

Slope Fields: Additional Notes

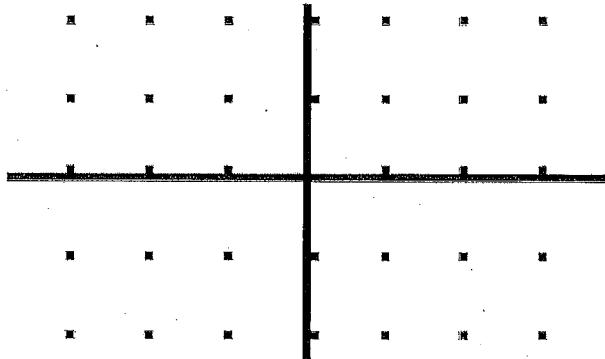
II. Finding Domain for Differential Equations

The domain of the solution to a differential equation is the largest open interval containing the initial value given for which both the differential equation and the solution are defined. This is called the “maximum interval of existence”

1. Solve the differential equation, and find the specific solution
2. Find the initial domain of the specific solution
3. Further restrict domain by:
 - a. Finding domain of differential equation
 - b. Be sure the initial condition is included in the domain
4. Graph solution and use slope fields to help determine/confirm domain
5. Incorporate restrictions to find the final domain

** Domain can only be ONE open interval that includes the initial value

1. Given $\frac{dy}{dx} = \frac{1}{x^2y}$, and $y(-1) = -2$
 - a) Find the particular solution
 - b) Find the domain.
 - c) Find the equation of tangent line through $(-1, -2)$
 - d) Use it to approximate $y(-1.1)$



2. Let $P(t)$ represent the number of wolves in a population at time t in years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k .
 - a) If $P(0) = 500$, find $P(t)$ in terms of t and k .
 - b) If $P(2) = 700$, find k .
 - c) Find $\lim_{t \rightarrow \infty} P(t)$

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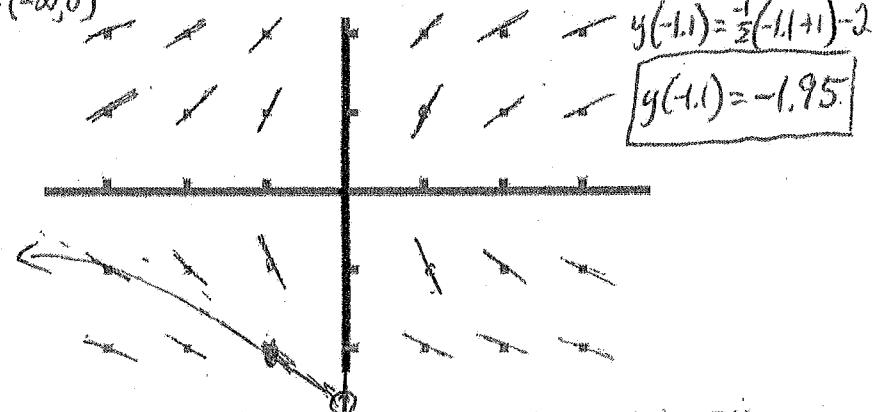
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2. Find the initial domain of the specific solution
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** Domain can only be ONE open interval that includes the initial value

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- c) Find the equation of tangent line through $(-1, -2)$ d) Use it to approximate $y(-1.1)$

$$\int y dy = \int \frac{dx}{x^2} = \int x^{-2} dx$$

$y^2 = -\frac{1}{x} + C$ $y^2 = \frac{-2}{x} + 2$ $y = \pm \sqrt{\frac{-2}{x} + 2}$ <small>*choose equation that satisfies $y(-1) = -2$</small> $y = -\sqrt{\frac{-2}{x} + 2}$	Domain: $(-\infty, 0)$
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2. Let $P(t)$ represent the number of wolves in a population at time t in years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k .

- a) If $P(0) = 500$, find $P(t)$ in terms of t and k .

b) If $P(2) = 700$, find k .

c) Find $\lim_{t \rightarrow \infty} P(t)$

$$P' = k(800 - P)$$

$$\frac{dP}{dt} = k(800 - P)$$

$$\int \frac{dP}{800 - P} = \int k dt$$

$$u = 800 - P$$

$$\frac{du}{dp} = -1 \quad dp = -du$$

$$\int \frac{-du}{u} = \int k dt$$

$$-\ln|u| = kt + C$$

$$-\ln|800 - P| = kt + C$$

$$\ln|800 - P| = -kt + C$$

$$800 - P = Ce^{-kt}$$

$$(0, 500)$$

$$(2, 700)$$

$$-P = Ce^{-kt} - 800$$

$$-500 = Ce^{-k(0)} - 800$$

$$300 = Ce^{-k(0)}$$

$$300 = C$$

$$-P = 300e^{-kt} - 800$$

$$-700 = 300e^{-kt} - 800$$

$$100 = 300e^{-k(2)}$$

$$\frac{1}{3} = e^{-k(2)}$$

$$\ln \frac{1}{3} = \ln e^{-k(2)}$$

$$\ln \frac{1}{3} = -2k$$

$$\frac{1}{3} \ln \frac{1}{3} = k$$

$$P(t) = -300e^{\frac{1}{3} \ln \frac{1}{3} t} + 800$$

$$\lim_{t \rightarrow \infty} -300e^{\frac{1}{3} \ln \frac{1}{3} t} + 800$$

$$= -300e^{\frac{1}{3} \ln \frac{1}{3} (\infty)} + 800$$

$$= -300(0) + 800$$

$$\lim_{t \rightarrow \infty} P(t) = \boxed{800}$$

A.P. Calculus AB Review for Test on Integrals of Logs/Exps (Chapter 6, 7, Slope fields)

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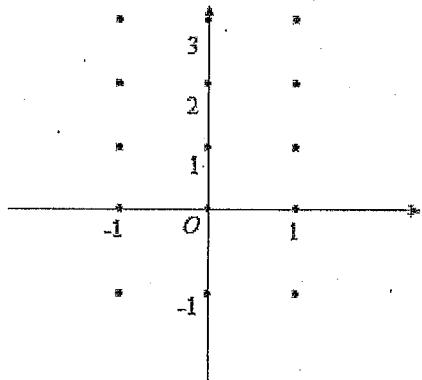
Calculators permitted.

1. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t . Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining.

- | | |
|---|---|
| <p>a. Write an equation for y, the amount of oil remaining in the well at any time t.</p> | <p>b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?</p> |
|---|---|
-
- c. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining. In order not to lose money, at what time t should oil no longer be pumped from the well?

2. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

- a) On the axes below, sketch a slope field for the given differential equation at the fifteen points indicated.



- b) While the slope field in part a is drawn at only fifteen points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.

- c) Find the particular solution, $y=f(x)$ to the given diff. equation with the initial condition $f(0)=3$.

- d) Find the equation of the line tangent to $y=f(x)$ at the point where $x=0$ and use it to approximate $f(0.1)$.

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3. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2(v + 16)$, with initial condition $v(0) = -50$.
- Use separation of variables to find an expression for v in terms of t , where t is measured in seconds.
 - Terminal velocity is defined as $\lim_{t \rightarrow \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.
 - It is safe to land when her speed is 20 feet per second. At what time, t , does she reach this speed?

4. Evaluate $\int \frac{x^2}{\sqrt{1-x^6}} dx$

5. $\int \frac{-4}{x^2 - 6x + 14} dx$

6. $\int \frac{-\csc(2x)\cot(2x)}{9^{\csc(2x)}} dx$

7. $\int \sec\left(\frac{x}{2}\right) dx$

Solution Key

1. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t . Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. (t, y)

- a) Write an equation for y , the amount of oil remaining in the well at any time t .

$$\int \frac{dy}{y} = \int k dt \quad y = Ce^{kt}$$

$$\ln y = kt + C \quad 1000000 = Ce^{k(0)}$$

$$y = e^{kt} \cdot e^C \quad 1000000 = Ce^0 \Rightarrow C = 1000000$$

$$y = 1000000e^{kt} \quad y = 1000000e^{\frac{1}{6}kt}$$

$$\frac{500000}{1000000} = 1000000e^{\frac{1}{6}k(6)} \quad y = 1000000e^{\frac{1}{6}k(6)}$$

- b. At what rate is the amount of oil in the well decreasing when there are

600,000 gallons of oil remaining? $y = 600,000$

Find $\frac{dy}{dt}$ when

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{dt} = \left(\frac{1}{6}\ln 0.5\right)(600,000) = -69344.718 \text{ gallons/gr.}$$

- b) It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining. In order not to lose money, at what time t should oil no longer be pumped from the well?

$$(t, 50,000) \quad 50000 = 100000e^{\frac{1}{6}kt} \quad 0.05 = e^{\frac{1}{6}kt}$$

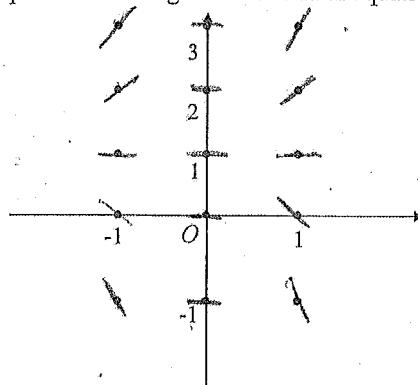
$$\frac{50000}{100000} = \frac{100000e^{\frac{1}{6}kt}}{100000} \quad \ln 0.05 = \ln e^{\frac{1}{6}kt}$$

$$\ln 0.05 = \frac{1}{6}kt \quad \frac{6\ln 0.05}{k} = t$$

$$t \approx 25.932 \text{ yrs. later}$$

2. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

- a) On the axes below, sketch a slope field for the given differential equation at the fifteen points indicated.



- b) While the slope field in part a is drawn at only fifteen points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative. Since x^2 is always positive, the determining factor is the y -value. When $y < 1$, then $\frac{dy}{dx} < 0$. Basically, all points below the line $y=1$ will have negative slope.

- c) Find the particular solution, $y = f(x)$ to the given diff. equation with the initial condition $f(0) = 3$.

$$\int \frac{dy}{y-1} = \int x^2 dx \quad e^{\ln(y-1)} = \frac{x^3}{3} + C \quad y-1 = Ce^{\frac{x^3}{3}}$$

$$y-1 = e^{\frac{x^3}{3}} \cdot C \quad 3-1 = Ce^{\frac{0^3}{3}} \quad 2=C \quad y-1 = 2e^{\frac{x^3}{3}}$$

- d) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.1)$.

Find ordered pair: $(0, 3)$

$$y = 2e^{\frac{0^3}{3}} + 1$$

$$y = 2(1) + 1 = 3$$

Find slope: $m=0$

$$\frac{dy}{dx} = x^2(y-1)$$

$$\frac{dy}{dx}(0,3) = 0^2(3-1) = 0$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 0(x - 0)$$

$$y = 3$$

$$y(0.1) = 3$$

⑥

3. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2(v+16)$, with initial condition $v(0) = -50$.

- a) Use separation of variables to find an expression for v in terms of t , where t is measured in seconds.

$$\int \frac{dv}{v+16} = \int -2 dt$$

$$\ln|v+16| = -2t + C$$

$$e^{\ln|v+16|} = e^{-2t+C}$$

$$v+16 = e^{-2t} \cdot e^C$$

$$v+16 = C e^{-2t}$$

$$(time, velocity)$$

$$(0, -50)$$

$$-50+16 = C e^{-2(0)}$$

$$-34 = C$$

$$V+16 = -34 e^{-2t}$$

$$v(t) = -34 e^{-2t} - 16$$

- b) Terminal velocity is defined as $\lim_{t \rightarrow \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

$$\lim_{t \rightarrow \infty} v(t) = \lim_{t \rightarrow \infty} -34 e^{-2t} - 16$$

$$\lim_{t \rightarrow \infty} \frac{-34}{e^{2t}} - 16$$

- c) It is safe to land when her speed is 20 feet per second. At what time, t , does she reach this speed?

$$(t, -20)$$

The skydiver is falling, so speed of 20 ft/s is in this situation $v(t) = -20 \text{ ft/s}$

$$v(t) = -34 e^{-2t} - 16$$

$$-20 = -34 e^{-2t} - 16$$

$$-4 = -34 e^{-2t}$$

$$\frac{4}{34} = e^{-2t}$$

$$\frac{2}{17} = e^{-2t}$$

$$\ln(\frac{2}{17}) = \ln e^{-2t}$$

$$\ln(\frac{2}{17}) = -2t$$

$$\frac{1}{2} \ln(\frac{2}{17}) = t$$

$$t = 1.07 \text{ seconds}$$

4. Evaluate $\int \frac{x^2}{\sqrt{1-x^6}} dx$

$$\int \frac{x^2}{\sqrt{(1-x^3)^2}} dx$$

$$u = x^3 \quad du = 3x^2 dx$$

$$\frac{du}{dx} = 3x^2 \quad \frac{du}{3x^2} = dx$$

$$\int \frac{x^2}{\sqrt{u^2-u^2}} \cdot \frac{du}{3x^2}$$

$$* \int \frac{du}{\sqrt{u^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

$$= \frac{1}{3} \int \frac{du}{\sqrt{u^2-u^2}}$$

$$= \frac{1}{3} \arcsin\left(\frac{u}{a}\right) + C$$

$$= \frac{1}{3} \arcsin\left(x^3\right) + C$$

5. $\int \frac{-4}{x^2 - 6x + 14} dx$ *complete the square

$$x^2 - 6x + \underline{\underline{\quad}} + 14 = \underline{\underline{\quad}}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

$$x^2 - 6x + 9 + 14 = 9$$

$$(x-3)^2 + 5$$

$$u = x-3 \quad du = dx$$

$$\frac{du}{dx} = 1 \quad a = \sqrt{5}$$

$$\int \frac{-4}{u^2 + a^2} du$$

$$* \text{actan rule}$$

$$= -4 \cdot \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$= -4 \cdot \frac{1}{\sqrt{5}} \arctan\left(\frac{x-3}{\sqrt{5}}\right) + C$$

$$= \frac{-4}{\sqrt{5}} \arctan\left(\frac{x-3}{\sqrt{5}}\right) + C$$

6. $\int \frac{-\csc(2x)\cot(2x)}{9\csc(2x)} dx$ *Recall

7. $\int \sec\left(\frac{x}{2}\right) dx$

- *Recall: $\int \sec u du = \ln|\sec u + \tan u| + C$

$$u = \frac{x}{2} \quad \frac{du}{dx} = \frac{1}{2} \quad dx = 2du \quad \int \sec u \cdot 2 du = 2 \int \sec u du$$

$$= 2 \ln|\sec u + \tan u| + C$$

$$u = -\csc(2x)$$

$$\frac{du}{dx} = -\csc(2x)\cot(2x) \cdot 2$$

$$dx = \frac{du}{-2\csc(2x)\cot(2x)}$$

$$+\frac{1}{2} \int 9^u du = +\frac{1}{2} \cdot \frac{9^u}{\ln 9} + C = \frac{9^{-\csc 2x}}{2 \ln 9} + C$$

$$= 2 \ln|\sec\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)| + C$$

1a) $y = Ce^{kt}$

$$1,000,000 = Ce^{k(0)}$$

$$\underline{1,000,000} = C$$

$$y = 1,000,000 e^{kt}$$

$$\frac{500,000}{1,000,000} = \frac{1,000,000 e^{k(6)}}{1,000,000}$$

$$0.5 = e^{6k}$$

$$\ln 0.5 = \ln e^{6k}$$

$$\frac{\ln 0.5}{6} = \frac{6k}{6} \ln e$$

$$\therefore \frac{\ln 0.5}{6} = k$$

(time, oil)

t y

^{finds}
 $C \leftarrow (0, 1, 000,000)$

^{find}
 $k \leftarrow (6, 500,000)$

(t , y)

$$y = 1,000,000 e^{\left(\frac{\ln 0.5}{6}\right)t}$$

$$\frac{600,000}{1,000,000} = 1,000,000 e^{\left(\frac{\ln 0.5}{6}\right)t}$$

$$0.6 = e^{\left(\frac{\ln 0.5}{6}\right)t}$$

$$\ln(0.6) = \ln e^{\left(\frac{\ln 0.5}{6}\right)t}$$

$$\frac{\ln(0.6)}{\frac{\ln 0.5}{6}} = \left(\frac{\ln 0.5}{6}\right)t \quad \text{ln e}$$

1b) $t = 4.422 \text{ yrs.}$

⑧ 2c) $\frac{dy}{dx} = x^2(y-1)$ Solve differential equation.
Find particular solution at (0,3)

$$\frac{dy}{dx} = \frac{(x^2)(y-1)}{1} \quad \left| \int \frac{1}{y-1} dy = \int x^2 dx \right.$$

$$\frac{1}{y-1} dy = \frac{x^2(y-1)}{y-1} dx \quad \left| \begin{array}{l} u = y-1 \\ \frac{du}{dy} = 1 \\ dy = du \end{array} \right. \quad \left| \begin{array}{l} \int \frac{1}{u} du \\ \ln|u| \end{array} \right.$$

$$\frac{1}{y-1} dy = x^2 dx \quad \left| \begin{array}{l} \ln|y-1| = \frac{x^3}{3} + C \\ \boxed{\ln|y-1| = \frac{x^3}{3} + C} \end{array} \right.$$

$$\left. \begin{array}{l} \ln|y-1| = \frac{x^3}{3} + C \\ e^{\ln|y-1|} = e^{\frac{x^3}{3} + C} \end{array} \right\} \text{at } (0,3) \quad y = Ce^{\frac{x^3}{3}} + 1 \quad \begin{array}{l} \text{plug in} \\ (0,3) \end{array}$$

$$|y-1| = e^{\frac{x^3}{3}} \cdot \boxed{e^C} \leftarrow C$$

$$|y-1| = Ce^{\frac{x^3}{3}}$$

$$y-1 = \pm Ce^{\frac{x^3}{3}}$$

$$3 = Ce^{\frac{0^3}{3}} + 1$$

$$3 = C(1) + 1$$

$$\underline{2 = C}$$

$$\boxed{y = 2e^{\frac{x^3}{3}} + 1}$$

2d) Find tangent line equation at $(1, 0)$ and
use it to approximate $f(1.2)$ ⑨

point: $(1, 0)$

slope: $\frac{dy}{dx} \Big|_{(1,0)} = x^2(y-1) \rightarrow (1)^2(0-1) = -1$

$$y = -1(\underline{x-1})$$

$$y(1.2) = -1(1.2-1) = -1(0.2) = \boxed{-0.2}$$

(10)
3a) $\frac{dv}{dt} = -2(v+16)$

$v(0) = -50$

* treat "t" as "x"

* treat "v" as "y"

$\frac{1}{v+16} dv = \frac{-2(v+16)}{v+16} dt$

$e^{\ln|v+16|} = e^{-2t+C}$

$|v+16| = e^{-2t} [e^C] < C$

$\frac{1}{v+16} dv = -2 dt$

$v+16 = \pm C e^{-2t}$

$v+16 = C e^{-2t}$ plug in

$v = C e^{-2t} - 16$

$-50 = C e^{-2(0)} - 16$

$-50 = C - 16$

$+16$

$+16$

$-34 = C$

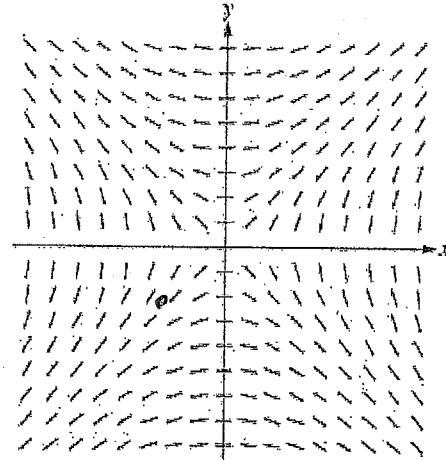
$u = v+16$	$\int \frac{1}{u} du$
$\frac{du}{dv} = 1$	$\ln u $
$dv = du$	$\ln v+16 = -2t + C$

3a) $v = -34e^{-2t} - 16$

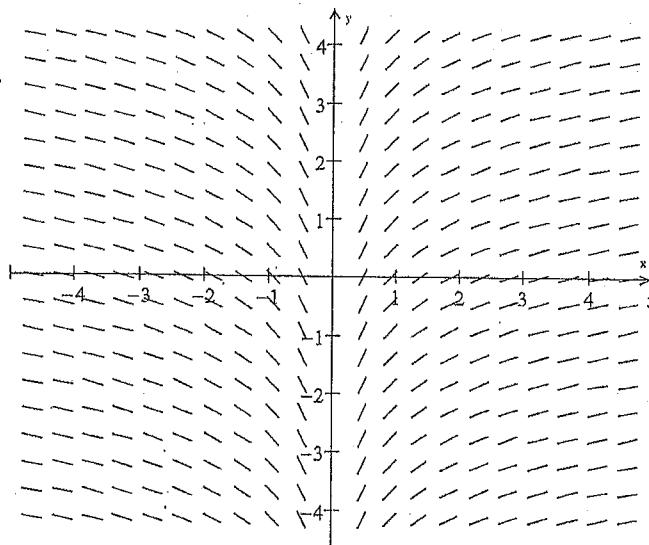
Differential Equation and Slope Fields Review WS #2

(11)

1. Given $\frac{dy}{dx} = \frac{x}{y}$, and $y(-3) = -2$
- Find the particular solution
 - Find the domain.
 - Find the equation of tangent line when $x = -\sqrt{6}$
 - Use it to approximate $y(-2.5)$



2. Consider the differential equation $\frac{dy}{dx} = \frac{1}{x}$.
- Find a particular solution to $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$
 - State its domain
 - Find the equation of tangent line when $x = e$
 - Use tangent line to approximate $y(2.8)$



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3. The rate of temperature decrease for a cup of coffee is given by equation $\frac{dy}{dt} = ky$ with t measured in minutes.

The initial temperature is 190°F and the temperature decreases to 76° after 5 minutes

a) Find the particular equation

b) Find how long it will take for the temperature to decrease to 50°

c) Find the average temperature of the coffee in the first 10 minutes

d) Using 4 equal intervals of right-handed Riemann sum, find the avg temperature of the coffee in the first 10 mins.

e) Find $\lim_{t \rightarrow \infty} y(t)$

4. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$ given $y(-2) = 1$

Differential Equation and Slope Fields Review WS #2

1. Given $\frac{dy}{dx} = \frac{x}{y}$, and $y(-3) = -2$
- Find the particular solution
 - Find the domain.
 - Find the equation of tangent line when $x = \sqrt{6}$
 - Use it to approximate $y(2.5)$

$$\int y dy = \int x dx$$

$$y^2 = x^2 - 5$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\frac{(-2)^2}{2} = \frac{(-3)^2}{2} + C$$

$$2 = \frac{9}{2} + C$$

$$\frac{-5}{2} = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} - \frac{5}{2}$$

a) $y = -\sqrt{x^2 - 5}$

b) $y \neq 0$ according to differential equation restriction and domain must include $x = -3$. Since $y \neq 0$, then $x = -\sqrt{5}$

c) Domain: $x < -\sqrt{5}$ or $(-\infty, -\sqrt{5})$

c) Find ordered pair

$$y = -\sqrt{(\sqrt{6})^2 - 5}$$

$$y = -\sqrt{6-5} = -1$$

point $(-\sqrt{6}, -1)$

$$\frac{dy}{dx} = \frac{x}{y} = \frac{-\sqrt{6}}{-1}$$

$$m = \sqrt{6}$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \sqrt{6}(x + \sqrt{6})$$

$$d) \text{ Since } y = \sqrt{6}(x + \sqrt{6}) - 1$$

$$y(-2.5) = \sqrt{6}(-2.5 + \sqrt{6}) - 1 = -1.124$$

2. Consider the differential equation $\frac{dy}{dx} = \frac{1}{x}$.

- Find a particular solution to $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$
- State its domain
- Find the equation of tangent line when $x = e$
- Use tangent line to approximate $y(2.8)$

$$\int dy = \int \frac{dx}{x}$$

Find ordered pair

$$c) y = \ln|x| + 1 = 1 + 1 = 2$$

point: $(e, 2)$

$$\frac{dy}{dx} = \frac{1}{x} = \frac{1}{e}, m = \frac{1}{e}$$

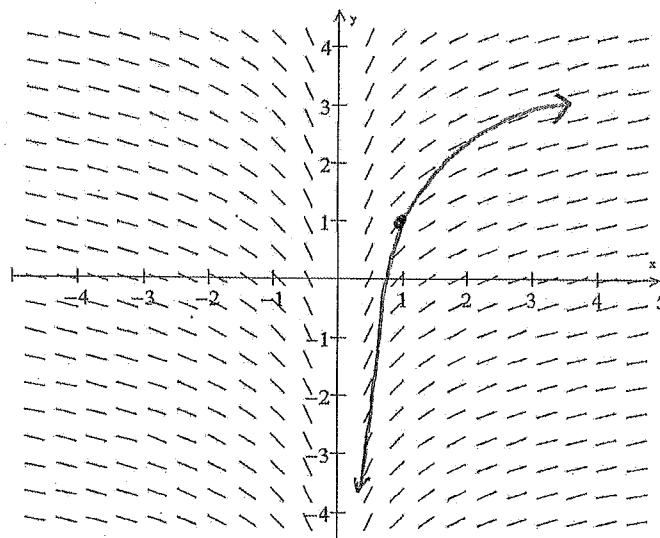
$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{e}(x - e)$$

$$d) \text{ Since } y = \frac{1}{e}(x - e) + 2,$$

$$y(2.8) = \frac{1}{e}(2.8 - e) + 2$$

$$y(2.8) = 2.030$$



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3. The rate of temperature decrease for a cup of coffee is given by equation $\frac{dy}{dt} = ky$ with t measured in minutes. The initial temperature is 190 °F and the temperature decreases to 76° after 5 minutes

(1) Find the particular equation (time, temp)	$\int \frac{dy}{y} = \int k dt$	$y = Ce^{kt}$ $190 = Ce^{k(0)}$ $190 = C$ $y = 190e^{kt}$	$76 = 190e^{k(5)}$ $\frac{76}{190} = e^{5k}$ $\frac{2}{5} = e^{5k}$ $\ln\left(\frac{2}{5}\right) = \ln e^{5k}$	$\ln\left(\frac{2}{5}\right) = 5k$ $\frac{1}{5}\ln\left(\frac{2}{5}\right) = k$
$(0, 190^\circ)$	$e^{\ln y} = e^{kt+c}$ $y = e^{kt} \cdot e^c$	$y = 190e^{kt}$		
$(5, 76)$			$\ln\left(\frac{2}{5}\right) = \ln e^{5k}$	$y = 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t}$

- b) Find how long it will take for the temperature to decrease to 50°

$(-, 50)$	$\ln\left(\frac{5}{19}\right) = \ln e^{\frac{1}{5} \ln\left(\frac{2}{5}\right)t}$	
$50 = 190e^{\frac{1}{5} \ln\left(\frac{2}{5}\right)t}$	$\ln\left(\frac{5}{19}\right) = \frac{1}{5} \ln\left(\frac{2}{5}\right)t$	$t = 7.285 \text{ mins.}$
$\frac{50}{190} = e^{\frac{1}{5} \ln\left(\frac{2}{5}\right)t}$	$7.285 = t$	

- c) Find the average temperature of the coffee in the first 10 minutes.

$$\text{Avg. value} = \frac{1}{b-a} \int_a^b y(t) dt$$

$$\text{Avg. temp} = \frac{1}{10-0} \int_0^{10} 190 e^{\frac{1}{5} \ln(\frac{1}{5})t} = \frac{1}{10} (870.9026) = 87.090^\circ F$$

$$\lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} 190e^{\frac{1}{5}\ln(\frac{1}{5})t} = 190e^{\frac{1}{5}\ln(\frac{1}{5})(\infty)} = 190e^{-\infty} = \frac{190}{e^{\infty}} = 0$$

- d) Using 4 equal intervals of right-handed Riemann sum, find the avg temperature of the coffee in the first 10 mins.

$$\text{width} = \frac{b-a}{n} = \frac{10-0}{4} = 2.5$$

$\int_0^{10} y(t) dt \approx 2.5 [y(2.5)] + 2.5 [y(5)] + 2.5 [y(7.5)] + 2.5 [y(10)]$
 $= 2.5 [y(2.5) + y(5) + y(7.5) + y(10)]$
 $= 2.5 [274.634] = 686.585$

4. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$ given $y(-2) = 1$

$$\begin{aligned} \int \frac{dy}{y} &= \int \frac{dx}{x} & | = C + 2 \\ \ln|y| &= \ln|x| + C & \frac{1}{2} = C \\ e^{\ln|y|} &= e^{\ln|x| + C} & y = \frac{1}{2}|x| \\ y &= e^{\ln|x|} \cdot e^C & \\ y &= C|x| & \text{Domain: } (-\infty, \infty) \end{aligned}$$

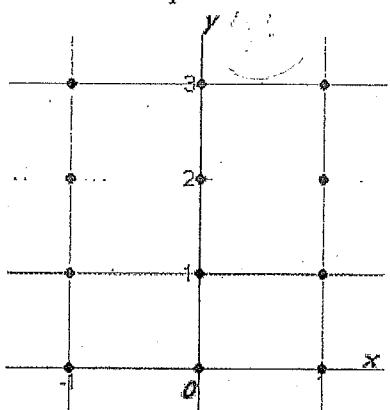
$$\text{Avg. temp.} = \frac{1}{10} \int_0^{10} y(t) dt$$

$$= \frac{1}{10}(686.585)$$

$$= 68.659^{\circ}\text{F}$$

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

a. Sketch a slope field through the indicated points.



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$

c. Write the equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$

d. Use the tangent line equation to estimate the value of $f(1.2)$

2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F . The initial temperature of the liquid is 160°F . After 5 minutes, the liquid's temperature is 60°F . How long will it take for its temperature to decrease to 30°F ?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

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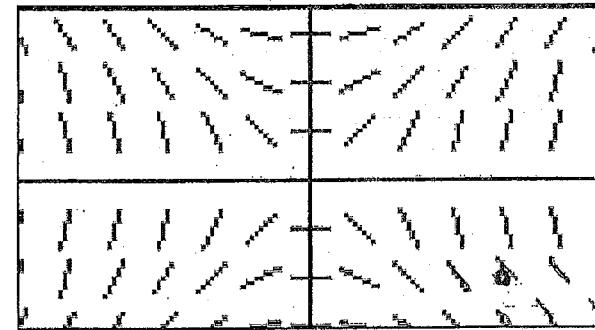
3. Match the slope field with the correct equation:

a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\frac{dy}{dx} = \frac{x}{y}$

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



3b. Find the particular solution given $y(5) = -2$

3c. Sketch the solution through the given point and find the domain

4. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$

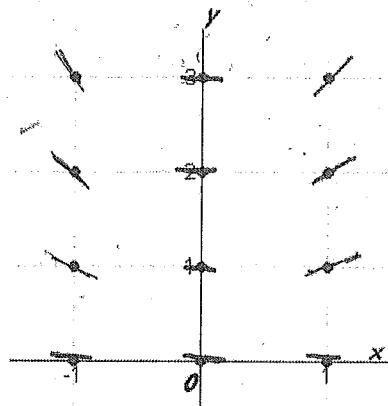
5. $\int (-\sin x)(3^{\cos x}) dx$

6. $\int \frac{8}{\sqrt{4-x^2+2x}} dx$ (hint: complete the square)

7. $\int \csc(1-3x) dx$

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

a. Sketch a slope field through the indicated points



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$

$$\begin{aligned}\int \frac{dy}{y} &= \int \frac{1}{2} x dx \\ \ln|y| &= \frac{1}{2} \left(\frac{x^2}{2}\right) + C \\ e^{\ln|y|} &= e^{\frac{1}{4}x^2 + C} \\ y &= e^{\frac{x^2}{4}} \cdot e^C \\ y &= Ce^{\frac{x^2}{4}}\end{aligned}$$

$$y = e^{\frac{x^2+1}{4}}$$

- c. Write the equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$

$$\begin{aligned}\frac{dy}{dx}(1,1) &= \frac{(1)(1)}{2} = \frac{1}{2} & \text{point: } (1, 1) \\ m &= \frac{1}{2} & y - 1 = \frac{1}{2}(x - 1) \\ y - y_1 &= m(x - x_1) & \boxed{y = \frac{1}{2}(x - 1) + 1}\end{aligned}$$

- d. Use the tangent line equation to estimate the value of $f(1.2)$

$$f(1.2) \approx \frac{1}{2}(1.2 - 1) + 1 = 1.1$$

$$\boxed{f(1.2) \approx 1.1}$$

2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F . The initial temperature of the liquid is 160°F . After 5 minutes, the liquid's temperature is 60°F . How long will it take for its temperature to decrease to 30°F ?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

(time, temperature)

$(0, 160)$

$(5, 60)$

$(\underline{\quad}, 30)$

$$u = y - 20 \quad \int \frac{dy}{y-20} = \int k dt$$

$$\begin{aligned}\frac{du}{dt} &= 1 \\ du &= dt \\ \int \frac{du}{u} &= \int dt\end{aligned}$$

$$e^{\ln|y-20|} = e^{-kt} \cdot e^C$$

$$y - 20 = C e^{-kt}$$

$$160 - 20 = C(1)$$

$$140 = C$$

$$y - 20 = 140 e^{-kt}$$

$$\ln|y-20| = kt + C$$

$$\ln|y-20| = kt$$

$$\ln|y-20| = k t$$

$$60 - 20 = 140 e^{5k}$$

$$40 = 140 e^{5k}$$

$$\frac{40}{140} = e^{5k}$$

$$\frac{2}{7} = e^{5k}$$

$$\ln(\frac{2}{7}) = \ln e^{5k}$$

$$\ln(\frac{2}{7}) = 5k$$

$$\frac{1}{5} \ln(\frac{2}{7}) = k$$

$$k(5)$$

$$60 - 20 = 140 e^{5k}$$

$$40 = 140 e^{5k}$$

$$\frac{40}{140} = e^{5k}$$

$$\frac{2}{7} = e^{5k}$$

$$\ln(\frac{2}{7}) = \ln e^{5k}$$

$$\ln(\frac{2}{7}) = 5k$$

$$\frac{1}{5} \ln(\frac{2}{7}) = k$$

$$k = \frac{1}{5} \ln(\frac{2}{7})$$

(18)

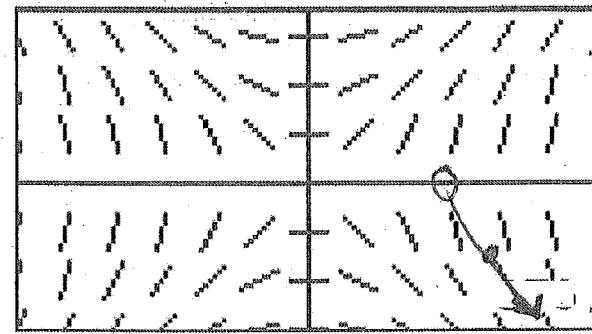
3. Match the slope field with the correct equation:

a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\frac{dy}{dx} = \frac{x}{y}$
when $y=0$, slope undefined
when $x=0$, slope = 0
at $(1,1)$, slope is positive

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



- 3b. Find the particular solution given $y(5) = -2$

$$\int y dy = \int x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + C$$

$$(-2)^2 = (5)^2 + C$$

$$-21 = C$$

$$4. \int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$$

$$\int \frac{2(e^x + e^{-x})}{u^2} \cdot \frac{du}{e^x - e^{-x}}$$

$$= 2 \int \frac{1}{u^2} du$$

$$= 2 \int u^{-2} du$$

$$6. \int \frac{8}{\sqrt{4-x^2+2x}} dx \quad (\text{hint: complete the square})$$

$$4-(x^2-2x+1)+1$$

$$4-(x^2-2x+1)+1$$

$$5-(x-1)^2$$

$$\int \frac{8}{\sqrt{5-(x-1)^2}} dx$$

$$\int \frac{8}{\sqrt{(\sqrt{5})^2-(x-1)^2}} dx$$

$$u = x-1$$

$$\frac{du}{dx} = 1$$

$$a = \sqrt{5}$$

$$8 \int \frac{du}{\sqrt{a^2-u^2}}$$

$$8 \cdot \arcsin\left(\frac{u}{a}\right) + C$$

$$= 8 \arcsin\left(\frac{x-1}{\sqrt{5}}\right) + C$$

- 3c. Sketch the solution through the given point, and find the domain \star the graph interval starts at $y=0$

$$0 = -\sqrt{x^2-21}$$

$$(0)^2 = (\sqrt{x^2-21})^2$$

$$0 = x^2 - 21$$

$$21 = x^2 \quad x = \pm \sqrt{21}$$

$$\boxed{\text{Domain: } (\sqrt{21}, \infty)}$$

5. $\int (-\sin x)(3^{\cos x}) dx$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{du}{-\sin x}$$

$$\int -\sin x \cdot 3^u \cdot \frac{du}{-\sin x}$$

$$\int 3^u du = \frac{3^u}{\ln 3} + C$$

$$= \frac{3^{\cos x}}{\ln 3} + C$$

7. $\int \csc(1-3x) dx$

$$u = 1-3x$$

$$\frac{du}{dx} = -3$$

$$dx = \frac{du}{-3}$$

$$\int \csc u \cdot du = -\frac{1}{3} \int \csc u du$$

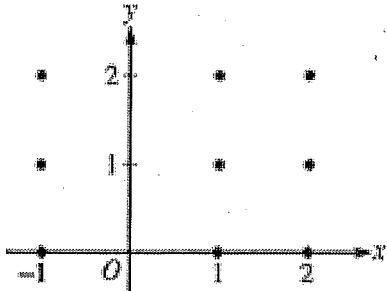
$$= -\frac{1}{3} \cdot \ln |\csc u + \cot u| + C$$

$$= \frac{1}{3} \ln |\csc(1-3x) + \cot(1-3x)| + C$$

Ch. 5&6 Test Review WS #4

1) Consider the differential equation $\frac{dy}{dx} = (x^3 - 3)(2y - 1)$

- a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated.



- b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 1$.

- c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

2) $\int_0^3 \frac{3e^x}{\sqrt{1+2e^x}} dx =$

(20)

- 3) Word Problem: The rise in population of a town is directly proportional to the number present at any given time t .

a) Write the differential equation and the general solution

- b) Population doubled in the 50 years between 1920 and 1970. In 1998, the population was 75,500. What was the population in 1920?

- 4) _____ Match the differential equation with the slope field graphed to the right.

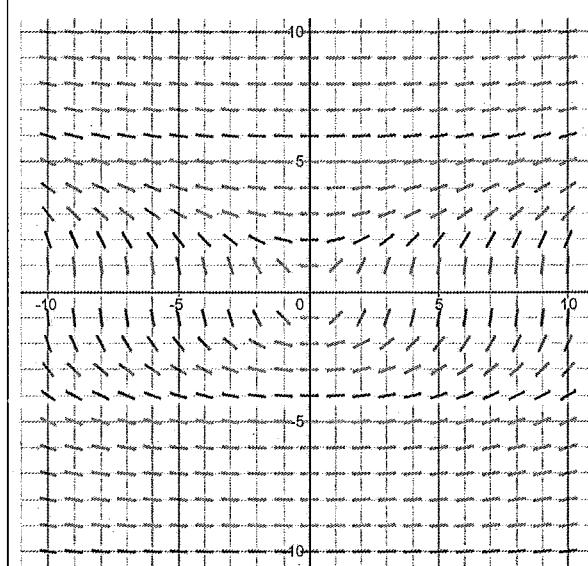
A) $\frac{dy}{dx} = \frac{x^2}{-y}$

B) $\frac{dy}{dx} = \frac{x}{y^2}$

C) $\frac{dy}{dx} = \frac{x^2}{y^2}$

D) $\frac{dy}{dx} = \frac{x}{y^3}$

E) $\frac{dy}{dx} = \frac{y^2}{x}$



5) $\int \sin\left(\frac{\pi x}{3}\right) - \csc(5x) dx =$

(21))

$$6) \int \frac{3x}{(4x^2)\sqrt{16x^4-7}} dx =$$

$$7) \int \frac{7}{x^2 + 16x + 67} dx =$$

8)

$$\int_1^3 \frac{2x^3 - 5}{x + 1} dx =$$

9)

$$\int \frac{5}{2x \ln x^3} dx$$

(Chapter 5) Derivative & Integral Rules Reference Sheet

Derivative Rules:

Power Rule:

$$\frac{d}{dx} x^n = nx^{n-1}$$

Trig Derivatives:

$$\frac{d}{dx} \sin u = \cos u * u'$$

$$\frac{d}{dx} \cos u = -\sin u * u'$$

$$\frac{d}{dx} \tan u = \sec^2 u * u'$$

$$\frac{d}{dx} \cot u = -\csc^2 u * u'$$

$$\frac{d}{dx} \sec u = \sec u \tan u * u'$$

$$\frac{d}{dx} \csc u = -\csc u \cot u * u'$$

$$\frac{d}{dx} e^u = e^u * u'$$

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

$$\frac{d}{dx} a^u = \ln a * a^u * u'$$

$$\frac{d}{dx} \log_a u = \frac{1}{\ln a} * \frac{u'}{u}$$

Integral Rules:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

More Trig Integral Rules:

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

Arc-Trig Integral Rules

$$17. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$16. \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$18. \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

Arc-Trig derivative Rules

$$19. \frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$20. \frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$21. \frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$22. \frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

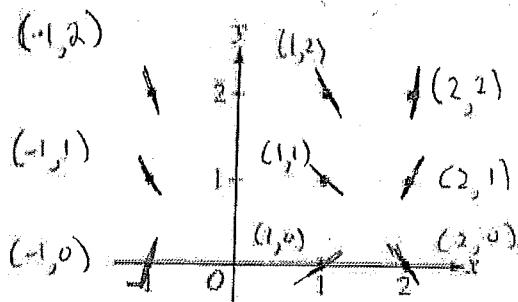
$$23. \frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u| \sqrt{u^2 - 1}}$$

$$24. \frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u| \sqrt{u^2 - 1}}$$

Ch. 5&6 Test Review WS #4

1) Consider the differential equation $\frac{dy}{dx} = (x^3 - 3)(2y - 1)$

- a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated.



- b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 1$.

$$\frac{dy}{dx} = \frac{(x^3 - 3)(2y - 1)}{1}$$

$$dy = (x^3 - 3)(2y - 1) dx$$

$$\int \frac{dy}{2y-1} = \int x^3 - 3 dx$$

$$u = 2y - 1 \quad \frac{du}{dy} = 2$$

$$\frac{du}{2} = dy$$

- c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

point: $(0, 1)$

$$\text{slope: } \left. \frac{dy}{dx} \right|_{(0,1)} = (0^3 - 3)(2 - 1) = -3$$

$$2) \int_0^3 \frac{3e^x}{\sqrt{1+2e^x}} dx =$$

$$\int \frac{3e^x}{(1+2e^x)^{1/2}} dx \quad \left. \frac{du}{dx} = \frac{du}{2e^x} \right. \quad u = 1+2e^x$$

$$\frac{du}{dx} = 2e^x$$

$$\int \frac{3e^x}{u^{1/2}} \cdot \frac{du}{2e^x}$$

$$\begin{aligned} \int \frac{1}{u} \cdot \frac{du}{2} &= \int x^3 - 3 dx \\ \frac{1}{2} \int \frac{1}{u} du &= \int x^3 - 3 dx \\ 2 \left(\frac{1}{2} \ln |2y-1| \right) &= \frac{x^4}{4} - 3x + C \\ \ln |2y-1| &= \frac{x^4}{4} - 3x + C \\ |2y-1| &= e^{\frac{x^4}{4} - 3x + C} \\ 2y-1 &= Ce^{\frac{x^4}{4} - 3x} \\ y &= \frac{1}{2}Ce^{\frac{x^4}{4} - 3x} + \frac{1}{2} \\ y &= Ce^{\frac{x^4}{4} - 3x} + \frac{1}{2} \end{aligned}$$

plug in $(0, 1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x - 0)$$

$$y = -3(x - 0) + 1$$

$$y(0.2) = -3(0.2) + 1$$

$$y(0.2) = 0.4$$

$$1 = Ce^0 + \frac{1}{2}$$

$$1 = C + \frac{1}{2}$$

$$\frac{1}{2} = C$$

$$y = \left(\frac{1}{2}\right)e^{\frac{x^4}{4} - 3x} + \frac{1}{2}$$

$$\begin{aligned} \left. \frac{3}{2} \int u^{-1/2} du \right|_0^3 &= \left. 3u^{1/2} \right|_0^3 = 3(1+2e^3)^{1/2} - 3(1+2e^0)^{1/2} \\ \left. \frac{3}{2} \cdot \left(\frac{u^{1/2}}{1/2} \right) \right|_0^3 &= \left. \frac{3}{2} \cdot 2u^{1/2} \right|_0^3 = \left. 3(1+2e^3)^{1/2} - 3(3)^{1/2} \right| \end{aligned}$$

3) Word Problem: The rise in population of a town is directly proportional to the number present at any given time t .

a) Write the differential equation and the general solution

$$P' = kP \Rightarrow P = Ce^{kt}$$

b) Population doubled in the 50 years between 1920 and 1970. In 1998, the population was 75,500.
What was the population in 1920?

(time, Population)	$P = Ce^{kt}$	$\frac{\ln 2}{50} = \frac{50k}{50}e$	$C = 25,606$
t		$\frac{\ln 2}{50} = k$	Population was ~25,606 in 1920
(0, C)	$2C = Ce^{k(50)}$		
(50, 2C)	$2 = e^{50k}$		
(78, 75,500)	$\ln 2 = \ln e^{50k}$	$P = Ce^{(\frac{\ln 2}{50})t}$	
		$75,500 = Ce^{(\frac{\ln 2}{50})(78)}$	

4) Match the differential equation with the slope field graphed to the right.

A) $\frac{dy}{dx} = \frac{x^2}{-y}$

i) slope = 0? (?)

B) $\frac{dy}{dx} = \frac{x}{y^2}$

ii) slope undefined? (?)

C) $\frac{dy}{dx} = \frac{x^2}{y^2}$

iii) pos. slope?

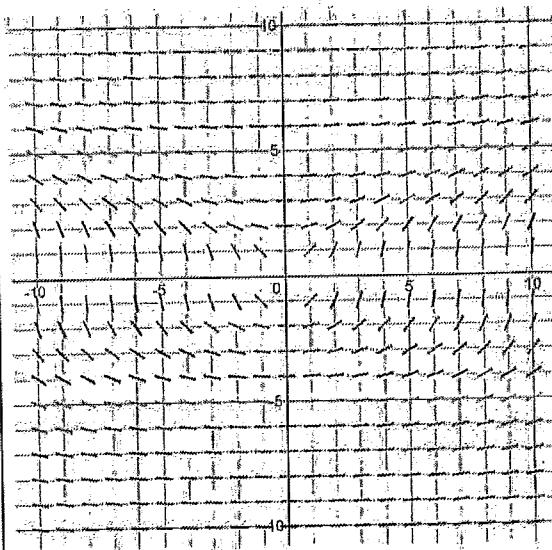
D) $\frac{dy}{dx} = \frac{x}{y^3}$

Q1, Q4 (when $x > 0$)

E) $\frac{dy}{dx} = \frac{y^2}{x}$

iv) neg. slope?

Q2, Q3 (when $x < 0$)



5) $\int \sin\left(\frac{\pi x}{3}\right) - \csc(5x) dx =$

$$\int \sin\left(\frac{\pi x}{3}\right) dx - \int \csc(5x) dx$$

$$u = \frac{\pi x}{3}$$

$$du = \frac{\pi}{3} dx \quad dx = \frac{3}{\pi} du$$

$$\frac{du}{dx} = \frac{1}{3}$$

$$\pi dx = 3du$$

$$\pi \cdot \frac{3}{\pi} du = 3du$$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$dx = \frac{1}{5} du$$

$$\frac{1}{5} \int \csc u du$$

$$\frac{1}{5} \int \csc u \cdot \frac{1}{5} du$$

$$\frac{1}{5} \int \csc u du$$

$$\int \csc u \cdot \frac{1}{5} du$$

$$\frac{1}{5} \int \csc u du$$

$$*\int \sin u du = -\cos u + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$\frac{3}{\pi} \int \sin u du - \frac{1}{5} \int \csc u du$$

$$\frac{3}{\pi} \cos\left(\frac{\pi x}{3}\right) - \left(-\frac{1}{5} \ln|\csc 5x + \cot 5x|\right) + C$$

$$\frac{3}{\pi} \cos\left(\frac{\pi x}{3}\right) + \frac{1}{5} \ln|\csc 5x + \cot 5x| + C$$

$$\star \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$$

$$6) \int \frac{3x}{(4x^2)\sqrt{16x^4-7}} dx$$

$$\int \frac{3x}{4x^2\sqrt{()^2-()^2}} dx \quad \begin{cases} a = \sqrt{7} \\ u = 4x^2 \\ \frac{du}{dx} = 8x \\ dx = \frac{du}{8x} \end{cases}$$

$$\int \frac{3x}{u\sqrt{u^2-a^2}} \frac{du}{8x} \rightarrow \frac{3}{8} \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\frac{3}{8} \cdot \frac{1}{\sqrt{7}} \operatorname{arcsec}\left(\frac{|4x^2|}{\sqrt{7}}\right) + C$$

$$\boxed{\frac{3}{8\sqrt{7}} \operatorname{arcsec}\left(\frac{|4x^2|}{\sqrt{7}}\right) + C}$$

8) synthetic division

* long division/synthetic division

$$\int_1^3 \frac{2x^3 - 5}{x+1} dx$$

$$\begin{array}{r} 3x^3 & 0 & 0 & -5 \\ \downarrow & -2 & 2 & -2 \\ 2x^3 & -2 & 2 & -7 \\ \hline & & & x+1 \end{array}$$

$$\int 2x^2 - 2x + 2 - \frac{7}{x+1} dx$$

$$\boxed{\frac{2x^3}{3} - \frac{2x^2}{3} + 2x - 7 \ln|x+1|}$$

$$\boxed{\frac{2}{3}(3)^3 - (3)^2 + 2(3) - 7 \ln|4| - \left(\frac{2}{3} - 1 + 2 - 7 \ln|2|\right)}$$

$$\star \int \frac{du}{a^2+u^2} = \frac{1}{a} \operatorname{arctan}\left(\frac{|u|}{a}\right) + C$$

$$7) \int \frac{7}{x^2 + 16x + 67} dx$$

* complete the square $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{16}{2}\right)^2} = \sqrt{8^2} = 64$

$$\begin{aligned} x^2 + 16x + 64 + 67 - 64 &= 3 \\ (x+8)(x+8) &= 3 \\ (x+8)^2 &= 3 \end{aligned}$$

$$\begin{cases} a = \sqrt{3} \\ u = x+8 \\ du = dx \end{cases}$$

$$\int \frac{7}{(x+8)^2 + 3} dx$$

$$\int \frac{du}{u^2+a^2}$$

$$\int \frac{dx}{(x+8)^2 + (\sqrt{3})^2}$$

$$\boxed{7 \cdot \frac{1}{\sqrt{3}} \operatorname{arctan}\left(\frac{x+8}{\sqrt{3}}\right) + C}$$

$$\boxed{\frac{7}{\sqrt{3}} \operatorname{arctan}\left(\frac{x+8}{\sqrt{3}}\right) + C}$$

9)

* u-substitution

$$\int \frac{5}{2x \ln x^3} dx \rightarrow \int \frac{5}{2x \cdot 3 \ln x} dx$$

$$\int \frac{1}{x \ln x} dx$$

$$\frac{5}{6} \int \frac{1}{x^2 u} \cdot x du \rightarrow \frac{5}{6} \int \frac{1}{u} du$$

u=lnx

$$\frac{du}{dx} = \frac{1}{x}$$

$$dx = x du$$

$$\frac{5}{6} \ln|u| + C$$

$$\boxed{\frac{5}{6} \ln|\ln x| + C}$$

OR

$$\boxed{\frac{40}{3} - 7 \ln 4 - 7 \ln 2}$$

(26)