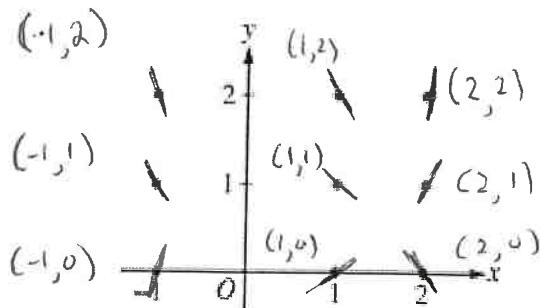


Ch. 5&6 Test Review WS #4

Key

1) Consider the differential equation $\frac{dy}{dx} = (x^3 - 3)(2y - 1)$

- a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated



- b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 1$.

$$\frac{dy}{dx} = \frac{(x^3 - 3)(2y - 1)}{1}$$

$$dy = (x^3 - 3)(2y - 1) dx$$

$$\int \frac{dy}{2y-1} = \int x^3 - 3 dx$$

$$\begin{cases} u = 2y - 1 \\ du = 2 dy \end{cases} \quad \begin{cases} dy = \frac{du}{2} \\ \frac{du}{2} = \frac{dy}{2y-1} \end{cases}$$

$$\int \frac{1}{u} \cdot \frac{du}{2} = \int x^3 - 3 dx$$

$$\frac{1}{2} \int \frac{1}{u} du = \int x^3 - 3 dx$$

$$2 \left(\frac{1}{2} \ln|2y-1| \right) = \frac{x^4}{4} - 3x + C$$

$$\ln|2y-1| = \frac{x^4}{4} - 3x + C$$

$$e^{\ln|2y-1|} = e^{\frac{1}{2}x^4 - 6x + C}$$

$$|2y-1| = e^{\frac{1}{2}x^4 - 6x + C}$$

$$2y-1 = Ce^{\frac{1}{2}x^4 - 6x}$$

$$y = \frac{1}{2}Ce^{\frac{1}{2}x^4 - 6x} + \frac{1}{2}$$

$$y = Ce^{\frac{1}{2}x^4 - 6x} + \frac{1}{2}$$

plug in $(0, 1)$

- c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

point: $(0, 1)$

$$\text{slope: } \left. \frac{dy}{dx} \right|_{(0,1)} = (0^3 - 3)(2 \cdot 1) = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x - 0)$$

$$y = -3(x - 0) + 1$$

$$y(0.2) = -3(0.2) + 1$$

$$y(0.2) = 0.4$$

$$1 = Ce^0 + \frac{1}{2}$$

$$1 = C + \frac{1}{2}$$

$$\frac{1}{2} = C$$

$$y = \left(\frac{1}{2}\right)e^{\frac{1}{2}x^4 - 6x} + \frac{1}{2}$$

2) $\int_0^3 \frac{3e^x}{\sqrt{1+2e^x}} dx =$

$$\int \frac{3e^x}{(1+2e^x)^{1/2}} dx \quad \left. \frac{du}{dx} = \frac{du}{2e^x} \right. \quad \left. \frac{3}{2} \int u^{-1/2} du \right.$$

$$\begin{cases} u = 1+2e^x \\ \frac{du}{dx} = 2e^x \end{cases}$$

$$\int \frac{3e^x}{u^{1/2}} \cdot \frac{du}{2e^x}$$

$$\begin{cases} \frac{3}{2} \cdot \left(\frac{u^{1/2}}{1/2} \right) \\ \frac{3}{2} \cdot \frac{2}{1} u^{1/2} \end{cases}$$

$$3u^{1/2} \rightarrow 3(1+2e^x)^{1/2} \Big|_0^3$$

$$\begin{cases} 3(1+2e^3)^{1/2} - 3(1+2e^0)^{1/2} \\ 3(1+2e^3)^{1/2} - 3(3)^{1/2} \end{cases}$$

3) Word Problem: The rise in population of a town is directly proportional to the number present at any given time t .

a) Write the differential equation and the general solution

$$P' = kP \rightarrow P = Ce^{kt}$$

b) Population doubled in the 50 years between 1920 and 1970. In 1998, the population was 75,500.

What was the population in 1920?

(time, Population)

t

(0, C)

(50, 2C)

(78, 75,500)

$$P = Ce^{kt}$$

$$2C = Ce^{k(50)}$$

$$2 = e^{50k}$$

$$\ln 2 = \ln e^{50k}$$

$$\frac{\ln 2}{50} = \frac{1}{50} \ln e$$

$$\frac{\ln 2}{50} = k$$

$$P = Ce^{\left(\frac{\ln 2}{50}\right)t}$$

$$75,500 = Ce^{\left(\frac{\ln 2}{50}\right)(78)}$$

$$C = 25,606$$

Population was ~25,606 in 1920

4) Match the differential equation with the slope field graphed to the right.

A) $\frac{dy}{dx} = \frac{x^2}{-y}$

i) slope=0? (?)

→ B) $\frac{dy}{dx} = \frac{x}{y^2}$

ii) slope undefined? (?)

C) $\frac{dy}{dx} = \frac{x^2}{y^2}$

iii) pos. slope?

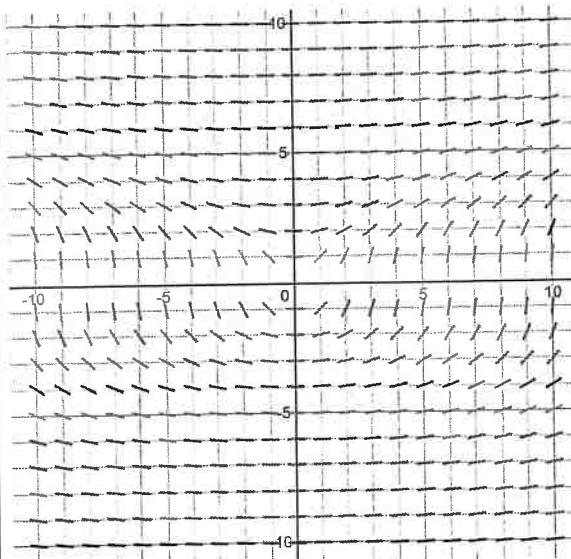
D) $\frac{dy}{dx} = \frac{x}{y^3}$

Q1, Q4 (when $x > 0$)

→ E) $\frac{dy}{dx} = \frac{y^2}{x}$

iv) neg. slope?

Q2, Q3 (when $x < 0$)



5) $\int \sin\left(\frac{\pi x}{3}\right) - \csc(5x) dx =$

$$\int \sin\left(\frac{\pi x}{3}\right) dx - \int \csc(5x) dx$$

$$u = \frac{\pi x}{3}$$

$$\frac{du}{dx} = \frac{\pi}{3}$$

$$\pi dx = 3du$$

$$dx = \frac{3du}{\pi}$$

$$\frac{du}{dx} = \frac{1}{5}$$

$$dx = \frac{du}{5}$$

$$\frac{1}{5} du$$

$$\frac{1}{5} \int \csc u du$$

$$\frac{1}{5} \int \csc u du$$

$$\int \csc u \cdot \frac{du}{5}$$

$$\frac{1}{5} \int \csc u du$$

$$\frac{1}{5} \int \csc u du$$

$$* \int \sin u du = -\cos u + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$\frac{3}{\pi} \int \sin u du - \frac{1}{5} \int \csc u du$$

$$-\frac{3}{\pi} \cos\left(\frac{\pi x}{3}\right) - \left(-\frac{1}{5} \ln|\csc 5x + \cot 5x| + C\right)$$

$$-\frac{3}{\pi} \cos\left(\frac{\pi x}{3}\right) + \frac{1}{5} \ln|\csc 5x + \cot 5x| + C$$

$$*\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \arcsin\left(\frac{|u|}{a}\right) + C$$

$$6) \int \frac{3x}{(4x^2)\sqrt{16x^4-7}} dx =$$

$$\int \frac{3x}{4x^2\sqrt{()^2-()^2}} dx \quad \begin{cases} a=\sqrt{7} \\ u=4x^2 \\ \frac{du}{dx}=8x \\ dx=\frac{du}{8x} \end{cases}$$

$$\int \frac{3x}{u\sqrt{u^2-a^2}} \cdot \frac{du}{8x} \rightarrow \frac{3}{8} \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\frac{3}{8} \cdot \frac{1}{\sqrt{7}} \arcsin\left(\frac{|4x^2|}{\sqrt{7}}\right) + C$$

$$\boxed{\frac{3}{8\sqrt{7}} \arcsin\left(\frac{|4x^2|}{\sqrt{7}}\right) + C}$$

8)

* long division / synthetic division

$$\int_1^3 \frac{2x^3 - 5}{x+1} dx$$

$$\begin{array}{r} 2x^3 \ 0 \ 0 \ -5 \\ \downarrow \quad -2 \ 2 \ -2 \\ 2x^2 - 2 \ 2 \ \underline{-7} \\ \underline{x+1} \end{array}$$

$$\int 2x^2 - 2x + 2 - \frac{7}{x+1} dx$$

$$\left[\frac{2x^3}{3} - \frac{2x^2}{2} + 2x - 7 \ln|x+1| \right]^3$$

$$\boxed{\frac{2}{3}(3)^3 - (3)^2 + 2(3) - 7 \ln|4| - \left(\frac{2}{3} - 1 + 2 - 7 \ln|2| \right)}$$

or

$$\boxed{\frac{40}{3} - 7 \ln 4 - 7 \ln 2}$$

$$*\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$7) \int \frac{7}{x^2 + 16x + 67} dx =$$

$$\begin{aligned} & * \text{complete the square} \quad \left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{16}{2}\right)^2 = 8^2 = 64 \\ & x^2 + 16x + \underline{64} + 67 - \underline{64} \quad \begin{cases} a=\sqrt{3} \\ u=x+8 \\ \frac{du}{dx}=1 \\ dx=du \end{cases} \\ & (x+8)(x+8) \quad 3 \end{aligned}$$

$$\int \frac{7}{(x+8)^2+3} dx$$

$$7 \int \frac{dx}{(x+8)^2+(\sqrt{3})^2}$$

$$7 \int \frac{du}{u^2+a^2}$$

$$7 \cdot \frac{1}{\sqrt{3}} \arctan\left(\frac{x+8}{\sqrt{3}}\right) + C$$

$$\boxed{\frac{7}{\sqrt{3}} \arctan\left(\frac{x+8}{\sqrt{3}}\right) + C}$$

9)

* u-sub

$$\int \frac{5}{2x \ln x^3} dx \quad \int \frac{5}{2x \cdot 3 \ln x} \rightarrow \int \frac{5}{6x \ln x} dx$$

$$\frac{5}{6} \int \frac{1}{x \ln x} dx$$

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$dx = x du$$

$$\frac{5}{6} \ln|u| + C$$

$$\boxed{\frac{5}{6} \ln|\ln x| + C}$$

(Chapter 5) Derivative & Integral Rules Reference Sheet

Derivative Rules:

Power Rule:

$$\frac{d}{dx} x^n = nx^{n-1}$$

Trig Derivatives:

$$\frac{d}{dx} \sin u = \cos u * u'$$

$$\frac{d}{dx} \cos u = -\sin u * u'$$

$$\frac{d}{dx} \tan u = \sec^2 u * u'$$

$$\frac{d}{dx} \cot u = -\csc^2 u * u'$$

$$\frac{d}{dx} \sec u = \sec u \tan u * u'$$

$$\frac{d}{dx} \csc u = -\csc u \cot u * u'$$

$$\frac{d}{dx} e^u = e^u * u'$$

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

$$\frac{d}{dx} a^u = \ln a * a^u * u'$$

$$\frac{d}{dx} \log_a u = \frac{1}{\ln a} * \frac{u'}{u}$$

Integral Rules:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$$

More Trig Integral Rules:

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

Arc-Trig Integral Rules

$$17. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$16. \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$18. \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

Arc-Trig derivative Rules

$$19. \frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$20. \frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$21. \frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$22. \frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$23. \frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$24. \frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$