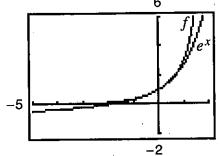


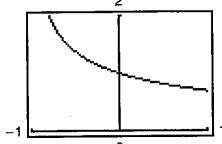
P.S. Problem Solving (page 395)

1. $a = 1, b = \frac{1}{2}, c = -\frac{1}{2}$

$$f(x) = \frac{1+x/2}{1-x/2}$$

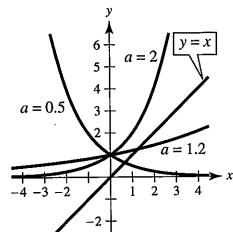


3. (a)



(b) 1 (c) Proof

5.



$y = 0.5^x$ and $y = 1.2^x$ intersect the line $y = x$;
 $0 < a < e^{1/e}$

7. $e - 1$

9. (a) Area of region A = $(\sqrt{3} - \sqrt{2})/2 \approx 0.1589$

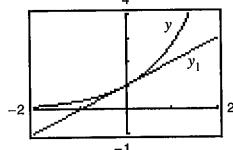
Area of region B = $\pi/12 \approx 0.2618$

(b) $\frac{1}{24}[3\pi\sqrt{2} - 12(\sqrt{3} - \sqrt{2}) - 2\pi] \approx 0.1346$

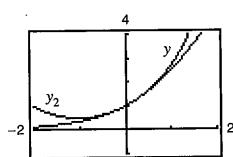
(c) 1.2958 (d) 0.6818

11. Proof 13. $2 \ln \frac{3}{2} \approx 0.8109$

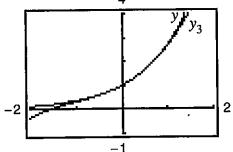
15. (a) (i)



(ii)

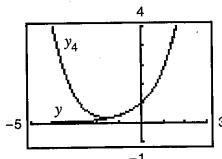


(iii)



(b) Pattern: $y_n = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$

$$y_4 = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$



(c) The pattern implies that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

Chapter 6**Section 6.1** (page 403)

1–11. Proofs

13. Not a solution 15. Solution

17. Solution 19. Solution

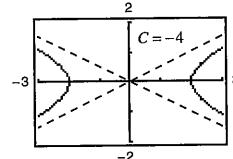
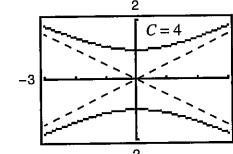
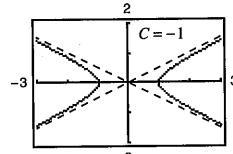
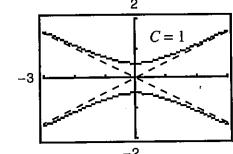
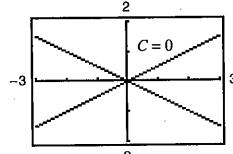
21. Not a solution

23. Solution 25. Not a solution

27. Not a solution

29. $y = 3e^{-x/2}$ 31. $4y^2 = x^3$

33.



35. $y = 3e^{-2x}$

37. $y = 2 \sin 3x - \frac{1}{3} \cos 3x$

39. $y = -2x + \frac{1}{2}x^3$ 41. $2x^3 + C$

43. $y = \frac{1}{2} \ln(1+x^2) + C$ 45. $y = x - \ln x^2 + C$

47. $y = -\frac{1}{2} \cos 2x + C$

49. $y = \frac{2}{5}(x-6)^{5/2} + 4(x-6)^{3/2} + C$ 51. $y = \frac{1}{2}e^{x^2} + C$

53.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	-4	Undef.	0	1	$\frac{4}{3}$	2

55.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

57. b

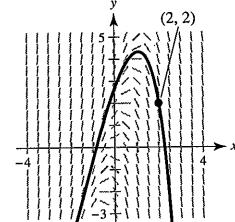
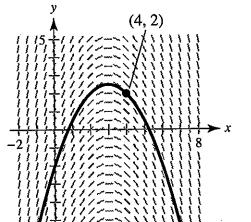
58. c

59. d

60. a

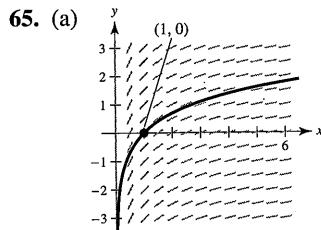
61. (a) and (b)

63. (a) and (b)



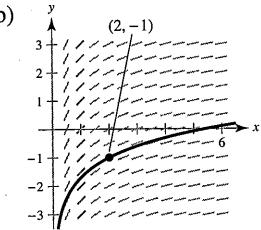
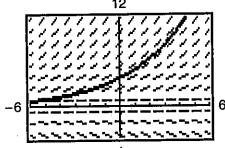
(c) As $x \rightarrow \infty, y \rightarrow -\infty$;
as $x \rightarrow -\infty, y \rightarrow -\infty$

(c) As $x \rightarrow \infty, y \rightarrow -\infty$;
as $x \rightarrow -\infty, y \rightarrow -\infty$



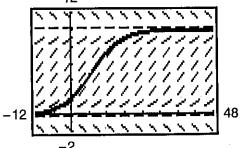
As $x \rightarrow \infty$, $y \rightarrow \infty$

67. (a) and (b)

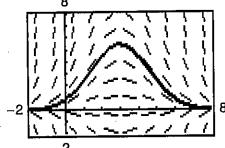


As $x \rightarrow \infty$, $y \rightarrow -1$

69. (a) and (b)



71. (a) and (b)



n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	2	2.2	2.43	2.693	2.992	3.332	3.715

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	4.146	4.631	5.174	5.781

n	0	1	2	3	4	5	6
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3
y_n	3	2.7	2.438	2.209	2.010	1.839	1.693

n	7	8	9	10
x_n	0.35	0.4	0.45	0.5
y_n	1.569	1.464	1.378	1.308

n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	1	1.1	1.212	1.339	1.488	1.670	1.900

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	2.213	2.684	3.540	5.958

79.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	3.0000	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ ($h = 0.2$)	3.0000	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ ($h = 0.1$)	3.0000	3.6300	4.3923	5.3147	6.4308	7.7812

81.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	0.0000	0.2200	0.4801	0.7807	1.1231	1.5097
$y(x)$ ($h = 0.2$)	0.0000	0.2000	0.4360	0.7074	1.0140	1.3561
$y(x)$ ($h = 0.1$)	0.0000	0.2095	0.4568	0.7418	1.0649	1.4273

83. (a) $y(1) = 112.7141^\circ$; $y(2) = 96.3770^\circ$; $y(3) = 86.5954^\circ$

(b) $y(1) = 113.2441^\circ$; $y(2) = 97.0158^\circ$; $y(3) = 87.1729^\circ$

(c) Euler's Method: $y(1) = 112.9828^\circ$; $y(2) = 96.6998^\circ$; $y(3) = 86.8863^\circ$

Exact solution: $y(1) = 113.2441^\circ$; $y(2) = 97.0158^\circ$; $y(3) = 87.1729^\circ$

The approximations are better using $h = 0.05$.

85. The general solution is a family of curves that satisfies the differential equation. A particular solution is one member of the family that satisfies given conditions.

87. Begin with a point (x_0, y_0) that satisfies the initial condition $y(x_0) = y_0$. Then, using a small step size h , calculate the point $(x_1, y_1) = (x_0 + h, y_0 + hF(x_0, y_0))$. Continue generating the sequence of points $(x_n + h, y_n + hF(x_n, y_n))$ or (x_{n+1}, y_{n+1}) .

89. False. $y = x^3$ is a solution of $xy' - 3y = 0$, but $y = x^3 + 1$ is not a solution.

91. True

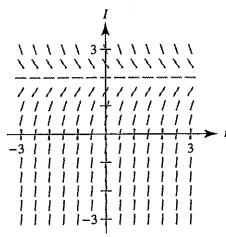
93. (a)

x	0	0.2	0.4	0.6	0.8	1
y	4	2.6813	1.7973	1.2048	0.8076	0.5413
y_1	4	2.56	1.6384	1.0486	0.6711	0.4295
y_2	4	2.4	1.44	0.864	0.5184	0.3110
e_1	0	0.1213	0.1589	0.1562	0.1365	0.1118
e_2	0	0.2813	0.3573	0.3408	0.2892	0.2303
r		0.4312	0.4447	0.4583	0.4720	0.4855

(b) If h is halved, then the error is approximately halved because r is approximately 0.5.

(c) The error will again be halved.

95. (a)



(b) $\lim_{t \rightarrow \infty} I(t) = 2$

97. $\omega = \pm 4$

99. Putnam Problem 3, Morning Session, 1954

Section 6.2 (page 412)

1. $y = \frac{1}{2}x^2 + 3x + C$

3. $y = Ce^x - 3$

5. $y^2 - 5x^2 = C$

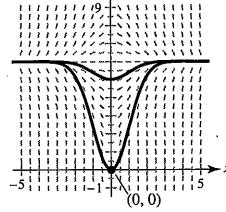
7. $y = Ce^{(2x^{3/2})/3}$

9. $y = C(1 + x^2)$

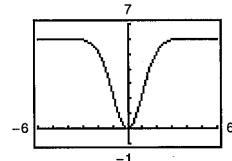
11. $dQ/dt = k/t^2$

$Q = -k/t + C$

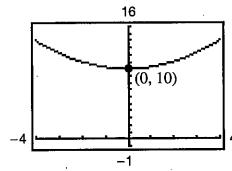
13. (a)



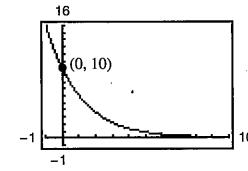
(b) $y = 6 - 6e^{-x^2/2}$



15. $y = \frac{1}{4}t^2 + 10$



17. $y = 10e^{-t/2}$



19. $\frac{8192}{4}$

21. $y = (1/2)e^{[(\ln 10)/5]t} \approx (1/2)e^{0.4605t}$

23. $y = 5(5/2)^{1/4}e^{[\ln(2/5)/4]t} \approx 6.2872e^{-0.2291t}$

25. C is the initial value of y , and k is the proportionality constant.27. Quadrants I and III; dy/dx is positive when both x and y are positive (Quadrant I) or when both x and y are negative (Quadrant III).29. Amount after 1000 yr: 12.96 g;
Amount after 10,000 yr: 0.26 g31. Initial quantity: 7.63 g;
Amount after 1000 yr: 4.95 g33. Amount after 1000 yr: 4.43 g;
Amount after 10,000 yr: 1.49 g35. Initial quantity: 2.16 g;
Amount after 10,000 yr: 1.62 g

37. 95.76%

39. Time to double: 11.55 yr; Amount after 10 yr: \$7288.48

41. Annual rate: 8.94%; Amount after 10 yr: \$1833.67

43. Annual rate: 9.50%; Time to double: 7.30 yr

45. \$224,174.18 47. \$61,377.75

49. (a) 10.24 yr (b) 9.93 yr (c) 9.90 yr (d) 9.90 yr

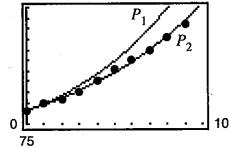
51. (a) $P = 2.21e^{-0.006t}$ (b) 2.08 million(c) Because $k < 0$, the population is decreasing.53. (a) $P = 33.38e^{0.036t}$ (b) 47.84 million(c) Because $k > 0$, the population is increasing.55. (a) $N = 100.1596(1.2455)^t$ (b) 6.3 h57. (a) $N \approx 30(1 - e^{-0.0502t})$ (b) 36 days

59. (a) Because the population increases by a constant each month, the rate of change from month to month will always be the same. So, the slope is constant, and the model is linear.

(b) Although the percentage increase is constant each month, the rate of growth is not constant. The rate of change of y is $dy/dt = ry$, which is an exponential model.

61. (a) $P_1 = 106e^{0.01487t} \approx 106(1.01499)^t$

(b) $P_2 = 107.2727(1.01215)^t$

(c) 

(d) 2029

63. (a) 20 dB (b) 70 dB (c) 95 dB (d) 120 dB

65. 379.2°F

67. False. The rate of growth dy/dx is proportional to y .

69. False. The prices are rising at a rate of 6.2% per year.

Section 6.3 (page 421)

1. $y^2 - x^2 = C$ 3. $15y^2 + 2x^3 = C$ 5. $r = Ce^{0.75s}$

7. $y = C(x + 2)^3$ 9. $y^2 = C - 8 \cos x$

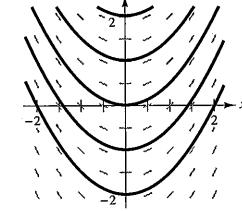
11. $y = -\frac{1}{4}\sqrt{1 - 4x^2} + C$ 13. $y = Ce^{(\ln x)^2/2}$

15. $y^2 = 4e^x + 5$ 17. $y = e^{-(x^2+2x)/2}$

19. $y^2 = 4x^2 + 3$ 21. $u = e^{(1-\cos v^2)/2}$ 23. $P = P_0e^{kt}$

25. $4y^2 - x^2 = 16$ 27. $y = \frac{1}{3}\sqrt{x}$ 29. $f(x) = Ce^{-x/2}$

31.



$y = \frac{1}{2}x^2 + C$

33. (a) $dy/dx = k(y - 4)$ (b) a (c) Proof

34. (a) $dy/dx = k(x - 4)$ (b) b (c) Proof

35. (a) $dy/dx = ky(y - 4)$ (b) c (c) Proof

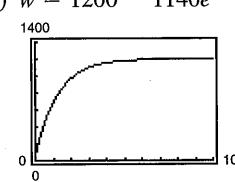
36. (a) $dy/dx = ky^2$ (b) d (c) Proof

37. 97.9% of the original amount

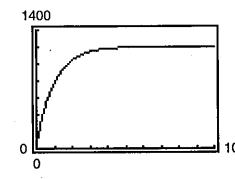
39. (a) $w = 1200 - 1140e^{-kt}$

(b) $w = 1200 - 1140e^{-0.8t}$

$w = 1200 - 1140e^{-0.9t}$



$w = 1200 - 1140e^{-t}$



(c) 1.31 yr; 1.16 yr; 1.05 yr (d) 1200 lb

