Summer Packet
AP CALCULUS BC

Success in AP Calculus BC is predicated on a solid and functional knowledge of all the Algebra, Precalculus and Trigonometry topics and skills that you have learnt in your previous math classes. Due to the scope of the AP Calculus curriculum there is very little to review/reteach previous skills; therefore, as you enter AP Calculus BC these topics are treated as assumed knowledge since you have already taken and passed the prerequisite classes. This packet is a compilation of questions intended to refresh your Algebra and Precalculus knowledge so you start the year well prepared for the challenging AP Calculus BC curriculum this year.

This packet is **NOT MANDATORY** however it is **STRONGLY RECOMMENDED** that you do complete it over the summer.

You will have a test on the contents of this packet on **Monday August 19th**.

We will be spending the first 3 days of the school year reviewing the contents of the packet so completing the packet over will allow you to determine the topics that you feel weakest on and need help with.

Furthermore, if you turn in a FULLY COMPLETED packet on Monday August 19th you will earn an extra 15% on your test (up to a maximum of 100%)

If you elect to do this packet you must follow the following format:

1. All work must be done in **INK** (black or blue pen.)
2. Questions must be submitted in numerical order.
3. You must show all work/reasoning, including any MC Questions.
4. You must leave at least one blank line between each question.
5. You may not do two columns of solutions on the same side of paper
6. You must not work with/copy from other students.

- Wherever should be solved algebraically **without a calculator**, leaving your answer in exact form.
- If a calculator is needed those answers should be accurate to **AT LEAST 3 decimal places.**
- All work must be shown to receive credit and all answers should be fully simplified. Some problems might not require a great deal of work in order to arrive at an answer; that's okay. Just show whatever you need to solve the problem.
- All intervals must be written in interval notation. For example, if your answer is $x \geq 2$, you need to write $[2, \infty)$. A great site for learning interval notation is: http://id.mind.net/~zona/mmts/miscellaneousMath/intervalNotation/intervalNotation.html

- All problems should be completed on <u>plain white paper</u>, not notebook paper, and certainly not written on this printout. You may use graph paper for the graphs if you wish but it's not required. Make sure all problems are labeled and easy to find.

- This handout should be turned in along with all of your solutions, stapled together. The <u>honor code must be signed</u> for the project to be accepted. Forgetting to sign the honor code is a loss of 5 points.

- Please write down on a separate sheet of paper any problems you had trouble with because you will have two days in class to ask any questions related to this project (not to work on it, just to go over any material you need help with) and then you will be tested on this material on the following day. Any problem in this project is subject to being on the first test.

- Don't forget that even though you will not have your textbooks over the summer, the internet has some great resources. A few good sites are listed below.
  <u>http://library.thinkquest.org/20991/alg2/index.html</u>   <u>http://www.khanacademy.com</u>
  <u>http://www.coolmath.com/algebra</u>   <u>http://www.brightstorm.com</u>
  <u>http://www.purplemath.com</u>

- A good site for learning about piecewise functions is:
  <u>http://www.analyzemath.com/Graphing/piecewise_functions.html</u>

- If there is a topic that you just need help with, try googling the topic. For example, if you don't remember how to solve an exponential equation, try googling "solving exponential equations" and a bunch of sites will probably pop right up with help and examples.

- Understand that everything in this packet is expected to be review. If you find that there is a lot in here that you are unable to do and you are leaving whole sections blank, you might want to rethink your decision to enter AP calculus. You can receive help with small questions from past topics, but you will not be re-taught entire units such as logarithms or trigonometry. You need to have some mastery of these topics before entering the class.

- If you have ANY questions or problems or just need a bit of help over the summer, please feel free to email Mr. Jones at <u>jonesSG@fultonschools.org</u> and I will help you as quickly as I can.

HONOR CODE: I confirm that all work contained in this project is my own. I have not collaborated with or asked questions of any other students or adults, with the exception of my teacher.

_____   _____
Signature   Date

1.

Algebraically and graphically solve $\dfrac{1-3x}{4} = \dfrac{x+6}{3} + \dfrac{1}{2}$

2.

Algebraically and graphically solve $\sqrt{x+1} + \sqrt{x-1} = \sqrt{2x+1}$

3.

Algebraically and graphically solve $|1 - 2x| = 3$

4.

Algebraically and graphically solve $6 > \dfrac{5-3x}{2} \geq -3$

5.

Algebraically and graphically solve $|2x - 5| \geq 9$

6.

Find the equation of the line perpendicular to the line $3x - y = -4$ passing through the point $(-2, 4)$

7.

Find the x- and y intercepts and test for symmetry for $x^2 + 4x + y^2 - 2y = 0$

8.

Find the slope of the line containing the points $(7, 4)$ and $(-3, 2)$. What is the distance between these points? What is their midpoint?

9.

Show that the points $A = (2, 5)$, $B = (6, 1)$, and $C = (8, -1)$ lie on a line by using slopes.

10.

Working Together to Get a Job Done Clarissa and Shawna, working together, can paint the exterior of a house in 6 days. Clarissa by herself can complete this job in 5 days less than Shawna. How long will it take Clarissa to complete the job by herself?

11.

Chemistry: Mixing Acids For a certain experiment, a student requires 100 cubic centimeters of a solution that is 8% HCl. The storeroom has only solutions that are 15% HCl and 5% HCl. How many cubic centimeters of each available solution should be mixed to get 100 cubic centimeters of 8% HCl?

12.

Physics: Uniform Motion   Refer to the figure below.
A man is walking at an average speed of 4 miles per hour
alongside a railroad track. A freight train, going in the
same direction at an average speed of 30 miles per hour,
requires 5 seconds to pass the man. How long is the freight
train? Give your answer in feet.

13.

For $f(x) = \dfrac{x^3}{x^2 - 4}$ find   (a) $f(-x)$ (b) $-f(x)$ (c) $f(x + 2)$ (d) $f(x - 2)$ (e) $f(2x)$

14.

Find the domain of $\sqrt{x + 2}$

15.

Find the domain of $f(x) = \dfrac{x}{x^2 + 2x - 3}$

16.

For $f(x) = \begin{cases} x & -4 \leq x < 0 \\ 1 & x = 0 \\ 3x & x > 0 \end{cases}$

(a) Find the domain of each function.
(c) Graph each function by hand.
(e) Verify your results using a graphing utility.

(b) Locate any intercepts.
(d) Based on the graph, find the range.

17.

Find the average rate of change from 2 to x for $f(x) = 3x - 4x^2$

18.

For $f(x) = \dfrac{1}{x^2 + 4}$; $g(x) = 3x - 2$

find   (a) $(f \circ g)(2)$ (b) $(g \circ f)(-2)$ (c) $(f \circ f)(4)$ (d) $(g \circ g)(-1)$

19.

Algebraically determine whether $h(x) = \dfrac{1}{x^4} + \dfrac{1}{x^2} + 1$ is even, odd or neither.
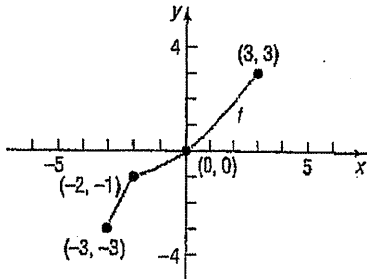
20.

Algebraically determine whether $g(x) = \dfrac{1 + x^2}{x^3}$ is even, odd or neither.

21.

For the following graph of the function f draw the graph of:

(a) $y = f(-x)$　　(b) $y = -f(x)$　　(c) $y = f(x + 2)$

(d) $y = f(x) + 2$　(e) $y = 2f(x)$　　(f) $y = f(3x)$

22.

Accurately graph $f(x) = 3x^2 - 4x - 1$ by completing the square and then using graphical transformations.

23.

Geometry Find the length and width of a rectangle whose perimeter is 20 feet and whose area is 16 square feet.

24.

Minimizing Cost Callaway Golf Company has determined that the daily per unit cost C of manufacturing x additional Big Bertha-type golf clubs may be expressed by the quadratic function

$$C(x) = 5x^2 - 620x + 20{,}000$$

(a) How many clubs should be manufactured to minimize the additional cost per club?

(b) At this level of production, what is the additional cost per club?

25.

Business The monthly revenue achieved by selling x boxes of candy is figured to be $x(5 - 0.05x)$ dollars. The wholesale cost of each box of candy is $1.50.

(a) How many boxes must be sold each month to achieve a profit of at least $60?

(b) Using a graphing utility, graph the revenue function.

(c) What is the maximum revenue that this firm could earn?

(d) How many boxes of candy should the firm sell to maximize revenue?

(e) Using a graphing utility, graph the profit function.

(f) What is the maximum profit that this firm can earn?

(g) How many boxes of candy should the firm sell to maximize profit?

(h) Provide a reasonable explanation as to why the answers found in parts (d) and (g) differ. Is the shape of the revenue function reasonable in your opinion? Why?

26.

Without using a calculator find ALL solutions (real and imaginary) for $2x^4 + 7x^3 - 5x^2 - 28x - 12 = 0$

27.

Write the 5th degree polynomial (in expanded form which has zeros of 5, 3+4i, 2-5i

28.

Algebraically and graphically solve $\dfrac{x(x^2 + x - 2)}{x^2 + 9x + 20} \leq 0$

29.

Find the inverse of the function $f(x) = \dfrac{2x + 3}{5x - 2}$. State the domain and range of f and f^{-1}

30.

Fully expand $\ln\left(\dfrac{x\sqrt[3]{x^2 + 1}}{x - 3}\right)$ as the sum and difference of logarithms.

31.

Express $2\log 2 + 3\log x - \dfrac{1}{2}\left[\log(x + 3) + \log(x - 2)\right]$ as a **single** logarithm.

32.

Find y as a function of x [C is a positive constant] if $\ln(y - 1) + \ln(y + 1) = -x + C$

33.

Find y as a function of x [C is a positive constant] if $e^{3y-C} = (x + 4)^2$

34.

Solve $5^{x+2} = 7^{x-2}$

35.

Solve $\log_6(x + 3) + \log_6(x + 4) = 1$

36.

Salvage Value   The number of years n for a piece of machinery to depreciate to a known salvage value can be found using the formula

$$n = \dfrac{\log s - \log i}{\log(1 - d)}$$

where s is the salvage value of the machinery, i is its initial value, and d is the annual rate of depreciation.
(a) How many years will it take for a piece of machinery to decline in value from $90,000 to $10,000 if the annual rate of depreciation is 0.20 (20%)?
(b) How many years will it take for a piece of machinery to lose half of its value if the annual rate of depreciation is 15%?

37.

Estimating the Date that a Prehistoric Man Died The bones of a prehistoric man found in the desert of New Mexico contain approximately 5% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, approximately how long ago did the man die?

38.

Logistic Growth The logistic growth model

$$P(t) = \frac{0.8}{1 + 1.67e^{-0.16t}}$$

represents the proportion of new computers sold that utilize the Microsoft Windows 98 operating system. Let $t = 0$ represent 1998, $t = 1$ represent 1999, and so on.
(a) What proportion of new computers sold in 1998 utilized Windows 98?
(b) Determine the maximum proportion of new computers sold that will utilize Windows 98.
(c) Using a graphing utility, graph $P(t)$.
(d) When will 75% of new computers sold utilize Windows 98?

39.

Without using a calculator find the exact value of $\sec\left(-\frac{\pi}{3}\right) - \cot\left(-\frac{5\pi}{4}\right)$
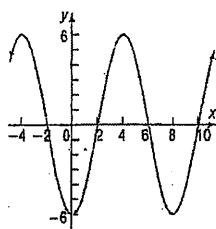
40.

Find the exact value of the 5 remaining trig functions if $\sin\theta = -\frac{5}{13}, \quad 3\pi/2 < \theta < 2\pi$

41.

Use amplitude period, and shift to sketch an accurate labelled graph of $y = -7\sin\left(\frac{\pi}{3}x + \frac{4}{3}\right) - 2$.(at least 2 periods).

42.

Find a function for the graph

43.

Spin Balancing Tires The radius of each wheel of a car is 16 inches. At how many revolutions per minute should a spin balancer be set to balance the tires at a speed of 90 miles per hour? Is the setting different for a wheel of radius 14 inches? What is this setting?

44.

Without using a calculator find the exact value of $\sin\left(-\frac{\pi}{12}\right)$

45.

Given $\csc\alpha = 2$, $\pi/2 < \alpha < \pi$, $\sec\beta = -3$, $\pi/2 < \beta < \pi$ without using a calculator find the exact value of :

(a) $\sin(\alpha + \beta)$ (b) $\cos(\alpha + \beta)$ (c) $\sin(\alpha - \beta)$ (d) $\tan(\alpha + \beta)$

(e) $\sin(2\alpha)$ (f) $\cos(2\beta)$ (g) $\sin\dfrac{\beta}{2}$ (h) $\cos\dfrac{\alpha}{2}$

46.

Without using a calculator find the exact value of $\tan\left[\sin^{-1}\left(-\dfrac{4}{5}\right)\right]$

47.

Without using a calculator find the exact value of $\sin\left(\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5}\right)$
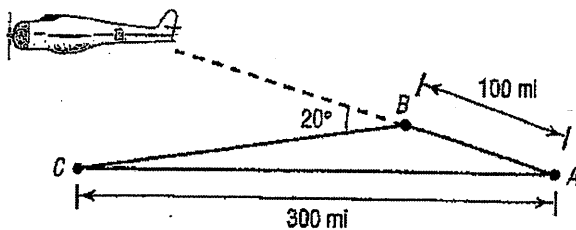
48.

Without using a calculator find the exact value of $\sin(2\theta) - \cos\theta - 2\sin\theta + 1 = 0$

49.

Without using a calculator find the exact value of $2\sin^2\theta - 3\sin\theta + 1 = 0$

50.

Correcting a Navigation Error Two cities A and B are 300 miles apart. In flying from city A to city B, a pilot inadvertently took a course that was 5° in error.
(a) If the error was discovered after flying 10 minutes at a constant speed of 420 miles per hour, through what angle should the pilot turn to correct the course? (Consult the figure.)
(b) What new constant speed should be maintained so that no time is lost due to the error? (Assume that the speed would have been a constant 420 miles per hour if no error had occurred.)
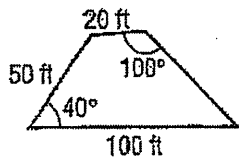
51.

Finding the Height of a Building Find the height of the building shown in the figure.
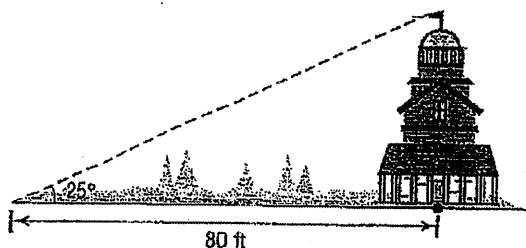
52.

Calculating the Cost of Land The irregular parcel of land shown in the figure is being sold for $100 per square foot. What is the cost of this parcel?

53.

Without using a calculator solve the system $\begin{cases} 3x - 4y = 4 \\ x - 3y = \frac{1}{2} \end{cases}$

54.

Without using a calculator solve the system $\begin{cases} x + 5y - z = 2 \\ 2x + y + z = 7 \\ x - y + 2z = 11 \end{cases}$

55.

Without using a calculator solve the system $\begin{cases} 2xy + y^2 = 10 \\ 3y^2 - xy = 2 \end{cases}$

56.

Use matrices to solve the system $\begin{cases} x - 3y + 3z - t = 4 \\ x + 2y - z = -3 \\ x + 3z + 2t = 3 \\ x + y + 5z = 6 \end{cases}$

57.

Evaluate $\displaystyle\sum_{k=1}^{10} (3k - 9)$

58.

Find the general formula for the arithmetic sequence where the 7th term is 31, and 20th term is 96.

59.

Find the sum of the infinite geometric series $\displaystyle\sum_{k=1}^{\infty} 3\left(-\frac{3}{4}\right)^{k-1}$

60.

Use binomial theorem to expand the expression $(2x + 3)^5$