Solving Differential Equations Task (part 2)

1)

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Solve the below differential equation:

2) $y' - xycos(x^2) = 0$ given $y(0) = e$ a) Find general solution b) Find particular solution

Solving Differential Equations: Additional Practice Problems

3) Solve the Differential Equation: $y \ln x^4 - xy' = 0$

4) a) Find the general solution b) Find the particular solution

$$yy' - 2e^{3x} = 0 \qquad y(0) = 5$$

Solving Differential Equations Task (Continued)

1) The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$\frac{dB}{dt} = \frac{100-B}{5}$

$5dB = (100-B)dt$

$\frac{dB}{100-B} = \frac{dt}{5}$

$\int \frac{dB}{100-B} = \frac{1}{5}\int dt$

$u = 100 - B$

$\frac{du}{dB} = -1$

$du = -dB$

$dB = -du$

$\int \frac{-1\,du}{u}$

$= -\ln|u|$

$-\ln|100-B| = \frac{1}{5}t + c$

$\ln|100-B| = -\frac{1}{5}t + c$

$e^{\ln|100-B|} = e^{-\frac{1}{5}t+c}$

$|100-B| = e^{-1/5 t} \cdot e^{c}$

$|100-B| = e^{-\frac{1}{5}t} \cdot C$

$|100-B| = Ce^{-\frac{1}{5}t}$

$100 - B = Ce^{-\frac{1}{5}t}$

$\overset{t}{(}\text{time}, \overset{B}{\text{Bird weight}})$

$(0, 20)$

$100 - Ce^{-\frac{1}{5}t} = B$

$B = 100 - Ce^{-\frac{1}{5}t}$ (general equation)

plug in $(0, 20)$

$20 = 100 - Ce^{-\frac{1}{5}(0)}$

$20 = 100 - C$

$C = 80$

$$\boxed{\begin{array}{c} B = 100 - 80e^{-\frac{1}{5}t} \\ B(t) = 100 - 80e^{-\frac{1}{5}t} \end{array}}$$

2) $y' - xy\cos(x^2) = 0 \quad y(0) = e$ find particular solution

$$\frac{dy}{dx} = xy\cos(x^2)$$

$$dy = xy\cos(x^2)\,dx$$

$$\frac{dy}{y} = x\cos(x^2)\,dx$$

$$\int \frac{1}{y}\,dy = \int x\cos(x^2)\,dx$$

$$u = x^2 \qquad dx = \frac{du}{2x}$$

$$\frac{du}{dx} = 2x$$

$$\int \cancel{x}\cos u \cdot \frac{du}{2\cancel{x}}$$

$$\int \frac{1}{y}\,dy = \frac{1}{2}\int \cos u\,du$$

$$\ln|y| = \frac{1}{2}\sin u + C$$

$$\ln|y| = \frac{1}{2}\sin(x^2) + C$$

$$e^{\ln|y|} = e^{\frac{1}{2}\sin(x^2) + C}$$

$$|y| = e^{\frac{1}{2}\sin(x^2)} \cdot e^{C}$$

$$|y| = Ce^{\frac{1}{2}\sin(x^2)}$$

$$\boxed{y = Ce^{\frac{1}{2}\sin(x^2)}} \leftarrow \text{general solution}$$

$$y = Ce^{\frac{1}{2}\sin(x^2)} \leftarrow \text{plug in } y(0) = e$$

$$e = Ce^{\frac{1}{2}\sin(0)^2}$$

$$e = Ce^{0}$$

$$e = C$$

$$y = e \cdot e^{\frac{1}{2}\sin(x^2)}$$

$$\boxed{y = e^{\frac{1}{2}\sin(x^2) + 1}}$$

Solving Differential Equations: Additional Practice Problems

3) Solve the Differential Equation: $\boldsymbol{y \ln x^4 - xy' = 0}$

$$y \ln x^4 - x\left(\frac{dy}{dx}\right) = 0$$

$$\frac{-x\,dy}{dx} = \frac{-y \ln x^4}{1}$$

$$-x\,dy = -y \ln x^4\,dx$$

$$x\,dy = y \ln x^4\,dx$$

$$\frac{dy}{y} = \frac{\ln x^4}{x}\,dx$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$$\int \frac{1}{y}\,dy = \int \frac{4 \ln x}{x}\,dx$$

$$4\int \frac{u}{x} \cdot x\,du \rightarrow 4\int u\,du$$

$$4\left(\frac{u^2}{2}\right)$$

$$\ln|y| = 2u^2 + C$$

$$\ln|y| = 2(\ln x)^2 + C$$

$$e^{\ln|y|} = e^{2(\ln x)^2 + C}$$

$$|y| = e^{2(\ln x)^2} \cdot e^{C}$$

$$|y| = e^{2(\ln x)^2} \cdot C$$

$$\boxed{y = Ce^{2(\ln x)^2}}$$

4) a) Find the general solution b) Find the particular solution

$$yy' - 2e^{3x} = 0 \qquad y(0) = 5$$

$$y\left(\frac{dy}{dx}\right) - 2e^{3x} = 0$$

$$\frac{y\,dy}{dx} = \frac{2e^{3x}}{1}$$

$$y\,dy = 2e^{3x}\,dx$$

$$\int y\,dy = \int 2e^{3x}\,dx \leftarrow u = 3x$$

$$\frac{du}{dx} = 3$$

$$dx = \frac{du}{3}$$

$$2\int e^{u} \cdot \frac{du}{3}$$

$$\int y\,dy = \frac{2}{3}\int e^{u}\,du$$

$$\frac{y^2}{2} = \frac{2}{3}e^{3x} + C \leftarrow \text{solve for } C\text{: } y(0) = 5$$

$$\frac{5^2}{2} = \frac{2}{3}e^{3(0)} + C$$

$$\frac{25}{2} = \frac{2}{3}(1) + C$$

$$\frac{25}{2} - \frac{2}{3} = C$$

$$\frac{75}{6} - \frac{4}{6} = C$$

$$\frac{71}{6} = C$$

$$\frac{y^2}{2} = \frac{2}{3}e^{3x} + \frac{71}{6}$$

$$2\left(\frac{y^2}{2} = \frac{2}{3}e^{3x} + \frac{71}{6}\right)$$

$$y^2 = \frac{4}{3}e^{3x} + \frac{142}{6}$$

$$y = \pm\sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}}$$

$$\boxed{y = \sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}} \quad \text{since } y(0) = 5}$$