Sequence: A **List** of numbers written in a specific order(finite or infinite)
We can think of sequences as functions
Convergent sequence: If $\lim_{x\to\infty} a_n$ exists approaches a real number
Divergent sequence: If $\lim_{x\to\infty} a_n$ does not exist and is infinite
In order for a limit for a rule of sequence to exist, the terms must be approaching a single finite value as **n** increases.

Comparative Growth Rates:
As $n\to\infty$, log < radical < polynomial < exponential < factorials
As $n\to\infty$, $\ln(x^2+1) < \sqrt[4]{400x+900} < 60x^5 + 20x^2 < 2^x < x^x < x!$

Monotonic sequence : Sequence that is either always increasing or always decreasing

Series – An Infinite Series is the **sum** of the terms in a sequence. S_n is the n^{th} partial Sum.

$$s_n = a_1 + a_2 + a_3 + \dots + a_n = \sum_{i=1}^{n} a_i$$

"i" is called the index. (index of summation)
A Series is **Convergent** if the sequence of Partial sum S_n is convergent and its limit is finite
A Series is **Divergent** if the sequence of Partial sum S_n is divergent and its limit is infinite
*If the sequence of partial sums is convergent , then the series will be convergent as well

Series is the limit of the partial sums: $\sum_{i=1}^{\infty} a_i = \lim_{n\to\infty} s_n$

Convergence theorem

If sum of series $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

Divergence test (n^{th} term test) *Always use this test first!*

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ will diverge

If $\lim_{n\to\infty} a_n = 0$, then this test is inconclusive (This series may converge or diverge. Use a different test to determine convergence)

Harmonic Series: $\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent series (p = 1)

Integral Test: Integral test involves finding definite integral of a rule of sequence to determine convergence/divergence.
Suppose f(x) is *continuous*, *positive*, and *decreasing* function on interval and that $f(n) = a_n$,

then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(x)dx$ either both **converge** (where the integral produces a Real number) or both **diverge** (where the integral produces either $\pm\infty$)
*Works as long as the function eventually must decrease and is positive for all terms
*Careful! The value of this particular integral is NOT the sum of the series. We will **NOT** be able to determine the sum of this series using this test.

Telescoping Series: Telescoping Series are series whose partial sums have terms that cancel and may eventually converge. When you find what you think might be a telescoping series, write out some terms until you see a pattern.
*These series require expanding and canceling terms
Telescoping series can be overt, such as $\sum\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$,
or disguised, such as $\sum\left(\frac{2}{4n^2-1}\right)$, requiring Partial Fraction Decomposition
*You **can** find the value of a telescoping series

P-Series Test: If k > 0, p is a constant, and n is a variable,

then $\sum_{n=k}^{\infty} \frac{1}{n^p}$ converges if p > 1 and diverges if $p \le 1$

(When p = 1, this series is a divergent Harmonic series)

Harmonic Series: $\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent series (p = 1)

Geometric Series Test (GST)
Geometric Series : (The terms are numbers raised to a variable power)

$\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$; $r = \frac{current\ term}{previous\ term}$

If Series $\sum_{n=1}^{\infty} ar^{n-1}$ converges, it converges to $S = \frac{a_1}{1-r}$

*You **CAN** find the actual Sum value of a geometric series.

Absolute Convergence: If series $\sum |a_n|$ converges then $\sum a_n$ also converges *This means if the absolute value of the series converge, then the original series must also converge.
*Also, this means this alternating series is not dependent on the alternator in order to converge.

Conditional Convergence: If $\sum a_n$ converges and $\sum |a_n|$ diverges *Conditional convergence means that the series will only converge on the condition that the alternator is needed to make the series convergent

Example: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ (The harmonic series requires an alternator in order become a converging series, therefore this has conditional convergence.)

Acronym for Convergence/Divergence Tests
Moses and the "PARTING C"

P – P series: Is the series in the form $\frac{1}{n^p}$?

A – Alternating Series: Does the series alternate signs? Are the terms getting smaller, and is n^{th} term = 0?

R – Ratio Test: Does the series contain terms that grow very large as n increases? (exponential or factorials)

T – Telescoping Series: Will all but a couple of terms in the series cancel out? (Expand and look for pattern)

I – Integral Test: Can you easily integrate the expression that defines the series? (Be sure function is Decreasing, Continuous, Positive)

N – n^{th} Term Divergence Test: Is the n^{th} term anything other than zero? (Use this test FIRST before using others)

G – Geometric Series Test: Is the series of the form $\sum_{n=1}^{\infty} ar^n$

C – Comparison Tests: Does the series resemble another kind of know series (p-series or geometric?) Can we use **Direct** or **Limit** Comparison Test?

Direct Comparison Test: This is a test where we identify a rule of sequence with a known convergence or divergence and use it to determine convergence/divergence of another series.

Suppose we have 2 series , $\sum a_n, \sum b_n \geq 0$ for all n and $a_n \leq b_n$ for all n values. Then:

*If $a_n \leq b_n$ and $\sum b_n$ is convergent, then $\sum a_n$ converges as well

*If $a_n \geq b_n$ and $\sum b_n$ diverges , then $\sum a_n$ diverges as well

*Determine and find a second (similar)series you can use to do comparison test.
* If the larger series converges, then the smaller series must also converge.
*If the smaller series diverges, then the larger series must also diverge
*Use this test when the rule of sequence is VERY SIMILAR to a known series

Ex) compare $\frac{n}{2^n}$ to $\frac{1}{2^n}$, $\frac{1}{n^3+1}$ to $\frac{1}{n^3}$, $\frac{n^2}{(n^2+3)^2}$ to $\frac{n}{(n^2+3)^2}$

***Careful**! If smaller series converges, we cannot conclude convergence for the larger series
If larger series diverges, we cannot conclude divergence for the smaller series.

Alternating Series Test (AST): An Alternating Series is a series whose terms alternate signs on consecutive terms.

Let $a_n > 0$. The Alternating Series $\sum_{n=0}^{\infty}(-1)^n a_n$ will converge if the following 2 conditions are met:

1. $\lim_{n\to\infty} a_n = 0$
2. $a_n \leq a_{n+1}$ for all n. (decreasing sequence)

Alternating Harmonic Series converges! : $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

Alternating Series Remainder:
Suppose an alternating series converges by AST (such that $\lim_{n\to\infty} a_n = 0$ and a_n is decreasing) , then

$|R_n| = |S - S_n| \leq |a_{n+1}|$

*This means that the **maximum error** for the n[th] term partial Sum S_n is no greater than the absolute value of the first unused term a_{n+1}

Limit Comparison Test: This is a variation of the Direct Comparison Test, and many times may be used instead of Direct Comparison Test to determine convergence/divergence.

If a_n, $b_n > 0$ and $\lim_{x\to\infty}\left|\frac{a_n}{b_n}\right|$ or $\lim_{x\to\infty}\left|\frac{b_n}{a_n}\right|$ equal any finite number, then either both $\sum a_n$ and $\sum b_n$ converge or diverge.

Ex) compare: $\frac{3n^2+2n-1}{4n^5-6x+7}$ to $\frac{1}{n^3}$ (you can disregard the leading coefficient and all non-leading terms, looking only at the condensed degree of the leading terms: $\frac{n^2}{n^5} = \frac{1}{n^3}$.

Ratio Test: Useful when rule of sequence contains powers and/or factorials (Taylor Series)

If $a_n > 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = N$ (where N is a real number), then

1. $\sum a_n$ converges absolutely (and hence converges) if $N < 1$
2. $\sum a_n$ diverges if $N > 1$ or $N = \infty$
3. The test is inconclusive if $N = 1$ (use another test)

*This test is good for series whose terms converge rapidly, those involving exponentials and/or factorials *Not a good test for series involving only polynomials or polynomials under radicals.

Root Test: This test is most useful if the rule of sequence is expressed in term of n[th] power, since these will simplify and cancel out when you take the n[th] root

If $\sum a_n$ is a series with non-zero terms and $\lim_{n\to\infty}\sqrt[n]{|a_n|} = N$

1. $\sum a_n$ converges absolutely (and hence converges) if $N < 1$
2. $\sum a_n$ diverges if $N > 1$ or $N = \infty$
3. The test is inconclusive if $N = 1$ (use another test)

Use this test for series involving nth powers. Ex) $\sum \frac{e^{2n}}{n^n}$

LaGrange Error Bound *This is similar to the Alternating Series Remainder. However, this method offers a way to determine the maximum error (remainder) when we do a Taylor polynomial approximation using a certain number of terms for a specific function.

$$R_n(x) = \left|\frac{\left|f^{(n+1)}(z)\right|}{(n+1)!}(x-c)^{n+1}\right| \leq \left|\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}(x-c)^{n+1}\right|$$

* The remainder for an n[th] degree polynomial is found by taking the (n +1)[st] (**first unused**) derivative at "z" *We are not expected to find the exact value of z. (If we could, then an approximation would not be necessary) *We want to maximize the (n+1)[st] derivative on the interval from [x , c] in order to find a safe upper bound for the $\left|f^{(n+1)}(z)\right|$ *The maximum error bound is the worst case scenario for the interval in which our actual approximation can live. **College Board will provide strictly increasing and decreasing functions. (So we only have to choose between f(c) and f(x) (the endpoints). This will allow us to determine the max value much more accurately.

Taylor Polynomial is a polynomial that will approximate other function's values in a region that is nearby the "center"
*a tangent line is essentially a first degree taylor polynomial.

n^{th} degree taylor polynomial:

$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$

Alternating Series Remainder:
Suppose an alternating series converges by AST. If the Series has Sum S, then $|R_n| = |S - S_n| \le |a_{n+1}|$

*This means that the **maximum error** for the n^{th} term partial Sum S_n is no greater than the absolute value of the first unused term a_{n+1}

Taylor Series: A General method for writing a power series representation for a function.

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$

$f^{(n)}$ represents the n^{th} derivative evaluated at f.

Maclaurin Series: is the special case of Taylor series when **c = 0.**

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$

Special Maclaurin Series:

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}$ IOC: All Reals

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \frac{(-1)^{n-1}x^{2n-2}}{(2n-2)!}$ IOC: All Reals

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^n}{n!}$ IOC: All Reals

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)}$ IOC: $-1 \le x \le 1$

Power Series: Written in form $\sum_{n=0}^{\infty} a_n(x-c)^n$ where **c** and $\mathbf{a_n}$ (coefficients) are numbers:
*Taylor and Maclaurin series are special cases of power series
For a power series centered at c, precisely one of the following is true:

1) The series converges only at c (ALL power series converge at least at their center) (Radius of convergence = 0)
2) The series converges for all x (function and infinite series have exact same values everywhere) → Radius = ∞
3) The series converges within a certain Radius of Convergence such that series converges for $|x - c| < R$
→The **interval of Convergence** (I.O.C.) is **[(c – R , c + R)]**

*Be sure to TEST convergence of endpoints
*Typically, you want to use the **RATIO TEST** to determine Radius of Convergence

Geometric Series below based on

$S = \frac{a_1}{1-r}$ IOC: $-1 < x < 1$

$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + \ldots(-1)^n x^n + \ldots$

IOC: $-1 < x < 1$

$\frac{1}{x} = \frac{1}{1-[-(x-1)]} = 1-(x-1)+(x-1)^2 - \ldots(-1)^n(x-1)^n$

IOC: $0 < x < 2$

$\ln x = \int \frac{1}{x}dx = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} \ldots \frac{(-1)^{n-1}(x-1)^n}{n}$

IOC: $0 < x \le 2$

(Ch. 6.1) Euler's Method: A numerical method used to approximate solutions to differential equations.
*Instead of using just one slope value as in tangent line approximation, we change the tangent lines with each step (of length Δx). This involves recalculating the point and slope after each step. This will produce a more accurate approximation than simply using the original tangent line.
*From a starting point, we calculate the slope at that point and then use the initial point and that slope to locate a new point, then repeat the process.

Euler's method (continued) ***Memorize this table***

x	y	$m = \frac{dy}{dx}$	$\Delta y = m(\Delta x)$	$y_{new} = y + \Delta y$

*Recall $\Delta x = \frac{b-a}{n}$

Ch. 7.3 (Shell Method) - uses cylindrical shells to evaluate the volume of a rotation: $V = 2\pi\int_a^b$ (shell radius)(shell height)dx

Vertical Axis of Revolution: $V = 2\pi\int_a^b$[horizontal x radius distance][(f(x)]dx

Horizontal Axis of Revolution: $V = 2\pi\int_a^b$[vertical y radius distance][(f(y)]dy

*Notice that the representative rectangles are **parallel** to the Axis of Revolution rather than **perpendicular**. This is opposite to the disc and washer methods.

Ch. 7.4 (Arc Length): Given function y = f(x), arc length $s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

Area of Surface of Revolution:
Derived from frustum surface area:
$S = 2\pi r l$ (l is slant height)

Horizontal AOR: $S = 2\pi\int_a^b [f(x)]\sqrt{1+[f'(x)]^2}\,dx$

Vertical AOR: $S = 2\pi\int_a^b x\sqrt{1+[f'(x)]^2}\,dx$

(Ch. 10.2 – 10.3) Parametric functions: where x and y coordinates on a graph are given in terms of a third variable "t" : x=f(t) and y = g(t) are parametric equations and **t** is called the parameter.

*Eliminate the parameter, and use substitution to write rectangular equation.

*Rectangular equation only shows path of graph

*Parametric Equation tracks more info: Includes the **path, speed**, and **direction of graph**

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

*Horizontal tangent occurs where $\frac{dy}{dt} = 0$

*Vertical tangent occurs where $\frac{dx}{dt} = 0$

*Beware of $\frac{0}{0}$, which is neither a horizontal nor vertical tangent

Parametric Arc Length: (s) $= \int_{t1}^{t2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Speed of particle: $\left|\vec{v}(t)\right| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Distance of particle $\int_{t_1}^{t_2} |v| = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

*Distance of particle **IS** the parametric arc length

* $\int speed$ is the **total distance traveled** along the curve (arc length)

* $\int velocity$ is the **displacement** (net change in position where positives and negatives cancel)

* **Final Position = Initial Position + Displacement**

Arc Length:

Arc length (s) $= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ or $\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric s $= \int_{t1}^{t2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ or $\int_{t1}^{t2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

$ds^2 = dx^2 + dy^2$

Area of a Surface of Revolution in Parametric Form

Revolution about **x-axis**: $S = 2\pi \int_{t_1}^{t_2} y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dx$ Revolution about **y-axis**: $S = 2\pi \int_{t_1}^{t_2} x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dx$

(Ch. 10.4 – 10.5) Polar Equations: Ordered pairs are expressed as (r, θ) with θ as the independent variable.

r = distance from origin θ= directed angle from polar axis

origin is called the pole. x-axis is called the polar axis

Polar to Rectangular

$x = r\cos\theta$

$y = r\sin\theta$

Rectangular to Polar

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

Polar Derivatives:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{y'(\theta)}{x'(\theta)}$$

Region bounded by a Polar Curve:

Area of a circular sector: $A = \frac{1}{2} r^2 \theta$

Polar Area Enclosed Region: $A = \int_a^b \frac{1}{2} r^2\, d\theta$

Arc Length Polar Curve: $s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + [r'(\theta)]^2}\, d\theta$

(Ch. 10.4) Special Polar Graphs

Circles: $r = a\cos\theta$ or $r = a\sin\theta$

*Traces out 1 rotation CCW from $[0, \pi]$

*coefficient **a** is the length of the diameter

*cosine graph symmetric to x-axis

*sine graph symmetric to y-axis

CCW = counter clockwise

Limacons: r = a ± b cos θ or r = a ± b sin θ (a > 0, b > 0)

* Traces out 1 rotation Clockwise from $[0, 2\pi]$

*constant + coefficient = outer radius

*constant – coefficient = inner radius

Graphs going through poles (origin):

Cardioid: **once**

Limacon with inner loop: **twice**

Dimpled Limacon: **none**

Rose Curves $r = a\cos(n\theta)$, $r = a\sin(n\theta)$

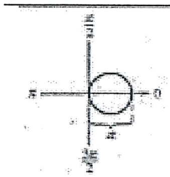
*coefficient **a** is the length of each petal

*If **n** is odd, then there are **n** petals on graph

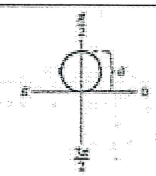
*If **n** is even, then there are **2n** petals on graph

*If **n** is odd, 1 rotation traces out CCW from $[0, \pi]$

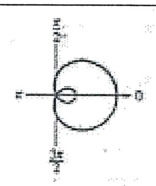
*If **n** is even, 1 rotation traces out CCW from $[0, 2\pi]$



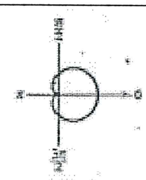
$r = a\cos\theta$ Circle



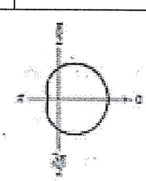
$r = a\sin\theta$ Circle



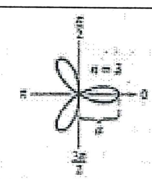
$\frac{a}{b} < 1$ Limaçon with inner loop



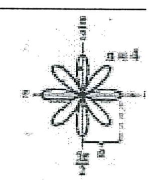
$\frac{a}{b} = 1$ Cardioid (heart-shaped)



$1 < \frac{a}{b} < 2$ Dimpled limaçon



$r = a\cos n\theta$ Rose curve



$r = a\cos n\theta$ Rose curve

(Ch.8.7) L'Hopital's Rule and Indeterminate Forms:

L'Hopital's Rule:

Suppose $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}\,or\pm\dfrac{\infty}{\infty}$ then $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$

*If direct substitution produces an indeterminate form of $\dfrac{0}{0}or\pm\dfrac{\infty}{\infty}$, we can apply **L'Hopital's Rule.**

Ch. 8.8 **Improper integrals:** $\int_a^\infty f(x)dx=\lim\limits_{b\to\infty}\int_a^b f(x)dx$

There are **other indeterminate forms** that need to be converted to $\dfrac{0}{0}or\pm\dfrac{\infty}{\infty}$ before applying **L'Hopital's Rule:**

1.If Indeterminate form is 0^0, 1^∞, or ∞^0 → rewrite as equation and use Log Differentiation
2. If Indeterminate form is $\infty-\infty$ → find common denominator, which will get the expression into a single quotient, ready to evaluate.
3. If Indeterminate form is $0\cdot\infty$ → rewrite as a quotient, bring ∞ or 0 down to denominator to create $\dfrac{0}{0}or\pm\dfrac{\infty}{\infty}$

(Ch. 8.8) Improper Integrals Improper integrals can have forms

$$\int_a^\infty f(x)dx,\ \int_{-\infty}^a f(x)dx,\ \int_{-\infty}^\infty f(x)dx$$

2 Categories of improper Integrals:
1) They have an infinite interval of integration
2) They have a disconuity on the interor of the interval of integration (or have both properties 1 and 2)

Steps to evaluate:
1) Rewrite the integral as a proper integral
2) Then use limits (see form in the above column)

*An improper integral that equals a finite value **converges** to that value

*An improper integral that does not equal a finite number is said to **diverge**.

(Ch. 8.5) Partial Fraction Decomposition: Process of rewriting a rational function into simpler rational functions.

Cover up Method for non - repeating Linear Factors in the denominator Steps:
1. Factor Denominator 2. Solve for x for each factor
3. Cover up a factor in the denominator of the problem
4. Plug in the x-value from #2 and solve for x.
5. Pair the value found with the factor from the cover up. $\left(\dfrac{value}{factor}\right)$
6. Repeat steps 2 – 3 for the other factors.

*In order for partial the fraction decomposition to work, the degree of **denominator** must be **greater than** that of the **numerator**. When it is not, we look to **test out long division method first**

(Ch. 6.3) Logistic Growth: The logistic differential equation is used to model logistic growth, such as population.

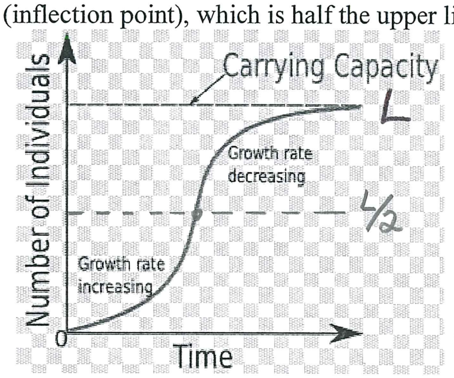
Logistic differential equation: $\dfrac{dy}{dt}=ky(L-y)$

Solution to differential equation: $y=\dfrac{L}{1+Ce^{-Lkt}}$

To help memorize solution, think **"Lice – Licked"**

L = carrying capacity or the **maximum** sustainable population.

L/2 =value when maximum growth rate is reached (inflection point), which is half the upper limit to growth



Determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$

Chapter 11: Vectors and Geometry of Space

vector is a directed line segment used to represent quantities that have both **magnitude** and **direction**.

Scalar quantity involve only magnitude

The directed line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q, and its length (magnitude) is denoted by $\|\overrightarrow{PQ}\|$

$\|\overrightarrow{PQ}\|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Vector $\vec{v}=\overrightarrow{PQ}$

Standard Unit Vectors

The **unit vectors** <1,0> and <0,1> are called standard unit vectors and are denoted by **i** = <1, 0> and **j** = <0, 1>

Dot Product: returns a scalar and not a vector

The dot product of $u=\langle u_1,u_2\rangle$ is $u\bullet v=u_1v_1+u_2v_2$

*Vectors are orthogonal (normal, perpendicular) if $u\bullet v=0$

Cross Product: The cross product yields a vector, also called the vector product

$$u\times v=\begin{vmatrix} + & - & + \\ i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$=\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}i-\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}j+\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}k$$

(Ch. 12) Vector-valued functions:

*For a given value of **t**, a Vector-Valued function returns a vector.

*The direction in which the curve is traced is called the **orientation** (we can show direction by drawing arrows on the curve)

*The **terminal point** from the vector function traces out the plane curve.

(12.3) Position Function for a Projectile: The path of a projectile launched from an initial height **h** with initial velocity $\mathbf{v_o}$ and angle of elevation θ is described by the vector function:

$$r(t)=(v_o cos\theta)t\boldsymbol{i}+\left[h+(v_o sin\theta)t-\tfrac{1}{2}gt^2\right]\boldsymbol{j}$$

g is the gravitational constant: **g = 32 ft/s² or 9.8m/s²**

Miscellaneous Topics and Notes

(Ch. 12.3) Velocity and Acceleration:

Position vector **R**= $\vec{R}(t)$= <x, y>

Velocity vector V $=\vec{v}(t) = \frac{dR}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle = \langle v_x, v_y \rangle$

Acceleration vector $\vec{a}(t) = \frac{d^2R}{dt^2} = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle = \langle a_x, a_y \rangle$

Magnitude of v is $|v| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Magnitude of a is $|a| = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$

Motion along a plane curve: Vector **R** velocity vector **v** and acceleration vector **a**
R = <x, y>

$V = \frac{dR}{dt} = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle = \langle v_x, v_y \rangle$

$a= \frac{d^2R}{dt} = \frac{dv}{dt} = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle = \langle a_x, a_y \rangle$

(Ch. 8.1)Basic Integration Rules:
list of procedures that you use to make an integral fit one of the basic rules.

1) Expand a function $(1+e^x)^2 = 1+2e^x+e^{2x}$
2) Separate the numerator $\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
3) Complete the square $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$
4) Long division $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
5) Add and subtract terms in numerator

$$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{x^2+2x+1}$$

6) Use trigonometric identities $\cot^2 x = \csc^2 x - 1$
7) Multiply and divide by Pythagorean conjugate

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$$

(Ch. 8.5) Partial Fractions: Partial Fraction Steps:

1. Partial fractions can only be done if the degree of the numerator is less than the degree of the denominator
2. Factor the denominator as much as possible and get the form of the partial fraction decomposition(using A and B)
3. Multiply the equation through with the denominator
4. Set the numerators equal to each other
5. Expand the right side of the equation and group like terms (with x) together and constants together
6. Set the coefficients of terms with variables equal and coefficients of terms of constant together
7. Solve the system of equation and find the values of A and B

Alternate method: Carefully pick the x's so that the unknown constants will drop out.

(Ch. 8.3) Trig Functions with Powers:
To break up the integral into manageable parts, use the following identities:

$\sin^2 x + \cos^2 x = 1$ Pythagorean identity

$\sin^2 x = \frac{1-\cos 2x}{2}$ Half-angle identity for $\sin^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$ Half-angle identity for $\cos^2 x$

Guidelines for Evaluating Integrals Involving Sine and Cosine

1) If the power of sine is odd and positive, save one sine and convert the rest to cosines.
2) If the power of cosine is odd and positive, save one cosine and convert the rest to sines.
3) If the powers of both the sine and cosine are even and nonnegative, use the half-angle identities to convert the integrand to odd powers of the cosine.

Guidelines for Evaluating Integrals Involving Secant and Tangent (Note: $1+\tan^2 x = \sec^2 x$)

1) If the power of secant is even and positive, save a secant-squared factor and convert the rest to tangents.
2) If the power of the tangent is odd and positive, save a secant-tangent and convert the rest to secants.
3) If there are no secants and the power of tangent is even and positive, convert a tangent-squared to a (secant-squared – 1). Expand and repeat as necessary.
4) If the integral is only secant with an odd positive power, use integration by parts.
5) If none of the first four guidelines apply, try to convert to sines and cosines.

(Ch. 8.5) Partial Fraction decomposition with Quadratic Functions:
When using partial fractions with linear factors, making good choices for values of x can immediately yield values for your coefficients. However, with quadratic factors, a system of linear equations usually has to be solved, regardless of the choice of x.

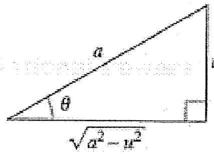
(Ch. 8.4) Trigonometric Substitution: Evaluate integrals involving the radicals $\sqrt{a^2-u^2}$, $\sqrt{a^2+u^2}$, $\sqrt{u^2-a^2}$
Objective is to eliminate the radical in the integrand (using Pythagorean Identities)

Trigonometric Substitution ($a > 0$)

1. For integrals involving $\sqrt{a^2-u^2}$, let

   $u = a \sin \theta.$

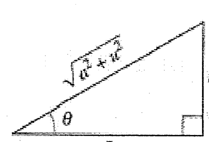
   Then $\sqrt{a^2-u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2.$



2. For integrals involving $\sqrt{a^2+u^2}$, let

   $u = a \tan \theta.$

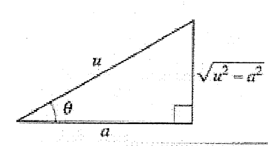
   Then $\sqrt{a^2+u^2} = a\sec\theta$, where $-\pi/2 < \theta < \pi/2.$



3. For integrals involving $\sqrt{u^2-a^2}$, let

   $u = a \sec \theta.$

   Then $\sqrt{u^2-a^2} = \pm a \tan\theta$, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$. Use the positive value if $u > a$ and the negative value if $u < -a$.



THEOREM 8.2 Special Integration Formulas ($a > 0$)

1. $\int \sqrt{a^2-u^2}\,du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2-u^2}\right) + C$
2. $\int \sqrt{u^2-a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2-a^2} - a^2 \ln\left|u + \sqrt{u^2-a^2}\right|\right) + C, \quad u > a$
3. $\int \sqrt{u^2+a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2 \ln\left|u + \sqrt{u^2+a^2}\right|\right) + C$