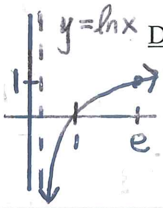
Calculus AB Fall Semester Summary / Formula Sheet

Even Functions: f(-x) = f(x) (Symmetry about y-axis)

- y = cos(x) is an even function

Odd Function: f(-x) = -f(x) (Symmetry about origin)

- y = sin(x) is an odd function



Domain: Defines where x-values exist in the function

a) Consider denominator (set denominator = 0)
b) Consider domain of numerator
   i) *For* $y = \ln(x)$ domain = $\{x > 0\}$
   ii) *For* $y = \sqrt{x}$ domain = $\{x \geq 0\}$
   iii) *For* $y = e^x$ domain = All Real Numbers

Vertical Asymptote: Set denominator = 0

Horizontal Asymptote:

(Same as finding Limits at Infinity, same as finding end behavior)

1) If degree of bottom is greater than degree of top, then $\lim_{x\to\infty} f(x) = 0$
   (This also means that H.A. is y = 0)
2) If degree of top is same as bottom, then the limit is the value of the leading coefficients: $\lim_{x\to\infty} f(x) = \frac{a}{b}$
3) If degree of top is greater than bottom, then the limit DNE, meaning that

$\lim_{x\to\infty} f(x) = +\infty$ or $\lim_{x\to\infty} f(x) = -\infty$

*Remember that function can still exist where there is a horizontal asymptote. Horizontal asymptote describes end behavior

$$\lim_{x\to-\infty} \frac{a}{\sqrt{b}} = \lim_{x\to-\infty} \frac{a/x}{-\sqrt{b/x}}$$

Finding Limits Algebraically

1) Plug in x-value.. If real number results, this is your limit (answer)
2) If DNE results, then factor, multiply by conjugate, or use trig limit rules to cancel out.
3) Plug in x-value again.

Trig Limit Definitions:

$$\lim_{x\to0} \frac{\sin ax}{bx} = \frac{a}{b} \qquad \lim_{x\to0} \frac{1-\cos(x)}{(x)} = 0$$

Extreme Value Theorem (EVT)

If f is continuous on $[a, b]$, then it has both a minimum and a maximum on that interval. **Steps**:

1) Find critical points (set f'(x) = 0).
2) Plug critical points and endpoints into f(x)
3) Compare y-values to determine abs max and abs min values.

Sum/Difference of Cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Removable discontinuity is where variable from denominator cancels out with numerator (hole exists in graph)

NonRemovable discontinuity is where variable does not cancel out in denominator (Vertical asymptote exits in graph)

Squeeze Theorem:

If h(x) ≤ f(x) ≤ g(x) for all x in an open interval containing c, and if

$\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then

$\lim_{x\to c} f(x) = L$

Intermediate Value Theorem (IVT):

If f is continuous on a closed interval [a, b] and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.

In other words, if a function is continuous on an interval, then it has to hit all of the y- values in between the endpoints somewhere in the interval.

Continuity Conditions:

For a function, f, to be continuous at c,

1. $f(c)$ is defined (the point exists)
2. $\lim_{x\to c} f(x)$ exists
   *This means $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$
3. $\lim_{x\to c} f(x) = f(c)$

This means the point and the limits are equal to each other

Finding equation of tangent line: Steps:

1.Find slope by finding f '(x), plugging in given x-value

2.If not given, find y-value using original function f(x)

3.Plug into point-slope: $y - y_1 = m(x - x_1)$

Piecewise functions:

*If piecewise function is continuous at a point, then set the two parts equal to each other.

*If piecewise function is differentiable at a point, then set the derivatives of each part equal to each other.

Comparative Growth Rates $\lim_{x\to\infty} \frac{f(x)}{g(x)}$

logs < radical < polynomial < exponential

Note: If a line is tangent to a curve at a point , 2 important properties hold true:

a)The Curve and the tangent line share the same ordered pair at that point (they meet at the point) (set 2 functions equal to each other)

b)The Curve and the tangent line share the same slope at that point as well. (set their derivatives equal to each other)

*To find where function is not continuous set the denominator of f(x) = 0

* To find where function is not differentiable, set the denominator of f ' (x) = 0

Limit Definition of Derivative:

1) General Definition
$$f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h}$$
2) Alternative Definition
$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

Derivative Rules:

Power Rule:

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - fg'}{g^2}$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

(multiply outer function's derivative with the inner function's derivative)

Avg. ROC vs. Instantaneous ROC

(ROC = Rate of change)

Avg. ROC (avg velocity) = $\frac{f(b) - f(a)}{b - a}$

Inst. ROC (Inst. Velocity) = f '(c)

*Remember: Average Rate of change has - nothing to do with the derivative. (Just find slope between the endpoints!)

$\frac{d}{dx}|u| = \frac{u}{|u|} * u'$, $u \neq 0$

Increasing velocity means acceleration is pos.

Decreasing velocity means acceleration is neg.

* Velocity s'(t) is the derivative of position function

* Acceleration s''(t) is the the 2^{nd} derivative of position function. (acceleration is the derivative of velocity, the rate of change of velocity)

*Speed = |velocity| (speed is always positive)

*Speed is increasing when velocity and acceleration. have the same sign.

*Speed is decreasing when velocity and acceleration have opposite signs.

Displacement how far you are from where you started

Distance: total amount you have traveled

Linear Particle Motion – PVA

1. To find when particle at rest, set v(t) = 0 and solve for t.
2. Create sign line, and test intervals with v(t). Pos(+) means particle is moving right. Neg(–) means particle is moving left.

Linear approximation: *in summary*,

a) Find equation of tangent line
   *$y - y_1 = m(x - x_1)$
b) Plug decimal value into tangent line equation to approximate function value: y(x) ≈ mx + b

Implicit Differentiation:

When finding derivative, use this method if y is not defined explicitly in terms of x (In other words: there are multiple y's in the equation or y is mixed together with x's)

Steps: 1. Find derivative of each term 2. For derivatives involving y, be sure to attach $\frac{dy}{dx}$ 3. Solve for $\frac{dy}{dx}$

Curve Sketching:

*First Derivative Test: used for finding increasing/decreasing intervals and relative extrema:

1. Find critical points (set f '(x) = 0) 2. Make Slope Sign Line
3. Test each interval to determine + slope (inc.) or – slope (dec)
4. Rel. max if f '(x) changes from + to –
5. Rel. min if f '(x) changes from – to +

*Critical points can come from numerator or denominator

Finding Concavity/POI: 1. Find critical points [set f ''(x) = 0]
2.. Make 2nd derivative concavity sign line 3. Test interval.
+ means concave up, – means concave down. 4. POI exists if graph is continuous and change in concavity at critical point

L'Hopital's Rule: If plugging in x-value to find limit results in indeterminate form $\left(\frac{0}{0} \; or \pm \frac{\infty}{\infty}\right)$

1) Apply L'Hopital's Rule (take derivative of top and derivative of bottom separately)

If $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{0}{0} or \pm \frac{\infty}{\infty}$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

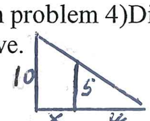
2) then try to find limit again (plug in x-value)

Sketching Derivative graphs: (Free Response #6)

Related Rates: a) Pythag Theoreom problems b) Problems involving geometric shapes c) Trig angle problems d) Similar Triangle Problems(see below)

Steps: 1)Write what's given 2)Write what you're finding 3)Write equation relating variables in problem 4)Differentiate with respect to t 5)Substitute and solve.

Ex: Similar triangle problem:



$\frac{5}{10} = \frac{y}{x+y}$

$\frac{dy}{dt}$ = R.O.C. of length of shadow; $\frac{dy}{dt} + \frac{dx}{dt}$ = R.O.C. of tip of shadow

Optimization steps: 1. Write equation for variable you want to optimize 2. Substitute to get equation in terms of one variable on the right side 3. Set derivative = 0 and solve.

Log Derivative Rules:

$\frac{d}{dx} \ln|u| = \frac{1}{u} * u'$

$\frac{d}{dx} \log_b u = \frac{1}{\ln b} * \frac{1}{u} * u'$

Exponential Derivatives Rules

$\frac{d}{dx} e^u = e^u * u'$

$\frac{d}{dx} b^u = \ln b * b^u * u'$

Trig Identities: $\sin^2\theta + \cos^2\theta = 1$

Double Angle: $\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Trig Derivatives:

$\frac{d}{dx}\sin u = \cos u * u'$ $\frac{d}{dx}\cos u = -\sin u * u'$

$\frac{d}{dx}\tan u = \sec^2 u * u'$ $\frac{d}{dx}\cot u = -\csc^2 u * u'$

$\frac{d}{dx}\sec u = \sec u \tan u * u'$ $\frac{d}{dx}\csc u = -\csc u \cot u * u'$

Finding Absolute Extrema: (Closed Interval)

1. Find first derivative 2. Find critical numbers
3. Test critical numbers and endpoints by plugging them into the original function 4. Determine absolute max and min.

2nd Derivative Test: Used for finding relative extrema (finds max/min, **not** POI) 1. Set f '(x) = 0 and find critical numbers. 2. Find f ''(x). 3. Plug critical points (from step 1) into f ''(x) and evaluate 4. **If positive**, then rel. min occurs (b/c f '(x) = 0 and f ''(x) > 0 and therefore concave up.)
5. If **negative**, then rel. min occurs (b/c f '(x) = 0 and f ''(x) < 0 and therefore concave down

"**Morgan's Method**" for evaluating and interpreting Derivative Graphs

X – x-ints
M – max & mins
P – POI

f(x)	f '(x)	f ''(x)
X		
M	X	
P	M	X
	P	M
		P

Rolle's Theorem: 3 Conditions

1. Continuous on [a,b] 2. Differentiable on (a,b)
3. f(a) = f(b) (endpoints have the same y-values)

Set derivative = 0 and solve for x

*Make sure x value resides between endpoints (a, b)

Mean Value Theorem (MVT) 2 conditions:

1. Continuous on [a,b] 2. Differentiable on (a,b)

Find Avg rate (avg slope) = $\frac{f(b) - f(a)}{b - a}$

Set f '(x) = Avg slope and solve for x.

*Make sure x value resides between endpoints (a, b)

Log/Exponent Properties:

ln(1) = 0

ln(e) = 1

$\ln(a^n) = n*\ln(a)$

ln(ab) = ln(a) + ln(b)

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

e≈ 2.718

$a^{\log_a x} = x$

$\log_a a^x = x$

Change of Base:

$\log_a x = \frac{\ln x}{\ln a}$

Exponent Properties:

$e^a e^b = e^{a+b}$ $(e^a)^b = e^{ab}$ $e^0 = 1$

$\ln e^x = x$ $e^{\ln x} = x$

Log Differentiation steps: (No logs in the problem initially!)

1) Take **ln** of both sides. 2) Expand right side.
3) Find derivative 4) Solve for $\frac{dy}{dx}$

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Inverse Trig Derivatives:

$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$ $\frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$ $\frac{d}{dx}arc\cot u = -\frac{u'}{1+u^2}$

$\frac{d}{dx}arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$ $\frac{d}{dx}arc\csc u = -\frac{u'}{|u|\sqrt{u^2-1}}$

Spring 2014 Calculus AB Topics Summary / Formula Sheet

Summation Formulas:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

Riemann Sum: Estimate Area under curve using rectangles

a) Right-handed sum
b) Left-handed sum
c) Mid-Point Rule

Trapezoid Rule: ½(width)(h_1 +2 h_2+ 2 h_3 + ... h_n)

width = $\frac{b-a}{n}$

Area using Limit Definition

$$\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(left\ endpoint + width * i)$$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Log Rule:

$$\int \frac{1}{u} du = \ln|u| + C$$

Exponential Rule: (Base e)

$$\int e^u du = e^u + C$$

Exponential Rule (base other than e)

$$\int a^u du = \frac{a^u}{\ln a} + C$$

Note: ln a is a constant

Integrals of Even/Odd Functions Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ Odd: $\int_{-a}^{a} f(x)dx = 0$

Trig Integrals:

$$\int \sin u\, du = -\cos u + C \qquad \int \cos u\, du = \sin u + C$$

$$\int \sec^2 u\, du = \tan u + C \qquad \int \sec u \tan u\, du = \sec u + C$$

$$\int \csc^2 u\, du = -\cot u + C \qquad \int \csc u \cot u\, du = -\csc u + C$$

More Trig Integrals:

$$\int \tan u\, du = -\ln|\cos u| + C \qquad \int \cot u\, du = \ln|\sin u| + C$$

$$\int \sec u\, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\, du = -\ln|\csc u + \cot u| + C$$

Trig properties:

$$\frac{\sin x}{\cos x} = \tan x \qquad \frac{\cos x}{\sin x} = \cot x$$

Integral properties

$$\int_a^a f(x)dx = 0 \qquad \int_a^b f(x)dx = -\int_b^a f(x)dx$$

CheckList Order for determining Method for Finding Integrals

(Check for correct method in the following order)
1.**Expand/Simplify/PowerRule** 2. Trig/Exponential/Log Rule
3. **U-Substitution** 4. U-Sub(Change of Variable) 5. Long Division/ Synthetic divison 6. ArcTrig (Complete the square) 7. IBP/Tab method

1st Fundamental Theorem of Calculus (FFTC)

$\int_a^b f(x)dx$ = F(b) – F(a) where F is the antiderivative of f.

* If a function is continuous on a closed interval, then it is integrable on that interval.

distance vs displacement

displacement = integral of velocity $\int_a^b v(t)dt$

distance = integral of absolute value of velocity $\int_a^b |v(t)|dt$

Mean Value Theorem for Integrals (Average Value Theorem)

If f is integrable on the closed interval [a, b], then the average value of f on the interval is $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$.

*This is derived from the Area of a rectangle:

Since height * width = Area, $f(c)*(b-a) = \int_a^b f(x)dx$

Therefore, $f(c) = \frac{\int_a^b f(x)dx}{b-a}$ or $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

2nd Fundamental Theorem of Calculus: (SFTC)

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x)$$

"a" is constant

*Highlights the inverse relationship between derivatives and integrals

PVA integral problem: Suppose f''(x) = 6x + 4
f'(0) = 3 and f(1) = 5. Find f(x)

$f''(x) = 6x + 4$ so $f'(x) = 3x^2 + 4x + C$.
Plug in (0, 3) to get $f'(x) = 3x^2 + 4x + 3$.
So $f(x) = x^3 + 2x^2 + 3x + K$.
Plug in (1, 5) to get that $f(x) = x^3 + 2x^2 + 3x - 1$

SFTC (Alternate)

$$\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x) - f(q(x)) \cdot q'(x)$$

Example of (SFTC)

$$\frac{d}{dx}\left[\int_{10}^{x^2} \sqrt{t-1}dt\right] = \sqrt{x^2-1} \cdot 2x = 2x\sqrt{x^2-1}$$

Differential Equations: Equations involving derivatives (Separation of Variables)

1)Rewrite y' as $\frac{dy}{dx}$ 2)Separate variables (y's on left, x's on right)

3)Take integral of both sides 4)Exponentiate both sides

5)Move constant of integration as leading coefficient

(ex. e^{kt+C} becomes $e^{kt}e^{C}$, then is written as Ce^{kt})

6. Solve for C (initial condition) 7. Solve for k

Varies directly or is directly proportional: use y = kx

Varies inversely or is inversely proportional: use $y=\frac{k}{x}$

Slope Fields: graphical approach to looking at solutions of a differential equation:

*Use the differential equation to match the individual slope segments which creates the slope fields (ex: $\frac{dy}{dx}=x-1$)

*Use the solution (integral) of the differential equation to match the shape and pattern of the slope field (ex: $y=\frac{1}{2}x^2-x+C$) (this solution will show graph patterms that resemble a parabola)

Exponential Growth/Decay

Be able to rewrite word problem in the form of differential equation: "The rate of increase of population is proportional to the population": Differential equation is $\frac{dP}{dt}=kp$

Exponential Growth example problem: The rate of increase of the population of a city is proportional to the population. If the population increases from 40,000 to 60,000 in 40 yrs, when will the population be 80,000?

(t, Population), (0, 40,000), (40, 60,000) (__ , 80,000)

$\frac{dP}{dt}=kp$ $\int\frac{dP}{dt}=\int kp$ $\int\frac{dP}{P}=\int kdt$ $\ln P=kt+C$ $e^{\ln P}=e^{kt+C}$

$P=Ce^{kt}$ C=40,000 $P=40{,}000e^{kt}$ $60{,}000=40{,}000e^{kt}$

$1.5=e^{kt}$ $\ln(1.5)=\ln e^{kt}$ $\ln(1.5)=k(40)$ $k=(\ln 1.5)/40$

$P=40{,}000e^{(\ln 1.5/40)t}$ $80{,}000=40{,}000e^{(\ln 1.5/40)t}$

$2=e^{(\ln 1.5/40)t}$ $\ln 2=\ln e^{(\ln 1.5/40)t}$ $\ln 2=\frac{\ln 1.5}{40}t$ Ans: t = 68.380yrs

Domain of solution to Differential Equation: the largest open interval containing the initial value for which the differential equation is defined

Arc Trig Integrals:

$$\int\frac{du}{\sqrt{a^2-u^2}}=\arcsin\frac{u}{a}+C \qquad \int\frac{du}{a^2+u^2}=\frac{1}{a}\arctan\frac{u}{a}+C$$

$$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}\text{arcsec}\frac{|u|}{a}+C$$

Completing the Square:

1) Group $x^2 + bx$
2) Add $\left(\frac{b}{2}\right)^2$
3) Factor

Example:
$x^2-6x+13$
$x^2-6x+\underline{9}+13-\underline{9}$
$(x-3)^2+4$

Area = $\int_a^b[f(x)-g(x)]dx$ If top – bottom, then integrate with respect to x

Area = $\int_a^b[f(y)-g(y)]dy$ If right – left, then integrate with respect to y

Volumes of Revolution: Disc Method

$$V=\pi\int_a^b R^2(dx\ or\ dy)$$

R = axis of revolution and function.
top – bottom if dx or right – left if dy

Horizontal axis: integrate with respect to x
Vertical axis: integrate with respect to y

Volumes of Revolution: Washer Method

$$V=\pi\int_a^b(R^2-r^2)(dx\ or\ dy)$$

R = axis of revolution and outer function
(top – bottom) or (right – left)

r = axis of revolution and inner function
(top – bottom) or (right – left)

Volumes with known cross sections

$$V=\int_a^b(\textit{Area}\text{ of cross section})(dx\ or\ dy)$$

*If Cross-section is perpendicular to x-axis, then Top – bottom dx
*If Cross-section is perpendicular to y-axis, then Right – Left dy

Area formula for cross sections:

1) Square: A = $(\text{base})^2$ 2) Rectangle: A = (base)(height)
3) Isosceles Right Triangle (hypotenuse on base) : $\frac{1}{4}(hypotenuse)^2$
4) Isosceles Right Triangle : A = $\frac{1}{2}(base)*(base)$
5) Equilateral Triangle : A = $\frac{\sqrt{3}}{4}(base)^2$
6) Semicircles: $A=\frac{\pi}{2}[radius]^2$ or $A=\frac{\pi}{8}[diameter]^2$

Integration by Parts: useful whenever

$$\int udv=uv-\int vdu$$

Steps: 1) Let u = L.I.P.E.T. (Preference order: Log/InverseTrig/Polynomial/Exponential/Trig)
2) find u, du, v, and dv
3) plug in and integrate

Tab Method: use whenever you have a polynomial (of degree higher than 1) multiplied by a function that you can antidifferentiate

Steps: 1) Create 3 columns: Signs | u | dv 2) Let u value be the **polynomial** 3) Let dv be the other portion 4) Find the derivative of polynomial (u) until reaching zero. 5) Find integral of dv the same number of times. 6) Assign alternating signs (+/ –). 7) Add the product of diagonal terms

Calculator Steps:

* *Remember – Calculator **ALWAYS** in **RADIAN** mode**

1. Evaluating derivatives
   Math 8 → nDeriv (y_1, x, value)
   [to enter y_1 go to VARS / Y-VARS / Function / 1 }
2. Evaluating Definite Integrals
   Math 9 → fnInt (y_1, x, lower bound, upper bound)
3. Evaluating Total Distance:
   Math 9 → fnInt (Abs (y_1), x, lower, upper)

Curve Sketching:

Evaluating f '(x) graph (velocity graph)

Portions of graph above x-axis indicates where f(x) is increasing (+ slope)
Portions of graph below x-axis indicates where f(x) is decreasing (– slope)
x-intercepts indicates critical points for potential max/min of f(x) graph
Area between graph and x-axis indicates accumulation of distance
Hills and valleys indicate POI 's
Increasing slope indicates where graph is concave up
Decreasing slope indicates where graph is concave down

Evaluating f ''(x) graph (acceleration graph)

Portions of graph above x-axis indicates concave up
Portions of graph below x-axis indicates concave down
x-intercepts indicates critical points for potential POI ' s
Area between graph and x-axis indicates accumulation of velocity