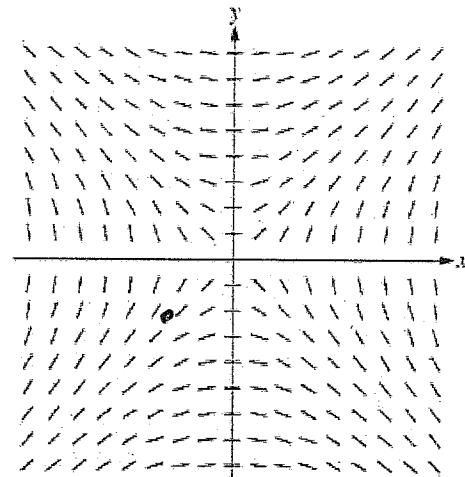
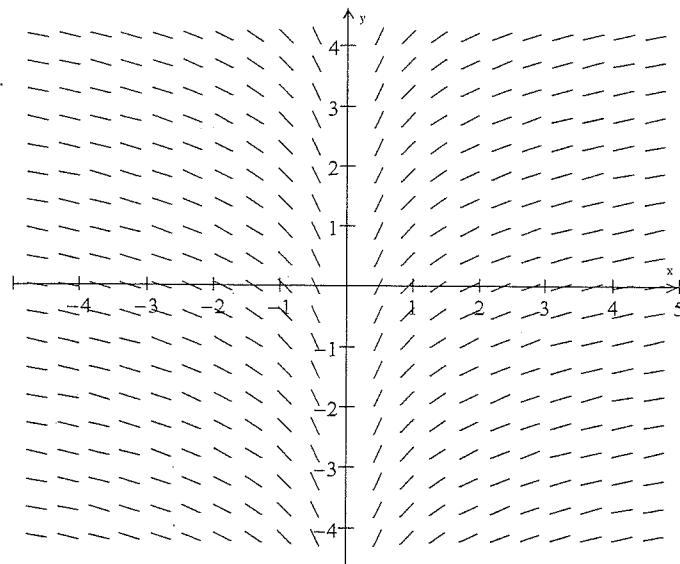


Differential Equation and Slope Fields Review WS #2

1. Given $\frac{dy}{dx} = \frac{x}{y}$, and $y(-3) = -2$
- Find the particular solution
 - Find the domain.
 - Find the equation of tangent line when $x = -\sqrt{6}$
 - Use it to approximate $y(-2.5)$



2. Consider the differential equation $\frac{dy}{dx} = \frac{1}{x}$.
- Find a particular solution to $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$
 - State its domain
 - Find the equation of tangent line when $x = e$
 - Use tangent line to approximate $y(2.8)$



3. The rate of temperature decrease for a cup of coffee is given by equation $\frac{dy}{dt} = ky$ with t measured in minutes. The initial temperature is 190°F and the temperature decreases to 76° after 5 minutes

a) Find the particular equation

b) Find how long it will take for the temperature to decrease to 50°

c) Find the average temperature of the coffee in the first 10 minutes

d) Using 4 equal intervals of right-handed Riemann sum, find the avg temperature of the coffee in the first 10 mins.

e) Find $\lim_{t \rightarrow \infty} y(t)$

4. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$ given $y(-2) = 1$

Differential Equation and Slope Fields Review WS #2

1. Given $\frac{dy}{dx} = \frac{x}{y}$, and $y(-3) = -2$
- Find the particular solution
 - Find the domain.
 - Find the equation of tangent line when $x = \sqrt{6}$
 - Use it to approximate $y(2.5)$

$$\int y dy = \int x dx$$

$$y^2 = x^2 - 5 \quad y = \pm \sqrt{x^2 - 5}$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$(-2)^2 = (-3)^2 \quad C = \frac{9}{2} + C$$

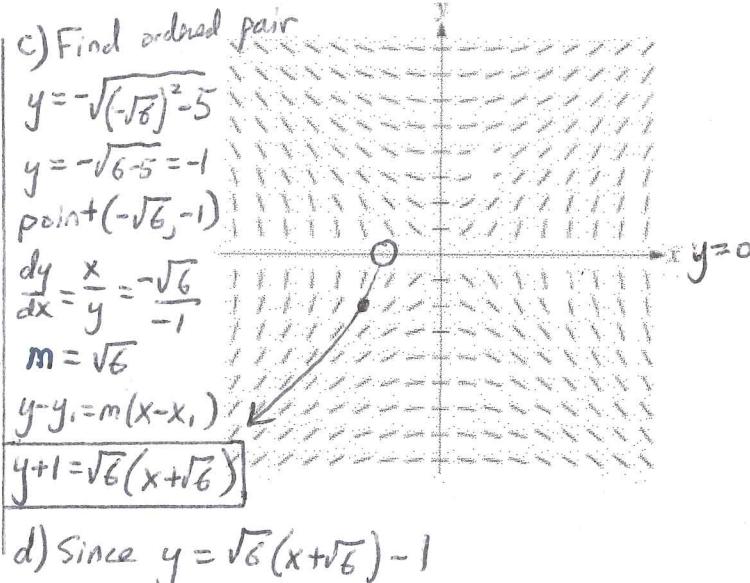
$$2 = \frac{9}{2} + C \quad C = -\frac{5}{2}$$

$$\frac{y^2}{2} = \frac{x^2}{2} - \frac{5}{2}$$

a) $y = -\sqrt{x^2 - 5}$

$y \neq 0$ according to differential equation restriction and domain must include $x = -3$. Since $y \neq 0$, then $x = -\sqrt{5}$

Domain: $x < -\sqrt{5}$ or $(-\infty, -\sqrt{5})$



2. Consider the differential equation $\frac{dy}{dx} = \frac{1}{x}$.

- Find a particular solution to $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$
- State its domain
- Find the equation of tangent line when $x = e$
- Use tangent line to approximate $y(2.8)$

$$\int dy = \int \frac{dx}{x}$$

$$y = \ln|x| + C$$

$$1 = \ln|1| + C \quad C = 0$$

a) $y = \ln|x| + 1$

b) $x \neq 0$
Domain: $(0, \infty)$

Find ordered pair

c) $y = \ln|e| + 1 = 1 + 1 = 2$
point: $(e, 2)$

$$\frac{dy}{dx} = \frac{1}{x} = \frac{1}{e}, \quad m = \frac{1}{e}$$

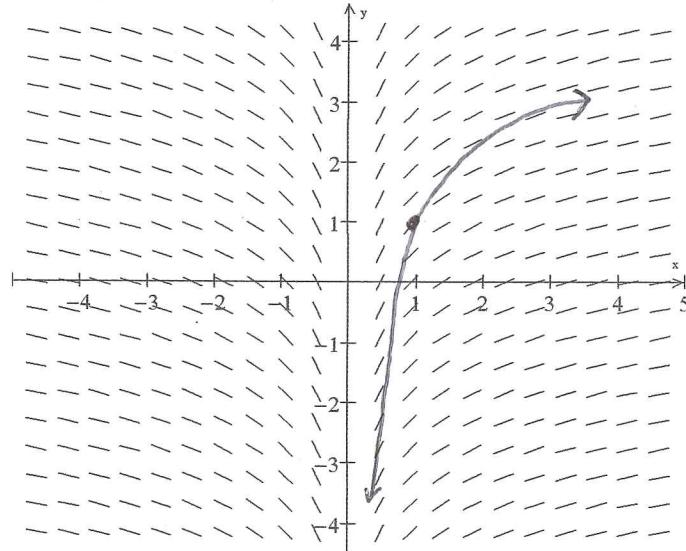
$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{e}(x - e)$$

d) Since $y = \frac{1}{e}(x - e) + 2$,

$$y(2.8) = \frac{1}{e}(2.8 - e) + 2$$

$y(2.8) = 2.030$



3. The rate of temperature decrease for a cup of coffee is given by equation $\frac{dy}{dt} = ky$ with t measured in minutes. The initial temperature is 190°F and the temperature decreases to 76° after 5 minutes

a) Find the particular equation

$$(\text{time, temp}) \quad (t, y)$$

$$(0, 190^{\circ})$$

$$(5, 76)$$

$$\int \frac{dy}{y} = \int k dt$$

$$y = Ce^{kt}$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$y = e^{kt} \cdot e^C$$

$$76 = 190e^{k(5)}$$

$$190 = Ce^{k(0)}$$

$$190 = C$$

$$y = 190e^{kt}$$

$$\ln\left(\frac{76}{190}\right) = \ln e^{5k}$$

$$\ln\left(\frac{2}{5}\right) = 5k$$

$$\frac{1}{5}\ln\left(\frac{2}{5}\right) = k$$

$$y = 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t}$$

b) Find how long it will take for the temperature to decrease to 50°

$$(-, 50)$$

$$50 = 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t}$$

$$\frac{50}{190} = e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t}$$

$$\ln\left(\frac{5}{19}\right) = \ln e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t}$$

$$\ln\left(\frac{5}{19}\right) = \frac{1}{5}\ln\left(\frac{2}{5}\right)t$$

$$7.285 = t$$

$$t = 7.285 \text{ mins.}$$

c) Find the average temperature of the coffee in the first 10 minutes

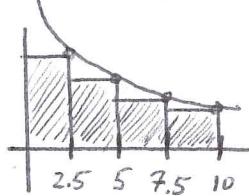
$$\text{Avg. value} = \frac{1}{b-a} \int_a^b y(t) dt$$

$$\text{Avg. temp} = \frac{1}{10-0} \int_0^{10} 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t} dt = \frac{1}{10} (870.9026) = 87.090^{\circ}\text{F}$$

$$e) \lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)t} = 190e^{\frac{1}{5}\ln\left(\frac{2}{5}\right)(\infty)} = 190e^{-\infty} = \frac{190}{e^{\infty}} = 0$$

d) Using 4 equal intervals of right-handed Riemann sum, find the avg temperature of the coffee in the first 10 mins.

$$\text{width} = \frac{b-a}{n} = \frac{10-0}{4} = 2.5$$



$$\int_0^{10} y(t) dt \approx 2.5 [y(2.5)] + 2.5 [y(5)] + 2.5 [y(7.5)] + 2.5 [y(10)]$$

$$= 2.5 [y(2.5) + y(5) + y(7.5) + y(10)]$$

$$= 2.5 [274.634] = 686.585$$

4. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$ given $y(-2) = 1$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + C$$

$$y = e^{\ln|x| + C}$$

$$y = C|x|$$

$$1 = C|-2|$$

$$\frac{1}{2} = C$$

$$y = \frac{1}{2}|x|$$

$$\text{Domain: } (-\infty, 0)$$

$$\text{Avg. temp.} = \frac{1}{10} \int_0^{10} y(t) dt$$

$$= \frac{1}{10} (686.585)$$

$$= 68.659^{\circ}\text{F}$$