

Chapter 5 Integral Review

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1) Evaluate the integral: $\int_1^{\infty} \frac{1}{x} dx.$

- (a) $\frac{1}{5e} - 1$
- (b) 0
- (c) ∞

- (d) $1 + \ln 5$
- (e) None of these

2) Evaluate the integral: $\int_1^{\infty} \frac{x+2}{x+1} dx.$

- (a) $\frac{x^2+4x}{x^2+2x} + C$
- (b) $2x+C$

- (d) $x + \ln|x+1| + C$
- (e) None of these

3) Evaluate the integral: $\int x \cot x^2 dx.$

- (a) $\frac{1}{2}x^2 \sec^2 x^2 + C$
- (b) $\frac{1}{4}x^2 \ln|\sin x^2| + C$

- (c) $x + C$
- (e) None of these

4) Evaluate the integral: $\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x} dx.$

- (a) $\frac{2}{2} \tan^2 x - \tan x + x + C$
- (b) $\sin x - \sec x + C$

- (c) $\sin x - \tan x + x + C$
- (e) None of these

5) Evaluate the integral: $\int \sec 2x dx.$

- (a) $\frac{1}{2} \ln|\sec 2x + \tan 2x| + C$

- (c) $\sin x - \tan x + x + C$
- (e) None of these

6) Evaluate the integral: $\int \frac{\ln \sqrt{x}}{x} dx.$

- (a) $\frac{1}{3} \ln x^3 + C$

- (c) $\sin x - \tan x + x + C$
- (e) None of these

7) Evaluate the integral: $\int \frac{1 - \sin \theta}{\cos \theta} d\theta.$

- (a) $\frac{1}{4}x^4 + C$

- (c) $\frac{1}{4} \arctan \frac{x^2}{4} + C$
- (e) None of these

8) Solve the differential equation: $\frac{dy}{dx} = \frac{3x}{1-x^2}.$

- (a) $\int \frac{x+3}{x^2+9} dx.$

- (c) $\frac{1}{2} \ln(x^2+9) + \arctan \frac{x}{3} + C$
- (e) None of these

10) Evaluate: $\int \frac{1}{\sqrt{8+2x-x^2}} dx.$

- (a) $\ln \sqrt{8+2x-x^2} + C$

- (c) $\frac{1}{2} \arctan \frac{x}{3} + C$
- (e) None of these

11) Evaluate: $\int \frac{x+2}{\sqrt{4-x^2}} dx.$

- (a) $\frac{1}{2} \sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$

- (c) $\ln|2-x| + C$
- (e) None of these

12) Evaluate: $\int \frac{5}{x^2+6x+13} dx.$

- (a) $5 \ln|x^2+6x+13| + C$

- (c) $\frac{5}{2} \arctan \frac{x+3}{2} + C$
- (e) None of these

13) Find the indefinite integral: $\int \frac{x}{16+x^4} dx.$

- (a) $\frac{1}{2} \arcsin \frac{x^2}{4} + C$

- (c) $\frac{1}{8} \arccsc \frac{x^2}{4} + C$
- (e) None of these

14) Evaluate the definite integral: $\int_1^{\sqrt{6}} \frac{-4x}{x^2} dx.$

- (a) -1

- (c) -6
- (e) None of these

15) Evaluate the integral: $\int_{-1}^{1} \frac{1}{2x\sqrt{4x^2-1}} dx.$

- (a) $\arcsin|2x| + C$

- (c) $\frac{1}{2} \arcsin|2x| + C$
- (e) None of these

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10) Evaluate: $\int \frac{dx}{\sqrt{8+2x-x^2}}$

- (b) $\arcsin \frac{x-1}{3} + C$
- (c) $\sqrt{8+2x-x^2} + C$

- (d) $\frac{1}{3} \arccsc \frac{x-1}{3} + C$
- (e) None of these

11) Evaluate: $\int \frac{x+2}{\sqrt{4-x^2}} dx.$

- (a) $\frac{1}{2} \sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$
- (b) $-\sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$

- (d) $x^2 + 2x + \arcsin \frac{x}{2} + C$
- (e) None of these

12) Evaluate: $\int \frac{5}{x^2+6x+13} dx.$

- (b) $5 \left(\frac{x^3}{3} + 3x^2 + 13x \right) + C$

- (d) $-\frac{5}{x} + \frac{5}{6} \ln|x| + \frac{5}{13}x + C$

- (e) None of these

13) Find the indefinite integral: $\int \frac{x}{16+x^4} dx.$

- (b) $\frac{1}{8} \arctan \frac{x^2}{4} + C$

- (c) $\frac{1}{4} \arctan \frac{x^2}{4} + C$

- (e) None of these

14) Evaluate the definite integral: $\int_1^{\sqrt{6}} \frac{-4x}{x^2} dx.$

- (a) -1

- (c) -6

- (e) -4

15) Evaluate the integral: $\int_{-1}^{1} \frac{1}{2x\sqrt{4x^2-1}} dx.$

- (a) $\arcsin|2x| + C$

- (c) $\frac{1}{2} \arcsin|2x| + C$

- (e) None of these

35) A deposit of \$1000 is made into a fund with an annual interest rate of 5 percent. Find the time (in years) necessary for the investment to triple if the interest is compounded continuously. Round your answer to 2 decimal places.

- (a) 30.00 years (b) 15.00 years
 (d) 21.97 years (e) None of these

16) Evaluate the integral: $\int \frac{3x^2 + 3x + 3}{x^2 + 1} dx$.

- (a) $3x + 3 \ln(x^2 + 1) + C$
 (b) $3 + \frac{3}{2} \ln(x^2 + 1) + C$
 (d) $3 + 3 \ln(x^2 + 1) + C$
 (e) None of these

17) Evaluate the integral: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

- (a) $2e^{\sqrt{x}} + C$
 (b) $\frac{1}{2}e^{\sqrt{x}} + C$
 (d) $\sqrt{x} e^{\sqrt{x}} + C$
 (e) None of these

18) Evaluate the indefinite integral: $\int \frac{1}{x^2 e^{5/x}} dx$.

- (a) $\frac{1}{5}e^{5/x} + C$
 (b) $\frac{1}{5}xe^{5/x} + C$
 (d) $\frac{1}{5}xe^{-5/x} + C$
 (e) None of these

19) Calculate the area of the region bounded by $y = e^{2x}$, $y = 0$, $x = 1$, $x = 4$.

- 20) Differentiate: $f(x) = \ln(e^{-x^2})$.
 (a) e^{-x^2}
 (b) $-2xe^{2x^2}$
 (d) $-2xe^{-x^2}$
 (e) None of these

21) Evaluate the integral: $\int x^3 e^x dx$.

- (a) $\left(\frac{x^2}{2}\right)3x^3/3 + C$
 (b) $\left(\frac{\ln 3}{2}\right)3x^4 + C$
 (d) $\frac{1}{2}(3x^4) + C$
 (e) None of these

22) A radioactive element has a half-life of 50 days. What percentage of the original sample is left after 60 days?

- (a) 43.53%
 (b) 49.56%
 (d) 25.00%
 (e) None of these

23) Evaluate the definite integral: $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{3}{(x - 3)\sqrt{x^2 - 6x + 9}} dx$.

24) Evaluate the indefinite integral: $\int \frac{1}{(x - 3)\sqrt{x^2 - 6x + 9}} dx$.

25) Use integration to find a general solution to the differential equation $y' = \frac{2}{\sqrt{1 - x^2}} + x$.

- (a) $\arcsin\left(\frac{x}{2}\right) + C$
 (b) $2 \arcsin x + \frac{x^2}{2} + C$
 (d) $2 \arcsin x^2 + \frac{x^2}{2} + C$
 (e) None of these

26) Use integration to find a general solution to the differential equation $y' = x\sqrt{x+1}$.

- (a) $\frac{2}{3}(x+1)^{3/2}\left(x - \frac{2}{3}\right) + C$
 (b) $\frac{2}{3}(x+1)^{5/2} - \frac{1}{3}(x+1)^{3/2} + C$
 (c) $(x+1)^{5/2} + (x+1)^{3/2} + C$
 (e) None of these

27) Use integration to find a general solution to the differential equation $y' = x\sqrt{x+1}$.

- (a) $\frac{2}{3}(x+1)^{3/2}\left(x - \frac{2}{3}\right) + C$
 (b) $\frac{2}{3}(x+1)^{5/2} - \frac{1}{3}(x+1)^{3/2} + C$
 (c) $(x+1)^{5/2} + (x+1)^{3/2} + C$
 (d) $2(x+1)^{3/2}(3x-2) + C$
 (e) None of these

28) Solve the differential equation: $\sqrt{x^2 - 1}y' = \frac{y}{x}$.

- 29) The rate of change of y with respect to x is inversely proportional to the square root of y .
- Write a differential equation for the given statement.
 - Solve the differential equation in part a.
- 30) The number of fruit flies increases according to the law of exponential growth. If initially there are 10 fruit flies and after 6 hours there are 24, find the number of fruit flies after t hours.
- $y = 10e^{\ln(12/5)t/6}$
 - $y = 10e^{\ln(12/5)t/6}$
 - $y = 10e^{-\ln(12/5)t/6}$
 - None of these

31) Find the general solution of the differential equation: $\frac{y'}{x} = \frac{e^x}{y}$.

- 32) Find the particular solution of the differential equation $\frac{dy}{dx} = 500 - y$ that satisfies the initial condition $y(0) = 7$.

Chapter 5 Integral Review Answer Key

- 1) $\int_{\ln 1}^{\ln 5} \frac{1}{x} dx = \ln|x| \Big|_{\ln 1}^{\ln 5} = \ln(5e) - \ln 1 = \ln 5 + \ln e - \ln 1 = \ln 5 + 1$ d
- 2) $\int \frac{x+2}{x+1} dx = \int \frac{u+1}{u} du = \int du + \int \frac{1}{u} du = u + \ln|u| + C = x+1 + \ln|x+1| + C$
 $u = x+1, du = dx$
 $= x + \ln|x+1| + C$ d
- 3) $\int x \cot x^2 dx = \frac{1}{2} \int \cot u du = \frac{1}{2} \ln|\sin u| + C = \frac{1}{2} \ln|\sin x^2| + C$ d.
 $u = x^2, du = 2x dx$
- 4) $\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x} dx = \int \frac{\cos^3 x - (1 - \cos^2 x)}{\cos^2 x} dx = \int \frac{\cos^3 x + \cos^2 x - 1}{\cos^2 x} dx$
 $= \int \cos x dx + \int dx - \int \sec^2 x dx = \sin x + x - \tan x + C$ c
- 5) $\int \sec 2x dx = \frac{1}{2} \int \sec u du = \frac{1}{2} \ln|\sec u + \tan u| + C = \boxed{\frac{1}{2} \ln|\sec 2x + \tan 2x| + C}$
 $u = 2x, du = 2dx$
- 6) $\int \frac{\ln \sqrt{x}}{x} dx = \int \frac{\ln x^{1/2}}{x} dx = \int \frac{1}{2} \frac{\ln x}{x} dx = \int \frac{1}{2} u du = \frac{1}{4} u^2 + C = \boxed{\frac{1}{4} (\ln x)^2 + C}$
 $u = \ln x, du = \frac{1}{x} dx$
- 7) $\int \frac{1 - \sin \theta}{\cos \theta} d\theta = \int \sec \theta d\theta - \int \tan \theta d\theta = \boxed{\ln|\sec \theta + \tan \theta| + \ln|\cos \theta| + C}$
- 8) $\frac{dy}{dx} = \frac{3x}{1-x^2}, \int dy = \int \frac{3x}{1-x^2} dx, y = -\frac{3}{2} \int \frac{1}{u} du = -\frac{3}{2} \ln|u| + C$ y = $-\frac{3}{2} \ln|1-x^2| + C$
 $u = 1-x^2, du = -2x dx$
- 9) $\int \frac{x+3}{x^2+9} dx = \int \frac{x}{x^2+9} dx + \int \frac{3}{x^2+9} dx = \frac{1}{2} \int \frac{1}{u} du + 3 \int \frac{1}{x^2+9} dx$
 $\begin{aligned} &\quad (u = x^2+9 \\ &\quad du = 2x dx) \\ &\quad = \frac{1}{2} \ln|u| + \frac{3}{3} \arctan \frac{x}{3} + C \end{aligned}$ c
 $= \frac{1}{2} \ln(x^2+9) + \arctan \frac{x}{3} + C$
- 10) $\int \frac{dx}{\sqrt{8+2x-x^2}} = \int \frac{dx}{\sqrt{9-(x-1)^2}} = \int \frac{du}{\sqrt{9-u^2}} = \arcsin \frac{u}{3} + C = \boxed{\arcsin \frac{x-1}{3} + C}$
 $\begin{aligned} &\quad (x^2-2x-8) \\ &\quad = -(x^2-2x+1-8-1) \\ &\quad = -((x-1)^2-9) = (9-(x-1)^2) \end{aligned}$

$$1) \int \frac{x+2}{\sqrt{4-x^2}} dx = \int \frac{x}{\sqrt{4-x^2}} dx + \int \frac{2}{\sqrt{4-x^2}} dx = -\frac{1}{2} \int u^{-\frac{1}{2}} du + 2 \int \frac{1}{\sqrt{4-u^2}} du \quad \boxed{E}$$

$$\begin{aligned} u &= 4-x^2 \\ du &= -2x dx \end{aligned} \quad = -u^{\frac{1}{2}} + 2 \arcsin \frac{x}{2} + C = -\sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$$

$$2) \int \frac{5}{x^2+6x+13} dx = 5 \int \frac{1}{(x+3)^2+4} dx = 5 \int \frac{1}{u^2+4} du = \frac{5}{2} \arctan \frac{u}{2} + C = \frac{5}{2} \arctan \frac{x+3}{2} + C$$

$$\begin{aligned} x^2+6x+9+13-9 &= (x+3)^2+4 \\ &= u^2 \end{aligned} \quad \begin{aligned} u &= x+3 \\ du &= dx \end{aligned}$$

C

$$3) \int \frac{x}{16+x^4} dx = \frac{1}{2} \int \frac{1}{16+u^2} du = \frac{1}{2} \cdot \frac{1}{4} \arctan \frac{u}{4} + C = \frac{1}{8} \arctan \frac{x^2}{4} + C \quad \boxed{B}$$

$$u = x^2, du = 2x dx$$

$$4) \int_1^{5e} \frac{-4x}{x^2} dx = \int_1^{5e} -\frac{4}{x} dx = -4 \ln|x| \Big|_1^{5e} = -4 \ln 5e - -4 \ln 1 = -4(\ln e^{\frac{1}{2}} - \ln 1) \quad \boxed{D}$$

$$= -4\left(\frac{1}{2} - 0\right) = -2$$

$$5) \int \frac{1}{2x\sqrt{4x^2-1}} dx = \int \frac{1}{2x(2)\sqrt{x^2-\frac{1}{4}}} dx = \frac{1}{4} \int \frac{1}{x\sqrt{x^2-\frac{1}{4}}} = \frac{1}{4} \cdot 2 \operatorname{arcsec} \frac{|x|}{\frac{1}{2}} + C = \frac{1}{2} \operatorname{arcsec} 2/x + C \quad \boxed{B}$$

B

$$6) \int \frac{3x^2+3x+3}{x^2+1} dx = 3 \int \frac{x^2+x+1}{x^2+1} dx = 3 \int \frac{x^2+1}{x^2+1} dx + 3 \int \frac{x}{x^2+1} dx = 3 \int dx + \frac{3}{2} \int \frac{1}{u} du$$

$$\begin{aligned} u &= x^2+1 \\ du &= 2x dx \end{aligned} \quad = 3x + \frac{3}{2} \ln(x^2+1) + C \quad \boxed{C}$$

C

$$7) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C \quad \boxed{A}$$

$$u = \sqrt{x}, du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}} dx$$

$$8) \int \frac{1}{x^2} e^{\frac{5}{x}} dx = \frac{1}{5} \int e^u du = \frac{1}{5} e^u + C = \frac{1}{5} e^{-\frac{5}{x}} + C \quad \boxed{C}$$

$$u = \frac{5}{x}, du = 5x^{-2} dx, \frac{1}{5} du = \frac{1}{x^2} dx$$

$$9) \int_1^4 e^{2x} dx = \frac{1}{2} \left[e^u \right]_2^4 = \frac{1}{2} (e^8 - e^2)$$

$$u = 2x, du = 2 dx$$

$$10) f(x) = \ln(e^{-x^2}) = -x^2 \ln e = -x^2 \quad f'(x) = -2x \quad \boxed{C}$$

$$11) \int x 3^x dx = \frac{1}{2} \int 3^u du = \frac{1}{2} \cdot \frac{3^u}{\ln 3} + C = \frac{3^x}{2 \ln 3} + C \quad \boxed{C}$$

$$u = x^2, du = 2x dx$$

$$\frac{dR}{dt} = KR \rightarrow \int \frac{1}{R} dR = K dt \rightarrow \ln R = Kt + C \rightarrow R = Ce^{Kt}$$

$$\frac{1}{2}C = Ce^{\frac{50K}{2}} \rightarrow \frac{1}{2} = e^{\frac{50K}{2}} \rightarrow \ln\left(\frac{1}{2}\right) = 50K \rightarrow K = \frac{1}{50} \ln\left(\frac{1}{2}\right) \rightarrow R = Ce^{\frac{1}{50} \ln\left(\frac{1}{2}\right)t}$$

$$R = e^{\frac{1}{50} \ln\left(\frac{1}{2}\right)(60)} \approx .43528 \rightarrow 43.53\% \quad \boxed{A}$$

$$23) \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{3}{\sqrt{4-x^2}} dx = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{3}{3\sqrt{\frac{4}{3}-x^2}} dx = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{1}{\sqrt{\frac{4}{3}-x^2}} dx = \arcsin\left[\frac{3x}{2}\right]_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}$$

$$= \arcsin\frac{3}{2\sqrt{3}} - \arcsin\frac{-3}{2\sqrt{3}} = \arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{-\sqrt{3}}{2} = \frac{\pi}{3} - \frac{-\pi}{3} = \frac{2\pi}{3} \quad \boxed{B}$$

$$24) \int \frac{1}{(x-3)\sqrt{x^2-6x+5}} dx = \int \frac{1}{(x-3)\sqrt{(x-3)^2-4}} dx = \int \frac{1}{u\sqrt{u^2-4}} = \frac{1}{2} \arcsin\frac{|u|}{2} + C$$

$$\begin{aligned} & x^2-6x+5 \\ &= x^2-6x+9+5-9 \\ &= (x-3)^2-4 \end{aligned} \quad \begin{aligned} u &= x-3 \\ du &= dx \end{aligned}$$

$$= \frac{1}{2} \arcsin\frac{|x-3|}{2} + C$$

$$25) A = Pe^{rt} \quad 3000 = 1000e^{0.05t} \rightarrow \ln 3 = 0.05t$$

$$A = 1000e^{0.05t} \quad 3 = e^{0.05t} \quad t = \frac{\ln 3}{0.05} \approx 21.97 \text{ years} \quad \boxed{D}$$

$$26) y = \int \frac{2}{\sqrt{1-x^2}} + x \quad y = 2\arcsin x + \frac{1}{2}x^2 + C \quad \boxed{B}$$

$$27) y' = \int x \sqrt{x+1} \quad y = \int (u-1) \cdot u^{\frac{1}{2}} du = \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C$$

$$\begin{aligned} u &= x+1 \\ du &= dx \end{aligned} \quad \begin{aligned} &= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C \quad \boxed{E} \\ &= (x+1)^{\frac{3}{2}} \left[\frac{2}{5}(x+1) - \frac{2}{3} \right] + C \end{aligned}$$

$$28) \sqrt{x^2-1} \frac{dy}{dx} = \frac{y}{x} \rightarrow \int \frac{1}{y} dy = \int \frac{1}{x\sqrt{x^2-1}} dx \rightarrow \ln|y| = \arccos x + C \rightarrow y = C e^{\arccos x}$$

$$29) a) \frac{dy}{dx} = \frac{k}{y} \quad b) \int y dy = \int k dx \rightarrow \frac{2}{3}y^{\frac{3}{2}} = Kx + C \rightarrow y^{\frac{3}{2}} = \frac{3}{2}Kx + C \rightarrow y = \left(\frac{3}{2}Kx\right)^{\frac{2}{3}} + C$$

$$30) \frac{df}{dt} = kf \rightarrow \int \frac{1}{f} df = \int k dt \rightarrow \ln|f| = kt + C \rightarrow f = Ce^{kt} \rightarrow 10 = Ce^0 = C \rightarrow f = 10e^{kt}$$

$$2f = 10e^{kt} \rightarrow \frac{12}{5} = e^{6k} \rightarrow \ln\frac{12}{5} = 6k \rightarrow K = \frac{1}{6} \ln\frac{12}{5} \rightarrow f = 10e^{\frac{1}{6} \ln\frac{12}{5} t} \quad \boxed{A}$$

$$\frac{dy}{dx} = \frac{e^x}{y} \rightarrow \int y dy = \int e^x dx \rightarrow \frac{1}{2}y^2 = \frac{1}{2}e^x + C = \frac{1}{2}e^x + C \rightarrow y^2 = e^x + C$$

$$32) \frac{dy}{dx} = 500-y \rightarrow \frac{1}{500-y} dy = dx \rightarrow \int -\frac{1}{u} du = \int dx \rightarrow -\ln|u| = x + C \rightarrow -\ln|500-y| = x + C \rightarrow \frac{1}{500-y} = Ce^x$$

$$500-y = Ce^{-x} \rightarrow y = 500-Ce^{-x}$$

$$\therefore \text{con} \sim 0.1 \text{ min} \rightarrow u = 500 - \frac{493}{e^x}$$