Name:____ **Period:** __________

Accel. Pre-Calculus

Unit 12 Packet

Trig Review

May 2022 Units 12Trig Review				
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Trigonometric Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities:

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Even/Odd Identities:

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

Sum & Difference Identities:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double-Angle Identities:

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

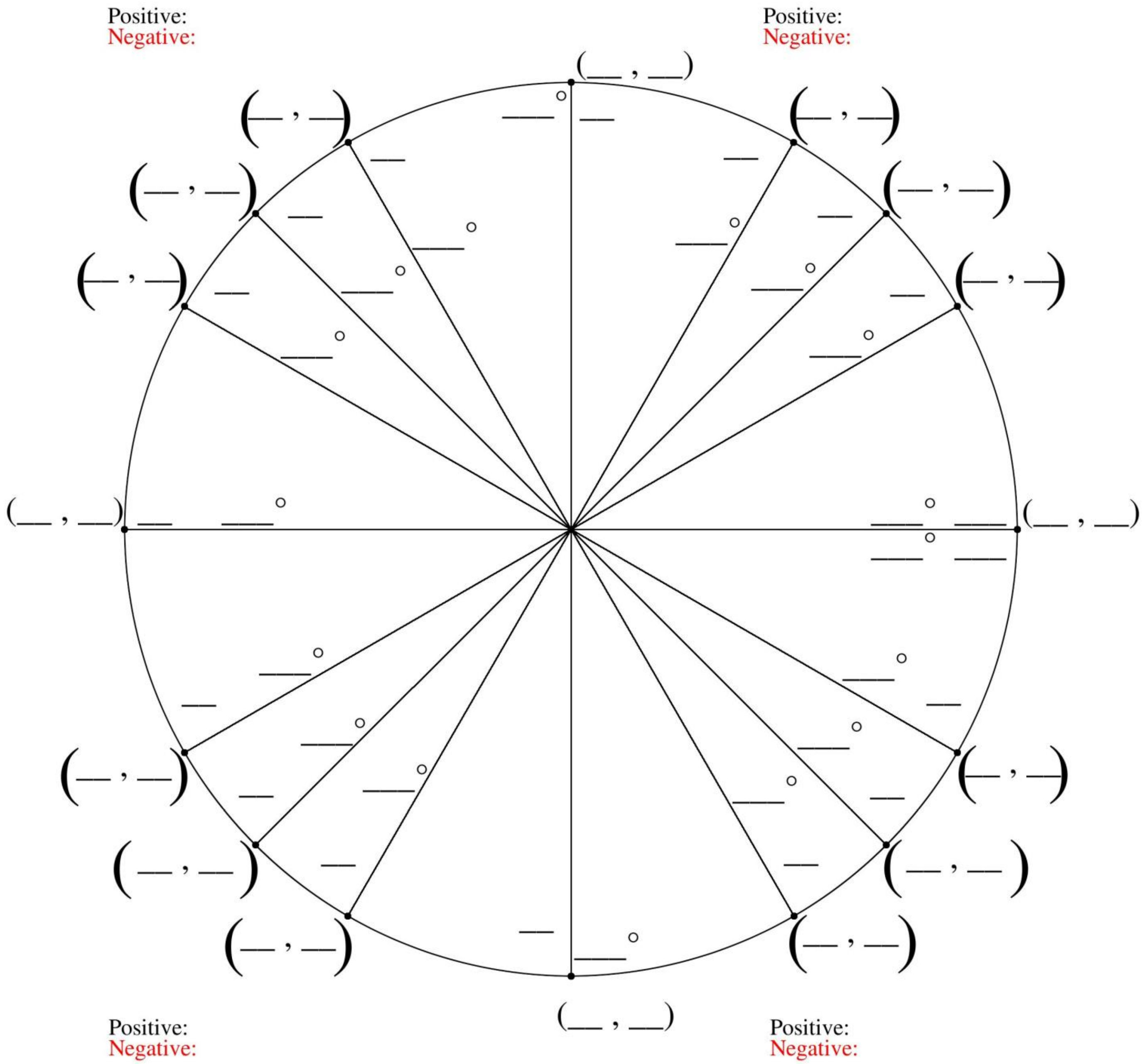
$= 1 - 2\sin^2\theta$

Half-Angle Identities:

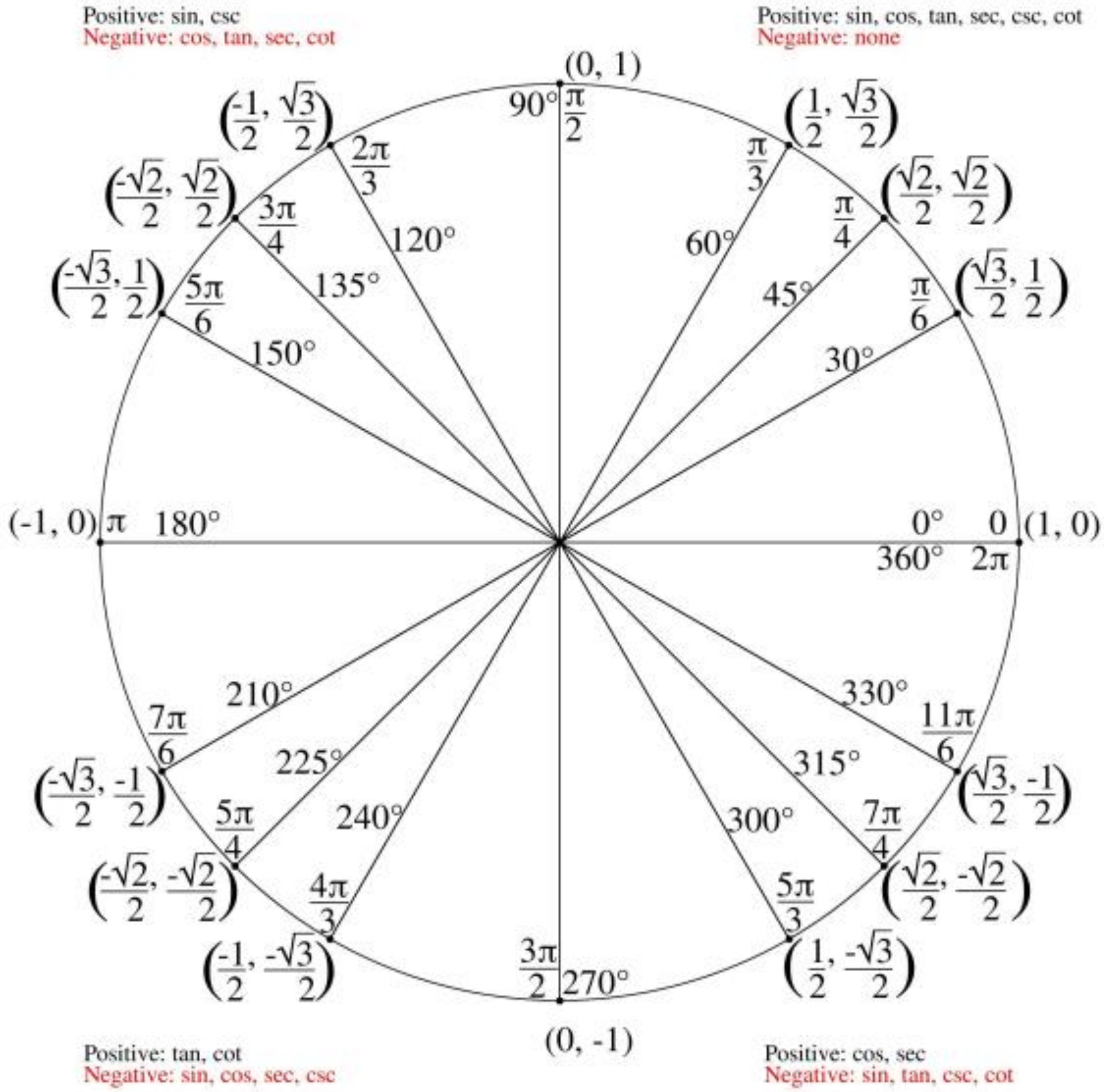
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Fill in The Unit Circle



The Unit Circle



Accelerated Precalculus
Investigation: What is a *radian*?

I. Lab Activity

Materials for each group of 3 students:

- One length of string
- Ruler (metric)
- Pencil/Pen

Steps:

1. Tie the string around a pencil. Tie a knot at the other end of the string.
2. Draw a point on the paper.
3. Have one student hold the knotted end of the string on the point drawn and another student hold the pencil.
4. With the string pulled taut, carefully draw a circle around the point.
5. Mark a point on the circumference of the circle.
6. Draw a radius connecting the point in step #5 to the center point.
7. Place one end of the string on the point in step #5. Wrap the string along the circle. Mark another point on the circle at the other end of the string. You have now marked off a length equal to one radius along the circumference
8. Now move the string so that it starts at the point found in #7 and wraps further around the circle. Mark the end of the string.
9. Continue mapping the length of the string around the circumference of the circle making sure that you mark the end of the string as you go.
10. Draw radii from the center of the circle to each of the marks that you made around the circumference of the circle. Label these angles, starting with angle #1, then angle #2, and so on.

Observations about the Circle & the Angles

a. How many of your angles are approximately the same measure? ____________

b. If you were to cut the circle in half (having only 180°), how many of these congruent angles can be completely (no partial angles) drawn within the half circle? ____________

c. Again thinking of only a half circle, how many of these congruent angles would it take to completely fill the half circle? (A decimal answer is appropriate here.) ____________

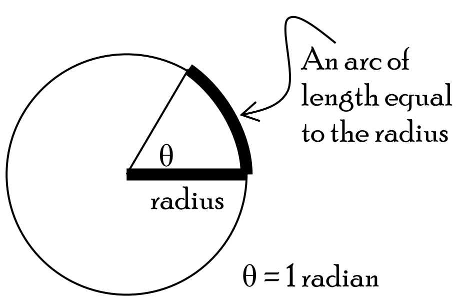
d. With your previous observation in mind, how many of these same angles could you then draw within the entire circle? ____________

II. Notes

A new *unit of measurement* for angles: a **radian** is the measure of an angle in a circle whose intercepted arc has a length equal to the radius.

Remember the formula relating the length of a radius of a circle to the circumference of the circle:

C = ____________________



About the center of a circle is _________ degrees.
Also about the center of a circle is _________ radians.

Therefore: _______ ° is equivalent to _______ radians

Or simplified, _______ ° = _______ **radians**
(This is our conversion unit!)

III. Practice

Plot the approximate location of each radian angle on the circle to the right, measured in standard position.

1. $\frac{7\pi}{6}$ radians

2. $\frac{11\pi}{3}$ radians

3. $-\frac{5\pi}{4}$ radians

4. $-\frac{3\pi}{2}$ radians

5. $-\frac{13\pi}{12}$ radians

6. 1 radian

7. 4.5 radians

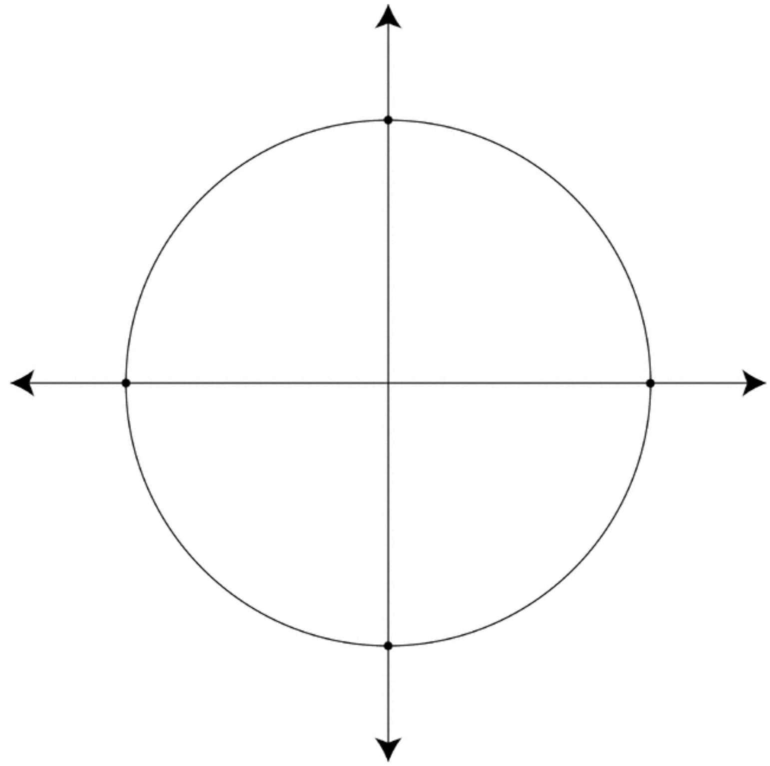
8. -2 radians

9. -1.2 radians

10. $\frac{\pi}{10}$ radians

11. $\frac{5\pi}{18}$ radians

12. $-\frac{8\pi}{15}$ radians



Convert each of the radian measurements above (#1 – 12) into degree measurements.

1b.

2b.

3b.

4b.

5b.

6b.

7b.

8b.

9b.

10b.

11b.

12b.

How were your approximations of the locations of the angles?

Accelerated Pre-Calculus Date: ________________

11.01: Building the Unit Circle

Unit Circle ____

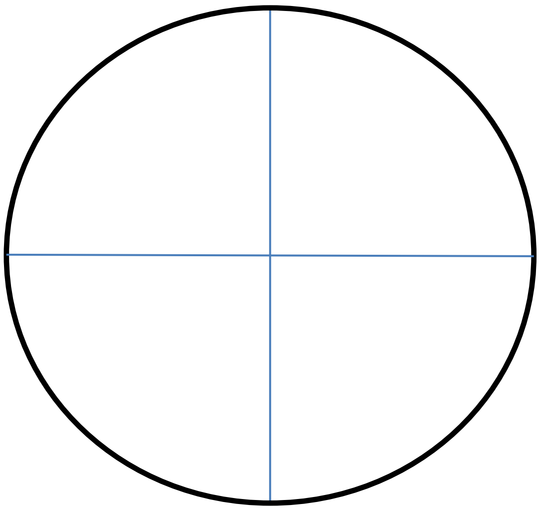
1) Draw a radius from the origin to a point on the <u>unit</u> circle in QI.

What is the length of the radius? ________ Why? _____________________________

2) Draw a line from that point perpendicular to the x-axis. What is the result? __________________

What are the coordinates of the point (and every point) on the circle? ______________

3) Label the reference angle, ϴ.



Define the following in terms of x and y:

sin ϴ = ______ cos ϴ = ______ tan ϴ = ______

csc ϴ = ______ sec ϴ = ______ cot ϴ =______

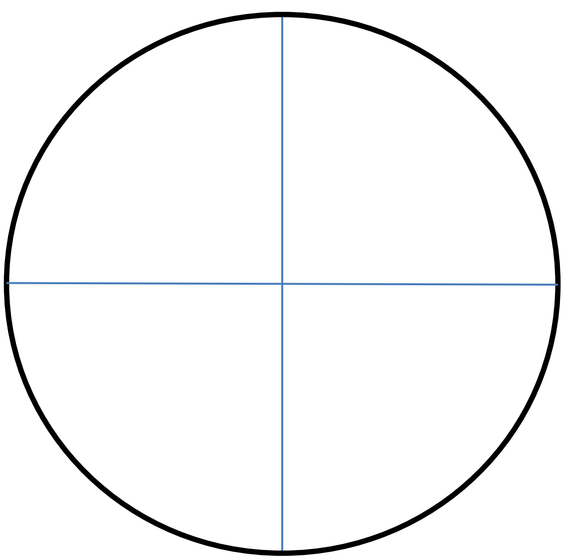
Given that (x, y) represents (cosϴ, sinϴ), use the coordinates to answer the following:

sin 90° = _____ tan $\frac{\pi}{2}$ = ______ cos π = ______

tan 270° =_____ tan 180° = _____ sec 2π = _____

Let's take a look at the unit circle a special triangle, 45 – 45 – 90.

Now draw ϴ = 45° on the unit circle. What is 45° in radians? ______



The coordinates for 45° are: _______________

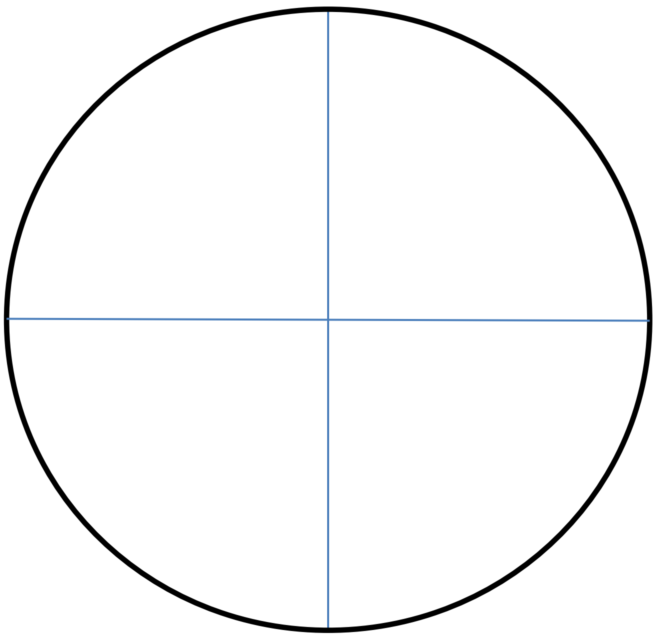
sin 45° =_______ cos 45° =_______

csc 45° =_______ sec 45° =_______

tan 45° =_______ cot 45° =_______

Let’s take a look at the unit circle and another special triangle, 30 – 60 – 90.

Now draw ϴ = 30° on the unit circle. What is 30° in radians? ______



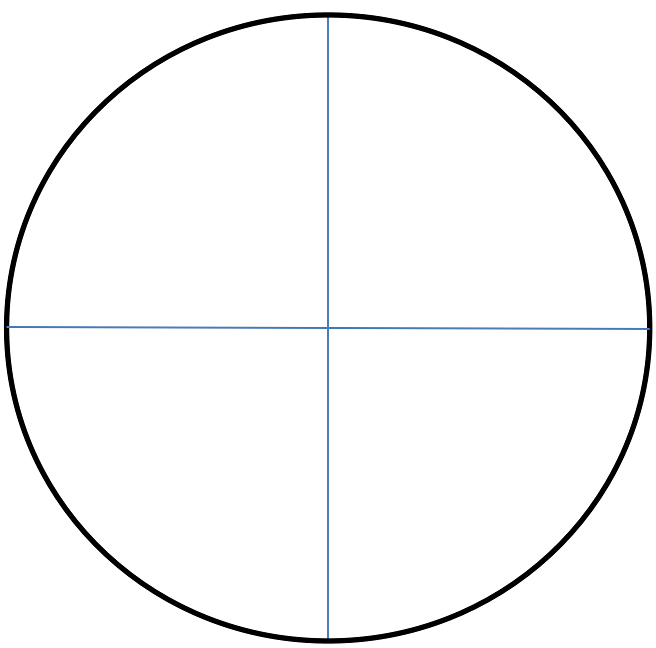
The coordinates for 30° are: ________________

sin 30° = _______ cos 30° = _______

csc 30° = _______ sec 30° = _______

tan 30° = _______ cot 30° = _______

Now draw ϴ = 60° on the unit circle. What is 60° in radians? ______



The coordinates for 60° are: ________________

sin 60° = _______ cos 60° = _______

csc 60° = _______ sec 60° = _______

tan 60° = _______ cot 60° = _______

Accelerated Precalculus **Date: ______________________**

HW: Unit Circle Trigonometry

Simplify the following using what you know about trigonometry and the unit circle.

1. $\cos\frac{3\pi}{4}$ **2.** $\cos 0$ **3.** $\sin\frac{2\pi}{3}$ **4.** $\sin\frac{11\pi}{6}$

5. $\cos\frac{7\pi}{6}$ **6.** $\cos\frac{5\pi}{3}$ **7.** $\sin\pi$ **8.** $\sin\frac{5\pi}{4}$

9. $\sec\frac{7\pi}{4}$ **10.** $\sec\frac{\pi}{2}$ **11.** $\csc\frac{4\pi}{3}$ **12.** $\csc\frac{5\pi}{6}$

13. $\sec\frac{\pi}{6}$ **14.** $\sec\frac{2\pi}{3}$ **15.** $\csc\frac{3\pi}{2}$ **16.** $\csc\frac{3\pi}{4}$

17. $\tan\frac{5\pi}{4}$ **18.** $\tan\frac{2\pi}{3}$ **19.** $\cot\frac{11\pi}{6}$ **20.** $\cot 0$

21. $\tan\frac{5\pi}{6}$ **22.** $\tan\pi$ **23.** $\cot\frac{3\pi}{4}$ **24.** $\cot\frac{4\pi}{3}$

25. $\csc\frac{5\pi}{3}$ **26.** $\tan\frac{3\pi}{2}$ **27.** $\cot\frac{2\pi}{3}$ **28.** $\sec\frac{5\pi}{4}$

State all angles in the interval $[\mathbf{0}, \mathbf{2\pi})$ that solve each of the following equations.

29. $\cos\theta = \frac{1}{2}$

30. $\cos\theta = -\frac{\sqrt{2}}{2}$

31. $\sin\theta = -\frac{1}{2}$

32. $\sin\theta = 1$

33. $\cos\theta = 0$

34. $\cos\theta = -\frac{\sqrt{3}}{2}$

35. $\sin\theta = \frac{\sqrt{2}}{2}$

36. $\sin\theta = \frac{\sqrt{3}}{2}$

37. $\sec\theta = -1$

38. $\sec\theta = -2$

39. $\csc\theta = -\frac{2\sqrt{3}}{3}$

40. $\csc\theta = -\sqrt{2}$

41. $\tan\theta = 1$

42. $\tan\theta = -\sqrt{3}$

43. $\cot\theta = \frac{\sqrt{3}}{3}$

44. $\cot\theta = 0$

45. $\sin\theta = 5$

46. $\csc\theta = 0$

Accelerated Precalculus **Date: ______________________**

Unit Circle Trigonometry & Coterminal Angles

State two co-terminal angles, one positive and one negative, with the given angle.

a. $\frac{7\pi}{10}$ **b.** $-\frac{2\pi}{9}$ **c.** $\frac{21\pi}{5}$ **d.** $-\frac{17\pi}{7}$

State the co-terminal angle between $[0, 2\pi)$ with the given angle.

e. $\frac{41\pi}{3}$ **f.** $-\frac{31\pi}{8}$ **g.** $\frac{59\pi}{10}$ **h.** $-\frac{33\pi}{7}$

Simplify the following using what you know about trigonometry, the unit circle, and coterminal angles.

1. $\cos\frac{17\pi}{3}$ **2.** $\cos 6\pi$ **3.** $\sin\frac{19\pi}{6}$ **4.** $\sin\frac{21\pi}{4}$

5. $\cos\left(-\frac{17\pi}{6}\right)$ **6.** $\cos\left(-\frac{5\pi}{3}\right)$ **7.** $\sin(-7\pi)$ **8.** $\sin\left(-\frac{23\pi}{3}\right)$

9. $\sec\frac{17\pi}{4}$ **10.** $\sec\frac{9\pi}{2}$ **11.** $\csc\frac{14\pi}{3}$ **12.** $\csc\frac{25\pi}{6}$

13. $\sec\left(-\frac{13\pi}{6}\right)$ **14.** $\sec\left(-\frac{17\pi}{3}\right)$ **15.** $\csc\left(-\frac{5\pi}{2}\right)$ **16.** $\csc\left(-\frac{3\pi}{4}\right)$

17. $\tan\frac{15\pi}{4}$ **18.** $\tan\frac{29\pi}{6}$ **19.** $\cot\frac{11\pi}{3}$ **20.** $\cot 8\pi$

21. $\tan\left(-\frac{17\pi}{6}\right)$ **22.** $\tan(-9\pi)$ **23.** $\cot\left(-\frac{9\pi}{4}\right)$ **24.** $\cot\left(-\frac{14\pi}{3}\right)$

State all angles that solve each of the following equations.

25. $\cos\theta = \frac{\sqrt{3}}{2}$

26. $\cos\theta = -\frac{\sqrt{3}}{2}$

27. $\sin\theta = \frac{1}{2}$

28. $\sin\theta = -1$

29. $\cos\theta = 1$

30. $\cos\theta = -\frac{1}{2}$

31. $\sin\theta = -\frac{\sqrt{2}}{2}$

32. $\sin\theta = -\frac{\sqrt{3}}{2}$

33. $\sec\theta = 1$

34. $\sec\theta = 2$

35. $\csc\theta = \frac{2\sqrt{3}}{3}$

36. $\csc\theta = \sqrt{2}$

37. $\tan\theta = -1$

38. $\tan\theta = \sqrt{3}$

39. $\cot\theta = -\frac{\sqrt{3}}{3}$

40. $\csc\theta = 1$

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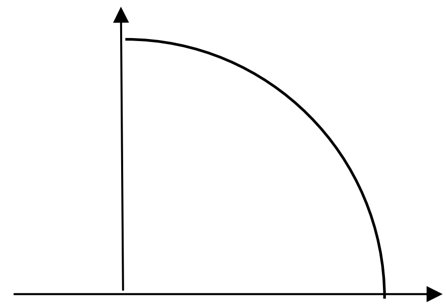
Pythagorean Identities



Use the following to develop the Pythagorean Identities.

1a. On the arc mark a point and label it (x, y).

b. Connect the origin to your point.

c. From your point draw a segment perpendicular to the x-axis, forming a right triangle.

d. Given that the radius = 1, use the Pythagorean Theorem to write an equation representing the sides of the triangle.

____

2a. Using your equation from #1d, substitute *x* and *y* with cos ϴ and sin ϴ, respectively. This is one of the Pythagorean Identities.

*________________________________

b. Solve for $\sin^2 \Theta$, and write this new form: ________________________________

c. Solve for $\cos^2 \Theta$, and write this form: ________________________________

3a. Using your equation from #2, divide **each term** by $\cos^2 \Theta$.

b. Simplify. Your equation should *not* contain any fractions. This is another Pythagorean Identity.

* ________________________________

c. Give 2 additional forms (see #2b-c): _____________________________ _________________________

4a. Using your equation again from #2, divide **each term** by $\sin^2 \Theta$.

b. Simplify. Your equation should *not* contain any fractions. This is also a Pythagorean Identity.

* _______________________________

c. Give 2 additional forms: _____________________________ __________________________

<u>Simplify each, using a Pythagorean Identity.</u>

1. $\frac{1+\tan^2 x}{\csc^2 x}$

2. $\frac{1-\cos^2 x}{\sin x}$

3. $\sin x - \sin^3 x$

4. $\cos^3 x + \cos x \sin^2 x$

5. $\frac{1-\sin^2 x}{\csc^2 x-1}$

6. $\frac{\sin^2 x+\tan^2 x+\cos^2 x}{\sec x}$

7. $\frac{\cot^2 x}{1+\csc x}$

8. $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$

9. $\frac{(\sec x+1)(\sec x-1)}{\sin^2 x}$

10. $\frac{1-\sin^2 x}{1-\cos^2 x} * \tan x$

<u>Answer Bank</u>: sinx cosx $\sin^2$x $\tan^2$x cotx

$\sec^2$x cscx – 1 sinx$\cos^2$x 2 secx

Accelerated Pre-Calculus Date: ____________________

Double Angle Identities

Double angle identities: sin 2θ, cos 2θ, and tan 2θ. Where do they come from? **Derive each.**

For sin 2θ, use the angle sum formula: sin (x + y) = sin x cos y + cos x sin y

***sin 2θ = ________________________**

For cos 2θ, use the angle sum formula: cos (x + y) = cos x cos y – sin x sin y

***cos 2θ = ________________________**

There are *two additional forms* for cos 2θ.
To find one, replace $\cos^2\theta$ with **$1-\sin^2\theta$**. **or cos 2θ = ________________________**

To find the other, replace $\sin^2\theta$ with **$1-\cos^2\theta$**. **or cos 2θ = ________________________**

For tan 2θ, use the angle sum formula: $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

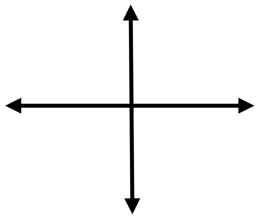
***tan 2θ = ________________________**

Practice

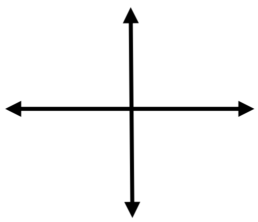
a. Draw and label the reference triangle in the appropriate quadrant for the given trig information about theta. (You will need to determine the length of the third side of the triangle.)

b. Find the exact values of sin 2θ, cos 2θ, and tan 2θ using the double-angle formulas.

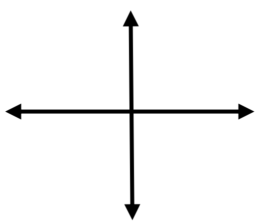
1. $\sin \theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$



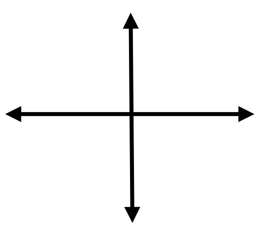
2. $\tan \theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$



3. $\cos \theta = \frac{15}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

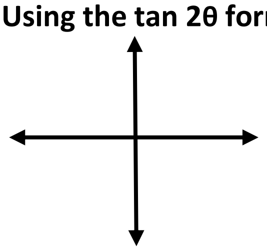


4. $\tan \theta = -2$ and $\cos \theta > 0$



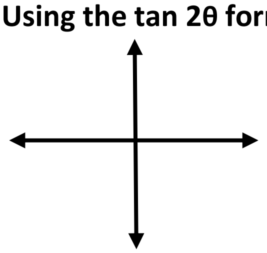
5. $\sin \theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$

Using the tan 2θ formula, find <u>only</u> tan 2θ



6. $\sin \theta < 0$ and $\cos \theta = -\frac{7}{25}$

Using the tan 2θ formula, find <u>only</u> tan 2θ



Accelerated Pre-Calculus

Name ___________________________________

Solving Trig Equations Review

Date ______________________ Period _______

Solve each equation for the variable on the interval from [0, 2π). Use Pythagorean and/or Double Angle Identities to rewrite as needed.

1. $2\sin^2 x = 1$

2. $2\sin^2 x + 3\sin x + 1 = 0$

3. $\sin^2 x + 1 = \cos x$

4. $\sec^2 x + \tan x - 1 = 0$

5. $\cot^2 x - \csc x = 1$

6. $5\sin x - 5 = \cos^2 x$

7. $\tan^2 x + 3\sec x + 3 = 0$

8. $3\sin x = 2\cos^2 x - 3\cot x\tan x$

9. $\cos 2\theta - 1 = 0$

10. $\sin 2\theta - \cos\theta = 0$

11. $\cos 2\theta + \cos\theta = 0$

12. $\sin\theta = \cos 2\theta - 1$

13. $\sin 2\theta - \tan\theta = 0$

14. $5\sin\theta - 2\cos^2\theta = 1$

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Quiz: Trigonometry Review Date: ________________ Period: _____

Evaluate.

1. $\tan\frac{4\pi}{3}$

2. $\sec\frac{3\pi}{4}$

3. $\sin\left(-\frac{5\pi}{6}\right)$

4. $\csc\frac{5\pi}{3}$

5. $\cot(-\pi)$

6. $\cos\frac{23\pi}{4}$

7. $\cot\frac{13\pi}{3}$

8. $\csc\left(-\frac{\pi}{2}\right)$

9. $\sec\frac{7\pi}{2}$

10. If $\sec\theta = -\frac{13}{12}$ and $\tan\theta > 0$, find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.

Solve each equation for the variable on the interval from $[\mathbf{0}, \mathbf{2\pi})$.

11. $4\cos^2\theta - 4\cos\theta = 3$

12. $1 = \cot^2\theta + \csc\theta$

13. $\sin 2x \cos x = \sin x$