Ch. 5 Integration Technique Checklist

1) **Power Rule** (Can you rearrange problem to rely on just power rule?)

*Some examples include: $\int (3-x)^2 \left(\frac{2}{\sqrt{x}}\right) dx$ and $\int \frac{2x(5-3x+x^4)}{3(\sqrt{x^7})} dx$

a) convert radicals to rational exponential form (example: $\sqrt{x^5} = x^{\frac{5}{2}}$)

b) move denominator variable to numerator

c) resolve parentheses, separate terms.

*typically, if there are multiple terms in denominator separated by addition or subtraction, power rule alone will not be enough to make progress. Proceed to Option #2

2) If unable to rely on just power rule, then explore **U-Substitution** options.

a) Big picture: We want to choose a u-value that will lead to a match with a **known Integral rule.** (Needs to be a perfect match outside of coefficient terms, and with no x-variables remaining)

b) If expression can be rewritten using parentheses, the u-value is usually the expression inside the set of parentheses.

c) u-value is more than just replacing an "x", and may involve replacing a significant portion of the expression.

d) For fractional expressions, the u-value usually comes from the denominator.

(potential notable exceptions are log functions like $\ln x$ and radical expressions like $\sqrt{x}$)

e) u-value are typically higher degree expressions when choosing between 2 expressions with different degrees.

2b) U-Substitution (using **change of variable**)

a) If the initial round of u-substitution is not enough to remove the remaining x's in the integrand, then explore option of rearranging the expression assigned to u, and solving for x.

b) Once we make that second set of substitutions, the problem is now purely in terms of u, and with all x's removed and replaced.

3) Rewrite rational expression using **Long Division** (synthetic division)

a) Condition needed to apply **long division** is the **numerator degree** $\geq$ **denominator degree**. (example: $\int \frac{2x^3-4x+1}{x^2+3} dx$)

b) For long division problems, we can apply **synthetic division** only if denominator degree is = 1 (linear degree) (example: $\int \frac{4x^3-7x+2}{x-5} dx$)

c) Once our rewrite is complete, we can typically find the antiderivative by using a combination of power rule and u-substitution across the different terms.