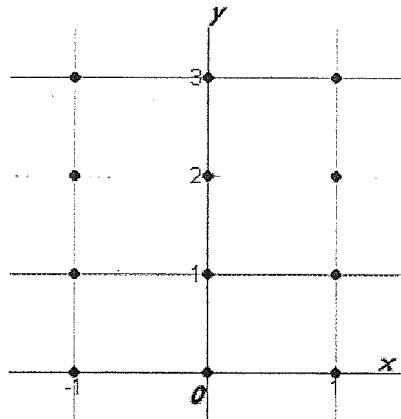


1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$
- Sketch a slope field through the indicated points
 - Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$



- Write the equation for the tangent line to the curve $y = f(x)$ through the point $(1,1)$
 - Use the tangent line equation to estimate the value of $f(1.2)$
2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F . The initial temperature of the liquid is 160°F . After 5 minutes, the liquid's temperature is 60°F . How long will it take for its temperature to decrease to 30°F ?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

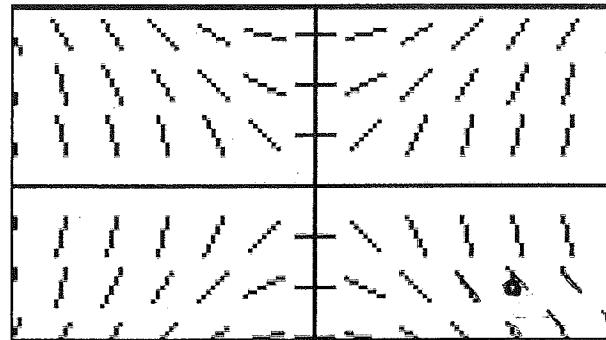
3. Match the slope field with the correct equation:

a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\frac{dy}{dx} = \frac{x}{y}$

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



3b. Find the particular solution given $y(5) = -2$

3c. Sketch the solution through the given point and find the domain

4. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$

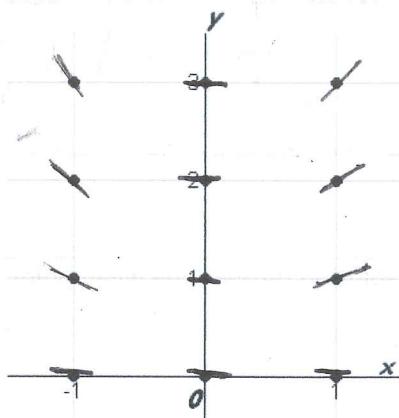
5. $\int (-\sin x)(3^{\cos x}) dx$

6. $\int \frac{8}{\sqrt{4-x^2+2x}} dx$ (hint: complete the square)

7. $\int \csc(1-3x) dx$

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

a. Sketch a slope field through the indicated points



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$

$$\begin{aligned}\int \frac{dy}{y} &= \int \frac{1}{2}x dx \\ \ln|y| &= \frac{1}{2}(\frac{x^2}{2}) + C \\ e^{\ln|y|} &= e^{\frac{1}{2}(x^2)} \cdot e^C \\ y &= e^{\frac{x^2}{4}} \cdot e^C \\ y &= Ce^{\frac{x^2}{4}}\end{aligned}$$

$$y = e^{\frac{x^2-1}{4}}$$

- c. Write the equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$

$$\begin{aligned}\frac{dy}{dx}_{(1,1)} &= \frac{(1)(1)}{2} = \frac{1}{2} & \text{point: } (1, 1) \\ m &= \frac{1}{2} \\ y - y_1 &= m(x - x_1)\end{aligned}$$

$$y - 1 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}(x - 1) + 1$$

- d. Use the tangent line equation to estimate the value of $f(1.2)$

$$f(1.2) \approx \frac{1}{2}(1.2 - 1) + 1 = 1.1$$

$$f(1.2) \approx 1.1$$

2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F . The initial temperature of the liquid is 160°F . After 5 minutes, the liquid's temperature is 60°F . How long will it take for its temperature to decrease to 30°F ?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

(time, temperature)

$(0, 160)$

$(5, 60)$

$(\underline{\quad}, 30)$

$$\int \frac{dy}{y-20} = \int k dt$$

$$\begin{aligned}u &= y - 20 \\ \frac{du}{dt} &= \frac{dy}{dt} \\ \frac{du}{dt} &= k \\ \int \frac{du}{u} &= \int k dt\end{aligned}$$

$$\begin{aligned}e^{\ln|y-20|} &= e^{kt} \cdot e^C \\ y - 20 &= C e^{kt}\end{aligned}$$

$$160 - 20 = C e^{k(0)}$$

$$140 = C(1)$$

$$140 = C$$

$$y - 20 = 140 e^{kt}$$

$$60 - 20 = 140 e^{5k}$$

$$40 = 140 e^{5k}$$

$$\frac{40}{140} = e^{5k}$$

$$\frac{2}{7} = e^{5k}$$

$$\ln(\frac{2}{7}) = \ln e^{5k}$$

$$\ln(\frac{2}{7}) = 5k$$

$$\frac{1}{5} \ln(\frac{2}{7}) = k$$

$$k(5)$$

$$60 - 20 = 140 e^{5k}$$

$$40 = 140 e^{5k}$$

$$\frac{40}{140} = e^{5k}$$

$$\frac{2}{7} = e^{5k}$$

$$\ln(\frac{2}{7}) = \ln e^{5k}$$

$$\ln(\frac{2}{7}) = 5k$$

$$\frac{1}{5} \ln(\frac{2}{7}) = k$$

$$\frac{1}{5} \ln(\frac{2}{7}) = k$$

$$y - 20 = 140 e^{5k}$$

$$30 - 20 = 140 e^{5k}$$

$$10 = 140 e^{5k}$$

$$\frac{1}{14} = e^{5k}$$

$$\ln(\frac{1}{14}) = \ln e^{5k}$$

$$\ln(\frac{1}{14}) = 5k$$

$$\frac{1}{5} \ln(\frac{1}{14}) = k$$

$$\frac{1}{5} \ln(\frac{1}{14}) = k$$

$$\frac{1}{5} \ln(\frac{2}{7}) t$$

$$t \approx 10.533 \text{ mins.}$$

3. Match the slope field with the correct equation:

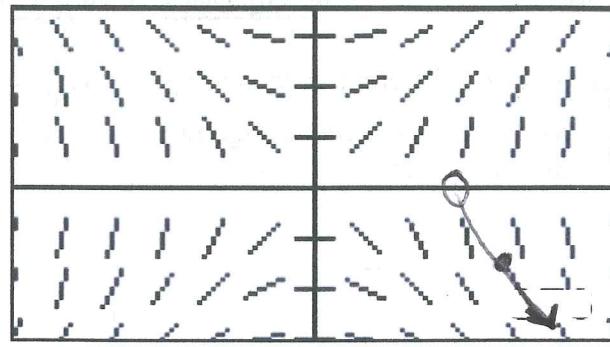
a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\frac{dy}{dx} = \frac{x}{y}$

when $y=0$, slope undefined
when $x=0$, slope = 0
at $(1,1)$ slope is positive

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



3b. Find the particular solution given $y(5) = -2$

$$\int y dy = \int x dx$$

$$y^2 = x^2 + C$$

$$y^2 = x^2 + C$$

$$(-2)^2 = (5)^2 + C$$

$$-21 = C$$

$$y^2 = x^2 - 21$$

$$y = \pm \sqrt{x^2 - 21}$$

$$y = -\sqrt{x^2 - 21}$$

4. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$

$$\frac{2(e^x - e^{-x})}{u^2} \cdot \frac{du}{e^x - e^{-x}}$$

$$= 2 \int \frac{1}{u^2} du$$

$$= 2 \int u^{-2} du$$

$$u = e^x + e^{-x}$$

$$\frac{du}{dx} = e^x(1) + e^{-x}(-1)$$

$$\frac{du}{dx} = e^x - e^{-x} \quad dx = \frac{du}{e^x - e^{-x}}$$

$$2\left(\frac{u^{-1}}{-1}\right) + C$$

$$= -\frac{2}{u} + C = \frac{-2}{e^x + e^{-x}} + C$$

6. $\int \frac{8}{\sqrt{4-x^2+2x}} dx$ (hint: complete the square)

$$4 - (x^2 - 2x + \underline{\underline{\underline{\underline{1}}}}) + \underline{\underline{\underline{\underline{1}}}}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$$

$$4 - (x^2 - 2x + 1) + 1$$

$$5 - (x-1)^2$$

$$\int \frac{8}{\sqrt{5-(x-1)^2}} dx$$

$$u = x-1$$

$$\frac{du}{dx} = 1$$

$$a = \sqrt{5}$$

$$\int \frac{8}{\sqrt{(\sqrt{5})^2 - (x-1)^2}} dx$$

$$\int \frac{8}{\sqrt{a^2 - u^2}} du$$

$$8 \int \frac{du}{\sqrt{a^2 - u^2}}$$

$$8 \cdot \arcsin\left(\frac{u}{a}\right) + C$$

$$= 8 \arcsin\left(\frac{x-1}{\sqrt{5}}\right) + C$$

- 3c. Sketch the solution through the given point and find the domain * the graph interval starts at $y=0$

$$0 = -\sqrt{x^2 - 21}$$

$$(0)^2 = (\sqrt{x^2 - 21})^2$$

$$0 = x^2 - 21$$

$$21 = x^2 \quad x = \pm \sqrt{21}$$

$$\boxed{\text{Domain: } (\sqrt{21}, \infty)}$$

5. $\int (-\sin x)(3^{\cos x}) dx$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{du}{-\sin x}$$

$$\int -\sin x \cdot 3^u \cdot \frac{du}{-\sin x}$$

$$\int 3^u du = \frac{3^u}{\ln 3} + C$$

$$\boxed{= \frac{3^{\cos x}}{\ln 3} + C}$$

7. $\int \csc(1-3x) dx$

$$u = 1-3x$$

$$\frac{du}{dx} = -3 \quad \int \csc u \cdot \frac{du}{-3} = -\frac{1}{3} \int \csc u du$$

$$dx = \frac{du}{-3} \quad = -\frac{1}{3} \cdot -\ln|\csc u + \cot u| + C$$

$$= \boxed{\frac{1}{3} \ln |\csc(1-3x) + \cot(1-3x)| + C}$$