BC Theater Extra Credit Spring 2018 Assignment #1:

Directions: Create and Solve 1 original FRQ . Then answer the following 4 FRQs, make corrections (red ink) and score FRQs. Due Wed (3/28)

1) (Calculator FRQ)

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

AB/BC Theater Extra Credit Spring 2018 Assignment #1:

Directions: Create and Solve 2 original FRQ's. Then answer the following 3 FRQs, make corrections (red ink) and score FRQs. Due Wed (3/28)

1) (Calculator FRQ)

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$H(t)$ (°C)	65	68	73	80	90

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(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

estimate rate of change by finding slope

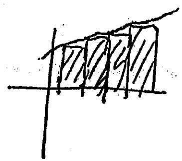
$\boxed{2}$ a) $H'(10) \approx \frac{H(12)-H(8)}{12-8} = \frac{80-73}{12-8} = \boxed{\frac{7}{4}\ °C/min}$

$\boxed{3}$ b) Avg. temperature $= \frac{1}{16}\int_0^{16} H(t)dt$ *use Avg. Value theorem $\frac{1}{b-a}\int_a^b f(t)dt$

Approximate using left Riemann Sum: (4 subintervals) $\int_0^{16} H(t)dt \approx 4(65)+4(68)+4(73)+4(80)$

$= 1144$

$\frac{1}{16}\int_0^{16} H(t)dt \approx \frac{1}{16}(1144) = \boxed{71.5°C}$



$\boxed{1}$ c) Since the graph of H is continually increasing (positive slope), the left Riemann Sum will be an underapproximation of the average temperature

$\boxed{3}$ d) Since the slopes of the values in the subinterval are increasing, this supports the claim that temperature is increasing at an increasing rate

slopes: $\frac{3}{4} < \frac{5}{4} < \frac{7}{4} < \frac{10}{4}$

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(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(a) $H'(10) \approx \dfrac{H(12) - H(8)}{12 - 8} = \dfrac{80 - 73}{4} = \dfrac{7}{4}$ °C/min

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer with units} \end{cases}$$

(b) Average temperature is $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$

$\displaystyle\int_0^{16} H(t)\,dt \approx 4 \cdot (65 + 68 + 73 + 80)$

Average temperature $\approx \dfrac{4 \cdot 286}{16} = 71.5$°C

$$3 : \begin{cases} 1 : \frac{1}{16}\int_0^{16} H(t)\,dt \\ 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{cases}$$

(c) The left Riemann sum approximation is an underestimate of the integral because the graph of H is increasing. Dividing by 16 will not change the inequality, so 71.5°C is an underestimate of the average temperature.

1 : answer with reason

(d) If a continuous function is increasing at an increasing rate, then the slopes of the secant lines of the graph of the function are increasing. The slopes of the secant lines for the four intervals in the table are $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, and $\frac{10}{4}$, respectively.

Since the slopes are increasing, the data are consistent with the claim.

OR

By the Mean Value Theorem, the slopes are also the values of $H'(c_k)$ for some times $c_1 < c_2 < c_3 < c_4$, respectively. Since these derivative values are positive and increasing, the data are consistent with the claim.

$$3 : \begin{cases} 1 : \text{considers slopes of four secant lines} \\ 1 : \text{explanation} \\ 1 : \text{conclusion consistent with explanation} \end{cases}$$

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

(Non-calculator)

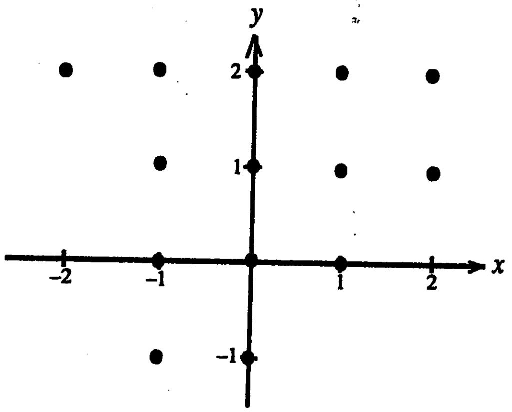
2) **Consider the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+4)}$.**

(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

(b) Sketch the solution curve that contains the point (–2, 2).

(c) Find a general solution to the differential equatiion.

(d) Find the particular solution to the differential equation that satisfies the initial condition $y(0) = 4$.



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(Non-calculator)

2) **Consider the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+4)}$.**

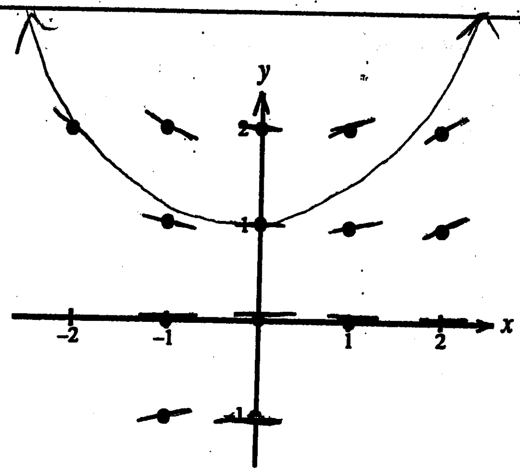
points:

$\boxed{1}$ **(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.**

$\boxed{1}$ **(b) Sketch the solution curve that contains the point (–2, 2).**

$\boxed{4}$ **(c) Find a general solution to the differential equatiion.**

$\boxed{3}$ **(d) Find the particular solution to the differential equation that satisfies the initial condition $y(0) = 4$.**



2) $\frac{dy}{dx} = \frac{xy}{x^2+4}$

c) $(x^2+4)\,dy = xy\,dx$

$\frac{dy}{y} = \frac{x}{x^2+4}\,dx$

$\int \frac{1}{y}\,dy = \int \frac{x}{x^2+4}\,dx$ $\quad u = x^2+4$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$\ln|y| = \int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|$

$\ln|y| = \frac{1}{2}\ln|x^2+4| + c$

$e^{\ln|y|} = e^{\ln|x^2+4|^{1/2} + c}$

$e^{\ln|y|} = e^{\ln|x^2+4|^{1/2}}\cdot e^{c}$

$|y| = |x^2+4|^{1/2}\cdot C$

$\boxed{y = C(x^2+4)^{1/2}}$ plug in (0,4)

↓

d) Find particular solution

$4 = C(0^2+4)^{1/2}$

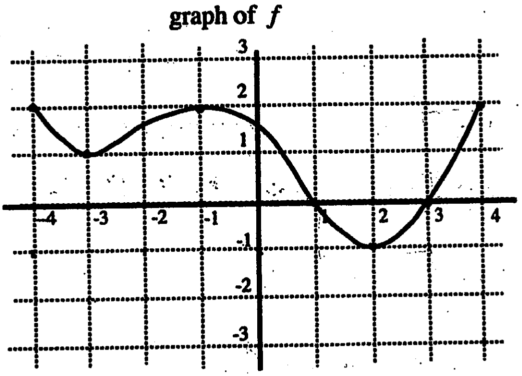
$4 = C(2)$

$2 = C$

$\boxed{y = 2(x^2+4)^{1/2}}$

(Non-calculator)

3) The graph of a differentiable function f on the closed interval $[-4, 4]$ is shown at the right. The graph of f has horizontal tangents at $x = -3, -1$ and 2.



Let $G(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

(a) Find $G(-4)$.

(b) Find $G'(-1)$.

(c) On which interval or intervals is the graph of G concave down. Justify your answer.

(d) Find the value of x at which G has its maximum on the closed interval $[-4, 4]$. Justify your answer.

(Non-calculator)

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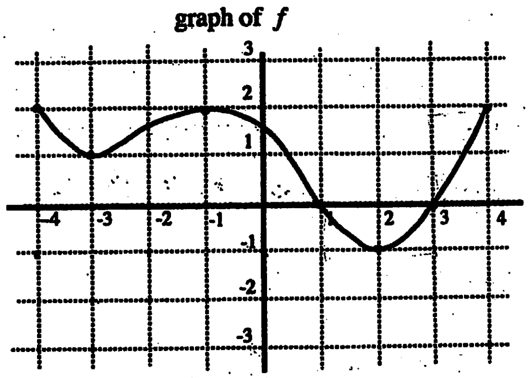

Let $G(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

points:

$\boxed{1}$ (a) Find $G(-4)$.

$\boxed{2}$ (b) Find $G'(-1)$.

$\boxed{3}$ (c) On which interval or intervals is the graph of G concave down. Justify your answer.

$\boxed{3}$ (d) Find the value of x at which G has its maximum on the closed interval $[-4, 4]$. Justify your answer.

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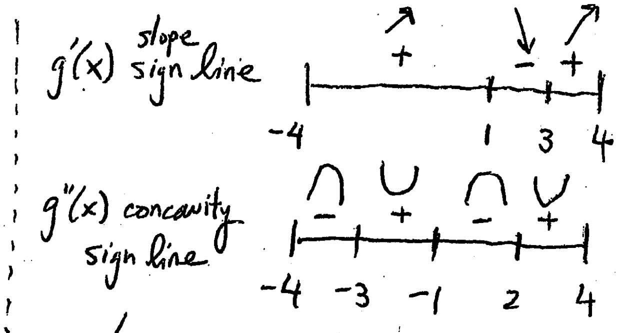
3) $\boxed{G(x) = \int_{-4}^{x} f(t)dt}$

$G'(x) = \frac{d}{dx}\int_{-4}^{x} f(t)dt = f(x)\cdot 1$ $\boxed{G'(x) = f(x)}$

$\boxed{G''(x) = f'(x)}$



a) $G(-4) = \int_{-4}^{-4} f(t)dt = \boxed{0}$

b) $G'(-1) = f(-1) = \boxed{2}$

c) G is concave down when slope of f is negative $(-4,-3) \cup (-1, 2)$

d) * Apply EVT to locate absolute maximum

i) test endpoints
ii) test relative $\underline{max}$, min

$G(-4) = \int_{-4}^{-4} f(t)dt = 0$

$G(4) = \int_{-4}^{4} f(t)dt \approx 7$

$G(1) = \int_{-4}^{1} f(t)dt \approx 6.5$

Absolute maximum is occuring at $x = 1$ in interval $[-4, 4]$

4) (Non-calculator)

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Question 6

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for f about $x = 2$.

(b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

4)

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Taylor Rule: $\frac{f^n(c)}{n!}(x-c)^n$

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(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

a) $$P_6(x) = \overbrace{7 + 0(x-2)}^{\text{tangent line}} + \frac{f''(2)}{2!}(x-2)^2 + \frac{f^4(2)}{4!}(x-2)^4 + \frac{f^6(2)}{6!}(x-2)^6$$

$$= 7 + \frac{1/3^2}{2!}(x-2)^2 + \frac{3!/3^4}{4!}(x-2)^4 + \frac{5!/3^6}{6!}(x-2)^6$$

general term

$$\boxed{P_6(x) = \underset{n=0}{7} + \underset{n=1}{\frac{1}{3^2}\cdot\frac{1}{2}(x-2)^2} + \underset{n=2}{\frac{3!}{3^4}\cdot\frac{1}{4!}(x-2)^4} + \underset{n=3}{\frac{5!}{3^6}\cdot\frac{1}{6!}(x-2)^6}} \cdots \frac{(2n-1)!}{3^{2n}\cdot(2n)!}(x-2)^{2n}$$

b) general term is $\underbrace{\frac{(2n-1)!}{3^{2n}\cdot(2n)!}}_{\text{coefficient}}(x-2)^{2n}$

$$\longrightarrow \frac{(2n-1)!}{3^{2n}\cdot(2n)!} = \frac{\cancel{(2n-1)!}}{3^{2n}\cdot(2n)\cdot\cancel{(2n-1)!}} = \boxed{\frac{1}{3^{2n}(2n)}}$$

c) $$f(x) = 7 + \sum_{n=1}^{\infty}\frac{1}{2n\cdot 3^{2n}}(x-2)^{2n}$$

*Ratio Test

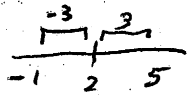
$$L = \lim_{n\to\infty}\left|\frac{(x-2)^{2(n+1)}}{2(n+1)\cdot 3^{2(n+1)}}\cdot\frac{2n\cdot 3^{2n}}{(x-2)^{2n}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(x-2)}^{2n+2}\cdot 2n\cdot\cancel{3^{2n}}}{(2n+2)\cdot\cancel{3}^{2n+2}\cdot\cancel{(x-2)^{2n}}}\right| = \lim_{n\to\infty}\left|\overbrace{\frac{2n}{2n+2}}^{\to =1}\cdot\frac{(x-2)^2}{3^2}\right| < 1$$

$$\left|\frac{(x-2)^2}{9}\right| < 1$$

$$|(x-2)^2| < 9$$

$$|x-2| < 3$$

$$|x-c| < r$$



$[(-1, 5)]$

Test $x=-1$:

$$7 + \sum_{n=1}^{\infty}\frac{1}{2n\cdot 3^{2n}}(-1-2)^{2n}$$

$$= \sum\frac{(-3)^{2n}}{2n\cdot 3^{2n}} = \frac{1}{2}\sum\frac{1}{n}$$ diverges (harmonic series)

Test $x=5$:

$$7 + \sum_{n=1}^{\infty}\frac{1}{2n\cdot 3^{2n}}(3)^{2n} = \sum\frac{\cancel{3^{2n}}}{2n\cdot\cancel{3^{2n}}}$$

$$= \frac{1}{2}\sum\frac{1}{n}$$ harmonic series diverge

$$\boxed{\text{I.O.C}: (-1, 5)}$$

Question 6

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \ge 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

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(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(a) $P_6(x) = 7 + \frac{1!}{3^2}\cdot\frac{1}{2!}(x-2)^2 + \frac{3!}{3^4}\cdot\frac{1}{4!}(x-2)^4 + \frac{5!}{3^6}\cdot\frac{1}{6!}(x-2)^6$

$$3 : \begin{cases} 1 : \text{polynomial about } x = 2 \\ 2 : P_6(x) \\ \quad \langle -1 \rangle \text{ each incorrect term} \\ \quad \langle -1 \rangle \text{ max for all extra terms,} \\ \qquad + \cdots, \text{ misuse of equality} \end{cases}$$

(b) $\frac{(2n-1)!}{3^{2n}}\cdot\frac{1}{(2n)!} = \frac{1}{3^{2n}(2n)}$

1 : coefficient

(c) The Taylor series for f about $x = 2$ is

$$f(x) = 7 + \sum_{n=1}^{\infty} \frac{1}{2n\cdot 3^{2n}}(x-2)^{2n}.$$

$$L = \lim_{n\to\infty}\left|\frac{\frac{1}{2(n+1)}\cdot\frac{1}{3^{2(n+1)}}(x-2)^{2(n+1)}}{\frac{1}{2n}\cdot\frac{1}{3^{2n}}(x-2)^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2n}{2(n+1)}\cdot\frac{3^{2n}}{3^2 3^{2n}}(x-2)^2\right| = \frac{(x-2)^2}{9}.$$

$L < 1$ when $|x-2| < 3$.

Thus, the series converges when $-1 < x < 5$.

When $x = 5$, the series is $7 + \sum_{n=1}^{\infty}\frac{3^{2n}}{2n\cdot 3^{2n}} = 7 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\frac{1}{n}$, the harmonic series, diverges.

When $x = -1$, the series is $7 + \sum_{n=1}^{\infty}\frac{(-3)^{2n}}{2n\cdot 3^{2n}} = 7 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\frac{1}{n}$, the harmonic series, diverges.

The interval of convergence is $(-1, 5)$.

$$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies interior of} \\ \quad \text{interval of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis/conclusion for} \\ \quad \text{both endpoints} \end{cases}$$