Accelerated Pre-Calculus

January & February 2022

Unit 6 – Vectors

Monday	Tuesday	Wednesday	Thursday	Friday
25 Applications with Matrices	25 Unit 5 Review	26 Unit 5 Review	27 Unit 5 Test- Matrices	28 6.01 Review Right Triangle Trig HW: 6.01 Practice
31 6.02 Introduction to Geometric Vectors • Geometric Vectors HW: 6.02 Practice	Feb 1 6.03 More Geometric Vectors • Vector Game • Direction HW: 6.03 Practice	2 6.04 Geometric and Algebraic Vectors Harry Potter Task HW: Finish 6.04 Task	3 6.05 Algebraic Vectors • Component Form • Operations • Unit Vectors HW: 6.05 Practice	4 6.06 More Algebraic Vectors • Magnitude • Direction HW 6.06 Practice
7 6.07 Angle with Vectors • Angle Between Vectors • Orthogonal Vectors • Parallel Vectors HW: 6.07 Practice	8 6.08 Quiz Review HW: Study for Quiz	9 **6.09 Quiz** **Vector Operations**	10 6.10 Bearings HW: Finish 6.10	11 6.11 Vector Applications Day 1 HW: 6.11 Application Practice Worksheet
14 6.12 Vector Applications Day 2 HW: 6.12 Application Day 2 Practice Wkst and Test Review	15 6.13 Test Review HW: 6.13 Test Review Worksheet	16 Test Review	17 **TEST: Vectors in 2 dimensions**	18

Vectors

2D Vectors: $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

1. **Component form** shows the vector from the *initial point* to the *terminal point* based on the <u>displacement</u> of its dimensional values:
   - 2D vector, from (x_1, y_1) to (x_2, y_2): $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

2. **Unit vector** is a vector of length 1. The standard unit vectors are $\vec{\imath} = \langle 1,0 \rangle$ and $\vec{\jmath} = \langle 0,1 \rangle$. A vector can be written as the *sum of unit vectors* by using its components as scalars of standard unit vectors:
   - 2D vector: $\vec{v} = a\vec{\imath} + b\vec{\jmath}$

3. **Magnitude** (length) of a vector:
   - 2D vector: $|\vec{v}| = \sqrt{a^2 + b^2}$

4. **Direction** of a vector:
   - 2D vector: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, add 180° if in quadrants 2 or 3.

5. Given the **magnitude** and the **direction** of a vector, it is possible to determine its components:
   - 2D vector with magnitude $|\vec{v}|$ and direction θ, $\vec{v} = |\vec{v}| \langle \cos\theta, \sin\theta \rangle = \langle |\vec{v}| \cos\theta, |\vec{v}| \sin\theta \rangle$

6. **Resultant vector** is the sum of two or more vectors.
   - Geometrically, this is shown with the *tip-to-tail* method, also known as the *triangle* method. The *parallelogram* method also can determine the resultant vector.
   - Algebraically, this is calculated by finding the sum of the corresponding components.
   - 2D vectors: $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$

7. **Scalar multiplication**:
   - 2D vector: $k\vec{v} = \langle ka, kb \rangle$

8. **Dot product** (inner product) is used to determine if two vectors are perpendicular:
   - 2D vectors: $\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$
   - For magnitude: $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$
   - 2 vectors are orthogonal (perpendicular) if their dot product equals 0.

9. **Angle between two vectors** can be found with a dot product:
   - 2D vectors: $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$

10. **Angles** have different ways of being measured:
    - **Standard Position** is measured from the positive x-axis, with positive angles opening counter-clockwise.
    - **True Bearing** or **Compass Bearing** is measured from North, with positive angles opening clockwise.
      True bearing measurement = 450° − Standard position measurement
      Standard position measurement = 450° − True bearing measurement
    - **Quadrant Bearing** is measured either from North or from South, opening toward East or toward West in such a way that the angle value is always acute.