Analytic Geometry **Solve by Completing the Square**

Key

Completing the square method allows us to factor an expression that is initially unfactorable

Steps for Completing the square:

1) Rearrange equation in standard form: $ax^2 + bx + c = 0$

2) divide each term in the equation by a if $a \neq 1$ (We need the new a value to be 1)

3) Move the constant to the other side of the equation.

4) Add spaces ___ to the equation: $x^2 + bx + \_\_\_ = c + \_\_\_$ "+ _ "

5) Find $\left(\frac{b}{2}\right)^2$ and enter this value into the blank spaces ___ on both sides of the equation

6) Rewrite left side in factored form and add the numbers on the right side

7) take the $\sqrt{}$ of both sides (don't forget $\pm$)

8) solve for x

I. Getting ready:

Find $\left(\frac{b}{2}\right)^2$ and factor the expression

1. $x^2 + 4x + \underline{4}$

$b=4$

$\left(\frac{4}{2}\right)^2 = 4$

2. $x^2 - 10x + \underline{25}$

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$

3. $x^2 + 8x + \underline{16}$

$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$

Examples: Solve by completing the square method:

1. $x^2 - 4x + 2 = 0$

$x^2 - 4x + \underline{4} = -2 + \underline{4}$

$\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$(x-2)(x-2) = 2$

$\sqrt{(x-2)^2} = \sqrt{2}$

$-2 \quad 4 \quad -2$

$1 \quad -4 \quad 1$

$x - 2 = \pm\sqrt{2}$

$\boxed{x = 2 \pm \sqrt{2}}$

$\left(\frac{12}{2}\right)^2$

6^2

2. $x^2 + 12x - 1 = 0$

$x^2 + 12x + \underline{36} = 1 + \underline{36}$

$(x+6)(x+6) = 37$

$\sqrt{(x+6)^2} = \sqrt{37}$

$x + 6 = \pm\sqrt{37}$

$\boxed{x = -6 \pm \sqrt{37}}$

Steps for **Completing the square:**

1) Rearrange equation in standard form: $ax^2 + bx + c = 0$
2) divide each term in the equation by a if a ≠1 (We need the new a value to be 1)
3) Move the constant to the other side of the equation.
4) Add spaces ___ to the equation: $x^2 + bx + \_\_\_ = c + \_\_\_$
5) Find $\left(\frac{b}{2}\right)^2$ and enter this value into the biank spaces ___ on both sides of the equation
6) Rewrite left side in factored form and add the numbers on the right side
7) take the $\sqrt{}$ of both sides (don't forget ±)
8) solve for x

Solve for x below using completing the square method:

3. $x^2 - 4x - 15 = 0$

$x^2 - 4x + 4 = 15 + 4$ $\quad \left(\frac{4}{2}\right)^2 = 2^2$

$(x-2)(x-2) = 19$

$\sqrt{(x-2)^2} = \sqrt{19}$

$x - 2 = \pm\sqrt{19}$

$\boxed{x = 2 \pm \sqrt{19}}$

4. $\frac{2x^2}{2} + \frac{32x}{2} - \frac{8}{2} = 0$ $\quad x^2 + 16x - 4 = 0$

$x^2 + 16x + 64 = 4 + 64$ $\quad \left(\frac{16}{2}\right)^2 = 8^2 = 64$

$\sqrt{(x+8)^2} = \sqrt{68}$

$x + 8 = \pm\sqrt{68}$

$\boxed{x = -8 \pm \sqrt{68}}$

5. $\frac{5x^2}{5} - \frac{20x}{5} + \frac{30}{5} = 0$ $\quad x^2 - 4x + 6 = 0$

$\boxed{5x^2 - 20x - 30 = 0}$ $\quad x^2 - 4x + \_ = -6 + \_$

$x^2 - 4x - 6 = 0$ $\quad \left(\frac{4}{2}\right)^2 = 2^2 = 4$

$x^2 - 4x + 4 = 6 + 4$

$(x-2)^2 = 10$

$x - 2 = \pm\sqrt{10}$

$x = 2 \pm \sqrt{10}$

6. $\frac{2x^2}{2} + \frac{16x}{2} = \frac{6}{2}$ $\quad x^2 + 8x = 3$

$x^2 + 8x + 16 = 3 + 16$ $\quad \left(\frac{8}{2}\right)^2 = 4^2 = 16$

$(x+4)^2 = 19$

$x + 4 = \pm\sqrt{19}$

$\boxed{x = -4 \pm \sqrt{19}}$

$3x^2 - 12x - 18 = 0$ *

Completing the Square WS #1 Homework

Solve for x below using completing the square method:

1. $\frac{3x^2}{3} - \frac{12x}{3} + \frac{18}{3} = 0$

$x^2 - 4x + 6 = 0$

$x^2 - 4x = -6$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 - 4x + 4 = -6 + 4$

$(x-2)^2 = -2$

$x - 2 = \pm\sqrt{-2}$

$x = 2 \pm \sqrt{-2}$

2. $x^2 + 24x - 4 = 0$

$\left(\frac{24}{2}\right)^2 = 12^2 = 144$

$x^2 + 24x + 144 = 4 + 144$

$(x+12)^2 = 148$

$x = \pm\sqrt{148}$

$x = -12 \pm \sqrt{148}$

$x^2 + 14x - 100$ *

3. $\frac{3x^2}{3} - \frac{12x}{3} - \frac{15}{3} = 0$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 - 4x - 5 = 0$

$x^2 - 4x + 4 = 5 + 4$

$(x-2)^2 = 9$

$\sqrt{(x-2)^2} = \pm\sqrt{9}$

$x - 2 = \pm 3$

$x = 2 \pm \sqrt{3}$ xx

2 ± 3

$x = 5, -1$

4. $x^2 + 14x - 100 = 0$

$\left(\frac{14}{2}\right)^2 = 7^2$

$x^2 + 14x + 49 = 100 + 49$

$(x+7)^2 = 149$

$x = \pm\sqrt{149} - 7$

$x = -7 \pm \sqrt{149}$

5. $\frac{3x^2}{3} - \frac{24x}{3} - \frac{3}{3} = 0$

$x^2 - 8x - 1 = 0$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

$x^2 - 8x + 16 = 1 + 16$

$(x-4)^2 = 17$

$x = 4 \pm \sqrt{17}$

6. $\frac{5x^2}{5} - \frac{20}{5} + \frac{60x}{5} = 0$

$x^2 - 4 + 12x = 0$

$x^2 + 12x - 4 = 0$

$\left(\frac{12}{2}\right)^2 = 6^2 = 36$

$x^2 + 12x + 36 = 4 + 36$

$(x+6)^2 = 40$

$x + 6 = \pm\sqrt{40}$

$x = -6 \pm \sqrt{40}$

$-16 + 2x^2 + 4x = 0$

7. $-16 + 2x^2 + 4x = 0$

$\frac{2x^2}{2} + \frac{4x}{2} - \frac{16}{2} = 0$

$x^2 + 2x - 8 = 0$

$x^2 + 2x = 8$

$x^2 + 2x + 1 = 8 + 1$

$(x+1)^2 = 9$

$x = -1 \pm 3$

8. $\frac{4x^2}{4} + \frac{64x}{4} - \frac{12}{4} = 0$

$x^2 + 16x - 3 = 0$

$x^2 + 16x - 3 = 0$

$\left(\frac{16}{2}\right)^2 = 8^2$

$x^2 + 16x + 64 = 3 + 64$

$(x+8)^2 = 67$

$x = -8 \pm \sqrt{67}$