Differential Equations Practice FRQ's

1)

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

2)

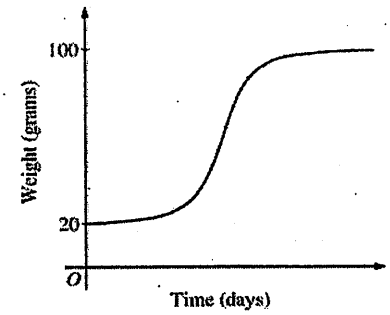
The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

3)

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

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1)

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(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(time, waste)
(t, W)

a) *tangent line approximation:

point: $(0, 1400)$ slope: $\frac{dW}{dt} = \frac{1}{25}(W-300)$

$\frac{dW}{dt} = \frac{1}{25}(1400-300) = 44$

slope: $m = 44$

$y - y_1 = m(x - x_1)$

$W - W_1 = m(t - t_1)$

$W - 1400 = 44(t - 0)$

$W = 44t + 1400$

$W(\frac{1}{4}) = 44(\frac{1}{4}) + 1400 = \boxed{1411 \text{ tons of waste}}$

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c) Find particular equation for $\frac{dW}{dt} = \frac{1}{25}(W-300)$ $W(0) = 1400$

$dW = \frac{1}{25}(W-300)\,dt$

$\frac{dW}{W-300} = \frac{1}{25}dt$

$\int \frac{1}{W-300}\,dW = \frac{1}{25}\int 1\,dt$

$u = W-300$

$\frac{du}{dW} = 1$ $dW = du$

$\int \frac{1}{u}\,du = \frac{1}{25}\int 1\,dt$

$\ln|u|$

$\ln|W-300| = \frac{1}{25}t + C$

$e^{\ln|W-300|} = e^{\frac{1}{25}t + C}$

$|W-300| = e^{\frac{1}{25}t} \cdot e^{C}$

$|W-300| = e^{\frac{1}{25}t} \cdot C$

$W - 300 = Ce^{\frac{1}{25}t}$

plug in $W(0) = 1400$

$W = Ce^{1/25\,t} + 300$

$1400 = Ce^{0} + 300$ $C = 1100$

$\boxed{W = 1100e^{1/25\,t} + 300}$

2)

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

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(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

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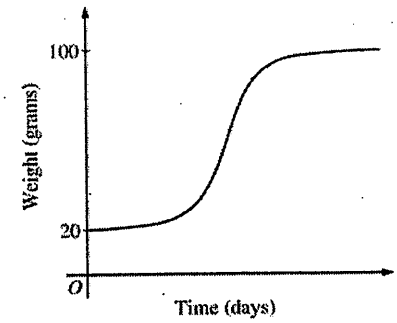

c) Find particular solution for $\frac{dB}{dt} = \frac{1}{5}(100-B)$ and $B(0)=20$

$dB = \frac{1}{5}(100-B)\,dt$

$\frac{dB}{100-B} = \frac{1}{5}dt$

$\int \frac{1}{100-B}\,dB = \frac{1}{5}\int 1\,dt$

$u = 100-B$

$\frac{du}{dB} = -1$

$dB = -du$

$\int \frac{1}{u}(-du) = \frac{1}{5}\int 1\,dt$

$-\ln|u| = \frac{1}{5}t + C$

$-\ln|100-B| = \frac{1}{5}t + C$

$\ln|100-B| = -\frac{1}{5}t + C$

$e^{\ln|100-B|} = e^{-1/5 t + C}$

$|100-B| = e^{-1/5 t} \cdot e^{C}$

$100-B = Ce^{-1/5 t}$

$B(0)=20$

$100 - Ce^{-1/5 t} = B$

$100 - Ce^{0} = 20$

$100 - C = 20$

$80 = C$

$B = 100 - Ce^{-1/5 t}$

$\boxed{B = 100 - 80e^{-1/5 t}}$

3)

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

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a) tangent line equation:

point: $(1,0)$ slope: $\frac{dy}{dx} = e^y(3x^2-6x)$

$\frac{dy}{dx} = e^0(3(1)^2 - 6(1))$

$\frac{dy}{dx} = 1(3-6) = -3$

$m = -3$

$y - y_1 = m(x - x_1)$

$y - 0 = -3(x-1)$

$y = -3(x-1)$

$y(1.2) \approx -3(1.2-1) = \boxed{-0.6}$

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b) Find particular solution: $\frac{dy}{dx} = e^y(3x^2-6x)$ point $(1,0)$

$dy = e^y(3x^2-6x)dx$

$\frac{dy}{e^y} = 3x^2 - 6x\,dx$

$\int e^{-y}dy = \int 3x^2 - 6x\,dx$

$u = -y$

$\frac{du}{dy} = -1$

$dy = -du$

$\int e^u(-du) = \int 3x^2 - 6x\,dx$

$-e^u = \frac{3x^3}{3} - \frac{6x^2}{2} + C$

$-e^{-y} = x^3 - 3x^2 + C$

solve for C: $(1,0)$

$e^{-y} = -x^3 + 3x^2 + C$

$e^0 = -1 + 3(1)^2 + C$

$1 = -1 + 3 + C$

$-1 = C$

$e^{-y} = -x^3 + 3x^2 - 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$\ln e^{-y} = \ln(-x^3 + 3x^2 - 1)$

$-y = \ln(-x^3 + 3x^2 - 1)$

$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

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(t, H)

a) *tangent line approximation (time, Internal Temperature)

point: $(\overset{t}{0}, \overset{H}{91})$ slope: $\frac{dH}{dt} = \frac{-1}{4}(H-27)$

$\frac{dH}{dt} = \frac{-1}{4}(91-27) = \frac{-1}{4}(64) = -16$ slope: $m=-16$

$y - y_1 = m(x - x_1)$

$H - H_1 = m(t - t_1)$

$H - 91 = -16(t - 0)$

$H = -16(t) + 91$

$H(3) = -16(3) + 91 = \boxed{43°\text{Celsius}}$

b) Find particular solution: (t, G)

$\frac{dG}{dt} = -(G-27)^{2/3}$ at $G(0) = 91$

$dG = -(G-27)^{2/3}dt$

$\frac{dG}{(G-27)^{2/3}} = -1dt$

$\int \frac{1}{(G-27)^{2/3}}dG = -\int 1dt$

$u = G - 27$

$\frac{du}{dG} = 1$ $dG = du$

$\int \frac{1}{u^{2/3}}du = -\int 1dt$

$\int u^{-2/3} = -\int 1dt$

$\frac{u^{1/3}}{1/3} = -t + C$

$3(G-27)^{1/3} = -t + C$

solve for C: $G(0) = 91$

$3(91-27)^{1/3} = -0 + C$

$3(64)^{1/3} = C$ $C = 3(4) = 12$

$3(G-27)^{1/3} = -t + 12$

$(G-27)^{1/3} = \frac{1}{3}(-t+12)$

$(G-27)^{1/3} = -\frac{1}{3}t + 4$

$G - 27 = (-\frac{1}{3}t + 4)^3$

$\boxed{G = (-\frac{1}{3}t + 4)^3 + 27}$

$G(3) = (-\frac{1}{3}(3) + 4)^3 + 27 = 27 + 27$

$\boxed{G(3) = 54°C}$