$\ln(x^y) = y \ln(x)$$

Precalculus Cheat Sheet

Factoring and Solving

Factoring Formulas

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x - a)^3$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

Square Root Method

If $x^2 = k$, then $x = \pm\sqrt{k}$

Completing the Square

For $ax^2 + bx + c = 0$, $a \neq 1$:
Divide by the coefficient of x^2 ,
then follow the steps below.

For $x^2 + bx + c = 0$, $a = 1$:

1. Move constant to RHS:

$$x^2 + bx = -c$$

2. Take half the coefficient of x , square it, and add it to both sides:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

3. Factor the LHS:

$$\left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

4. Apply Square Root Method:

$$x + \frac{b}{2} = \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

5. Solve for x :

$$x = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

Quadratic Formula

If $ax^2 + bx + c = 0$, $a \neq 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, then:

Two distinct Real solutions.

If $b^2 - 4ac = 0$, then:

One Real solution.

If $b^2 - 4ac < 0$, then:

Two Complex solutions.

Absolute Value Equation

For some positive number a .

If $|x| = a$, then:

$$x = a \quad \text{or} \quad x = -a$$

Absolute Value Inequality

For some positive number a .

If $|x| \leq a$, then:

$$-a \leq x \leq a$$

If $|x| < a$, then:

$$-a < x < a$$

If $|x| \geq a$, then:

$$x \leq -a \quad \text{or} \quad x \geq a$$

If $|x| > a$, then:

$$x < -a \quad \text{or} \quad x > a$$

Solving Rational Expressions

1. Find the Least Common Denominator (LCD) amongst the rational expressions.

2. Make note of what x -values make the denominator zero.

3. Multiply both sides of the denominator by the LCD to clear the expression of all fractions.

4. Solve for x .

5. Check your solutions!

Solving Radical Expressions

1. Isolate the radical to one side of the equation.

2. Remove the radical by raising both sides of the equation to a power that is equal to the index of the radical.

3. If a radical still remains, repeat the previous two steps.

4. Solve for x .

5. Check your solutions!
Remember that even-root radicals are nonnegative:

If n is even, then $\sqrt[n]{x} > 0$

Finding an Inverse Function

If $f(x)$ is a 1-1 function:

1. Replace $f(x)$ with y .

2. Interchange x and y .

3. Solve for y .

4. Replace y with $f^{-1}(x)$.

5. Check your work!

- $f(x)$ & $f^{-1}(x)$ are symmetric about the diagonal line $y = x$.

- Domain/Range:

$$D_f = R_{f^{-1}}$$

$$R_f = D_{f^{-1}}$$

- Composite Property:

$$(f \circ f^{-1})(x) = x$$

$$(f^{-1} \circ f)(x) = x$$

Precalculus Cheat Sheet

Functions and Graphs

Circle

$$(x - x_o)^2 + (y - y_o)^2 = r^2$$

Graph is a circle with radius $|r|$ and center point (x_o, y_o)

Constant Function

$$y = f(x) = a$$

Graph is a horizontal line passing through the point $(a, 0)$

Line

$$y = f(x) = mx + b$$

Graph is a line with slope m and y -intercept $(0, b)$

Slope:

The slope of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-Intercept Form:

$$y = mx + b$$

The equation of the line with slope m and y -intercept $(0, b)$.

Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

The equation of the line with slope m and passing through the point (x_1, y_1) .

Quadratic Function 1

$$y = f(x) = ax^2 + bx + c$$

The graph is a parabola that opens upward if $a > 0$ or opens downward if $a < 0$.

Vertex:

$$(x, y) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

Quadratic Function 2

$$y = f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens upward if $a > 0$ or opens downward if $a < 0$.

Vertex:

$$(x, y) = (h, k)$$

Logarithms

Definition:

$$y = \log_a(x) \Leftrightarrow a^y = x$$

$$y = \ln(x) \Leftrightarrow e^y = x$$

Common Logarithm:

$$\log(x) = \log_{10}(x)$$

Natural Logarithm:

$$\ln(x) = \log_e(x), e \approx 2.7182..$$

Domain of $\log_a(x)$ & $\ln(x)$:
 $x > 0$

x-intercept:

$$\log_a(1) = 0, \quad \ln(1) = 0$$

Algebra with Functions

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Composition of Functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

Translations

For a function $y = f(x)$:

Vertical Shifts:

If $y = f(x) + k$, then:
Shift $y = f(x)$ up k units.

If $y = f(x) - k$, then:

Shift $y = f(x)$ down k units.

Horizontal Shifts:

If $y = f(x - h)$, then:
Shift $y = f(x)$ right h units.

If $y = f(x + h)$, then:

Shift $y = f(x)$ left h units.

Reflections:

If $y = -f(x)$, then:
Reflect $y = f(x)$ across x -axis.

If $y = f(-x)$, then:

Reflect $y = f(x)$ across y -axis.

If $y = -f(-x)$, then:

Reflect $y = f(x)$ across origin.

Stretching & Compressing

Vertically:

For a function $y = af(x)$.

If $|a| > 1$, then:

Vertically stretch $y = f(x)$ by a factor of a .

If $0 < |a| < 1$, then:

Vertically compress $y = f(x)$ by a factor of a .

Horizontally:

For a function $y = f(cx)$.

If $|c| > 1$, then:

Horizontally compress $y = f(x)$ by a factor of c .

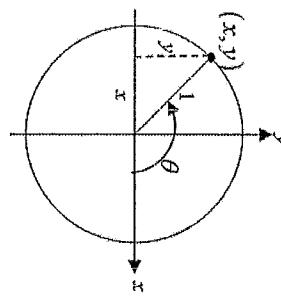
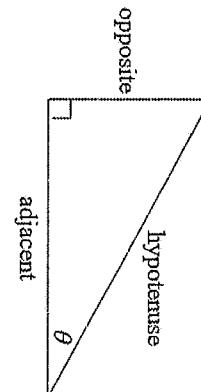
If $0 < |c| < 1$, then:

Horizontally stretch $y = f(x)$ by a factor of c .

Definition of the Trig Functions**Right triangle definition**

For this definition we assume that

$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$

**Unit Circle Definition**For this definition θ is any angle.

$$\begin{aligned}\sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc(\theta) &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec(\theta) &= \frac{1}{\cos(\theta)} \\ \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} & \cot(\theta) &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

Facts and Properties**Period**

The domain is all the values of θ that can be plugged into the function.

$\sin(\theta)$, θ can be any angle

$\cos(\theta)$, θ can be any angle

$\tan(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\sec(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\csc(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\cot(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\tan(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\sec(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\csc(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\cot(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

Range
The range is all possible values to get out of the function.

$$-1 \leq \sin(\theta) \leq 1$$

$$-1 \leq \cos(\theta) \leq 1$$

$$-\infty < \tan(\theta) < \infty$$

$$-\infty < \cot(\theta) < \infty$$

$$\sec(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1$$

$$\csc(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1$$

Formulas and Identities**Half Angle Formulas**

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2(\theta) = \frac{1}{1 - \cos(2\theta)}$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

$$\sec^2(\theta) = \csc^2(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \mp \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{1}{1 \mp \tan(\alpha)\tan(\beta)}$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

$$\sec^2(\theta) = \csc^2(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$\sin(2\theta) = 1 - 2\sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Product to Sum Formulas

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Formulas and Identities**Tangent and Cotangent Identities**

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

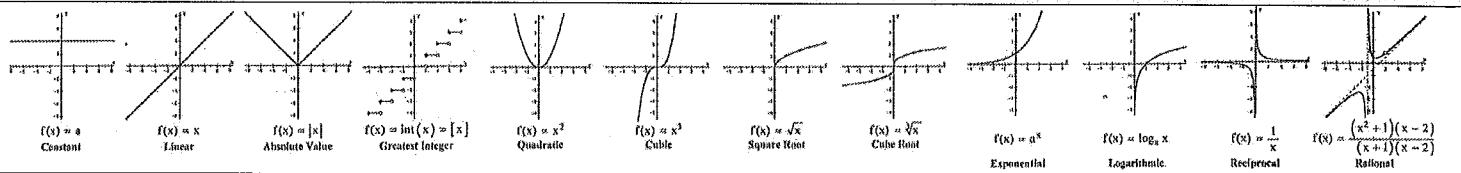
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

PRECALCULUS

UNIT 1 CHEAT SHEET

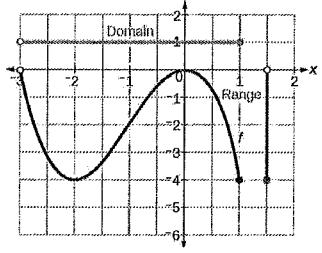
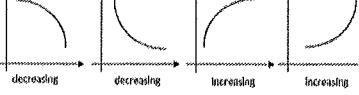
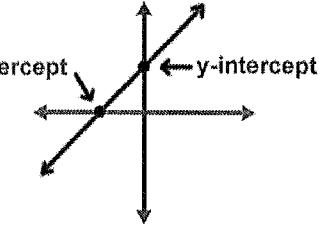
Parent Functions



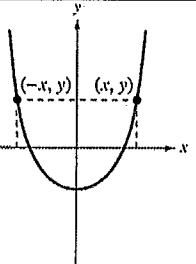
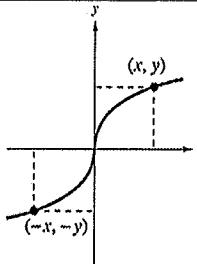
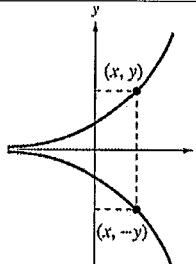
Transformations

Transformation	Function	Description	Transformation	Function	Description
Horizontal Shift	$f(x + h)$	Shift left h units		$a f(x)$, $a > 1$	Stretch vertically by a factor of a
	$f(x - h)$	Shift right h units		$a f(x)$, $0 < a < 1$	Compress vertically by a factor of a
Vertical Shift	$f(x) + k$	Shift up k units		$f(ax)$, $a > 1$	Compress horizontally by a factor of $\frac{1}{a}$
	$f(x) - k$	Shift down k units		$f(ax)$, $0 < a < 1$	Stretch horizontally by a factor of $\frac{1}{a}$
Reflection	$-f(x)$	Reflect across x-axis			
	$f(-x)$	Reflect across y-axis			

Key Features of Functions

Function Value	Domain/Range	Increasing/Decreasing	Y-intercept/Zeros/Roots
<p>Given $g(x) = 3x + 7$ Solve for $g(4)$</p> <p>Take the value inside parameters (the parenthesis) and plug in every time you see your variable $g(4) = 3(4) + 7$ $g(4) = 19$</p>	<p>Domain: values of X that the function can have Range: values of Y that the function can have</p> 	<p>Increasing (Directly Proportional): $as y \uparrow, then x \uparrow$ $as y \downarrow, then x \downarrow$</p> <p>Decreasing (Indirectly Proportional): $as y \downarrow, then x \uparrow$ $as y \uparrow, then x \downarrow$</p> 	<p>y-intercept: Where the function hits the y-axis. It should be in the form $(0, y)$</p> <p>zero: Where the function hits the x-axis. It should be in the form $(x, 0)$</p> 

Symmetry and Even/Odd

Property	Testing Symmetry						
 <p>Symmetric to y-axis Even function</p>  <p>Symmetric to origin Odd function</p>  <p>Symmetric to x-axis Not a function</p>	<p>Use the following test. If you return the EXACT same equation, it has symmetry "with respect to" the test</p> <table style="margin-left: 20px;"> <tr> <td>x-axis</td> <td>Replace $y \rightarrow -y$</td> </tr> <tr> <td>y-axis</td> <td>Replace $x \rightarrow -x$</td> </tr> <tr> <td>origin</td> <td>Replace $y \rightarrow -y$ Replace $x \rightarrow -x$</td> </tr> </table>	x-axis	Replace $y \rightarrow -y$	y-axis	Replace $x \rightarrow -x$	origin	Replace $y \rightarrow -y$ Replace $x \rightarrow -x$
x-axis	Replace $y \rightarrow -y$						
y-axis	Replace $x \rightarrow -x$						
origin	Replace $y \rightarrow -y$ Replace $x \rightarrow -x$						

PRECALCULUS

UNIT 1 CHEAT SHEET

Even/Odd/Neither

Change the sign of ALL "x".

$$f(-x)$$

Follow the test to see what your function is.

Even	Return original: $f(-x) = f(x)$
Odd	Return ALL signs changed: $f(-x) = -f(x)$
Neither	Return something funky

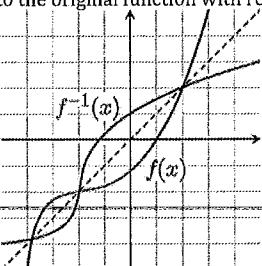
Compositions

Property	Blob Method										
The Composition Function $(f \circ g)(x) = f(g(x))$ <small>This is read "f composition g" and means to copy the f function down but where ever you see an x, substitute in the g function.</small> $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$ $f \circ g = 2(4x^3 + 1)^2 + 3$	<p>Given</p> <table border="0" style="width: 100%;"> <tr> <td style="width: 50%;">$f(x) = 3x + 7$</td> <td style="width: 50%;">$g(x) = x^2 - x$</td> </tr> </table> <p>Solve for $[g \circ f](x)$</p> <table border="0" style="width: 100%;"> <tr> <td style="width: 50%; text-align: right; padding-right: 10px;">First Term</td> <td>BLOB</td> </tr> <tr> <td></td> <td>$g(x) = \bullet^2 - \bullet$</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">Second Term</td> <td>PARENTHESIS</td> </tr> <tr> <td></td> <td>$f(x) = (3x + 7)$</td> </tr> </table> <p>Now, follow the first term. Copy everything down exactly as you see it, UNTIL you get to a blob. When you see a blob, copy down a parenthesis AND everything inside the parenthesis</p> $[g \circ f](x) = (3x + 7)^2 - (3x + 7)$	$f(x) = 3x + 7$	$g(x) = x^2 - x$	First Term	BLOB		$g(x) = \bullet^2 - \bullet$	Second Term	PARENTHESIS		$f(x) = (3x + 7)$
$f(x) = 3x + 7$	$g(x) = x^2 - x$										
First Term	BLOB										
	$g(x) = \bullet^2 - \bullet$										
Second Term	PARENTHESIS										
	$f(x) = (3x + 7)$										

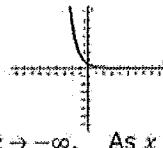
Real World Compositions

Procedures	Creating Equations from Word Problems
1. Create at least 2 equations from the word problem 2. Using your equations, perform compositions (both ways) 3. Plug in and solve! (remember, to check if you are solving for x or y)	http://goo.gl/2SEGco

Inverses

What is an inverse?	Solving inverses
An inverse can pass the horizontal line test. An inverse is symmetric to the original function with respect to the line $y = x$. 	Algebraic: 1. Change your function name to "y" 2. Switch you "y" and "x" 3. Solve for new "y" Graphical: 1. Make a table and put function coordinate points in 2. Switch you "x" and "y" in the coordinate points 3. Plot points and connect dots 4. Is it symmetric with respect to $y = x$?

Continuity, Limits, End Behavior

Continuity	Limits	End Behavior
Continuity – can I draw the function without picking up my pencil? My limit EQUALS my function at that point $\lim_{x \rightarrow c} f(x) = f(c)$ 1. Test the limit from the left 2. Test the limit from the right 3. Does the limit exist? 4. Test the function value 5. Does the limit equal the function?	Limits – APPROACH! Limits are NOT functions The limit from the left must equal the limit from the right for the limit to exist $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$	End Behavior – What is the graph doing at the ends of the graph! Use limit notation  As $x \rightarrow -\infty$, $y \rightarrow \infty$, As $x \rightarrow \infty$, $y \rightarrow 0$.