

**CCGPS Analytic Geometry**  
**Dilations**

A dilation is a transformation (notation  $D_k$ ) that produces an image that is the same shape \_\_\_\_\_ as the original but it is a different size. A dilation stretches or shrinks the original figure.

The description of a dilation includes the scale factor (or ratio) and the center of dilation. The center of dilation is a fixed point about which all points are expanded or contracted. It is the only point that remains the same under a dilation.

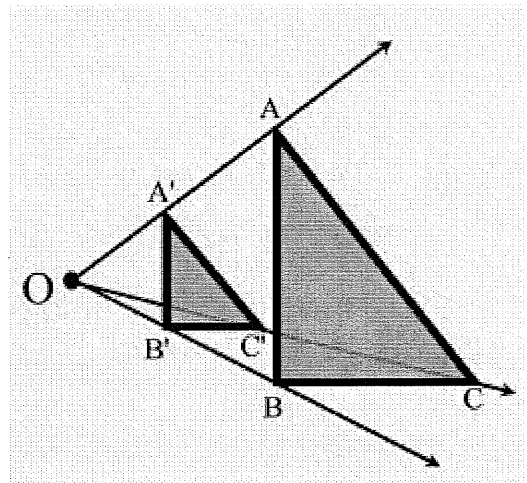
A dilation of scalar factor  $k$  whose center of dilation is the ORIGIN is written:

$$D_k(x,y) = (kx, ky)$$

If the scale factor,  $k$ , is greater than 1, the image is an enlargement (a stretch)

If the scale factor,  $k$ , is between 0 and 1, the image is a reduction (a shrink)

What do you think happens when the scale factor is 1?



**Properties of Dilations:**

1. Angle measures remain the same
2. Parallel Lines remain parallel
3. points stay on the same lines
4. midpoint remains the same in each figure
5. orientation (lettering order remains the same)

However, distance is NOT preserved. (lengths of segments are not the same in all cases except a scale factor of 1.)

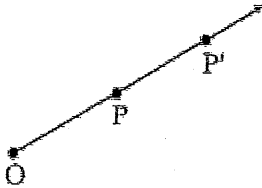
DEFINITION: A **dilation** is a transformation of the plane,  $D_k$ , such that if  $O$  is a fixed point,  $k$  is a non-zero real number and  $P'$  is the image of point  $P$ , then  $O$ ,  $P$ , and  $P'$  are collinear and  $\frac{OP'}{OP} = k$ .

<p><b>Examples:</b></p> <p><b>1.</b></p> <p><math>P'</math> is the image of <math>P</math> under a dilation about <math>O</math> of ratio 2.</p> <p><math>OP' = 2OP</math> and <math>\frac{OP'}{OP} = 2</math></p>	<p><b>2.</b></p>	<p><math>\triangle A'B'C'</math> is the image of <math>\triangle ABC</math> under a dilation about <math>O</math> of ratio <math>\frac{1}{2}</math>.</p> <p><math>OA' = \frac{1}{2}OA</math></p> <p><math>OB' = \frac{1}{2}OB</math></p> <p><math>OC' = \frac{1}{2}OC</math></p>
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DEFINITION: A **dilation** is a transformation of the plane,  $D_k$ , such that if  $O$  is a fixed point,  $k$  is a non-zero real number and  $P'$  is the image of point  $P$ , then  $O$ ,  $P$ , and  $P'$  are collinear and  $\frac{OP'}{OP} = k$ .

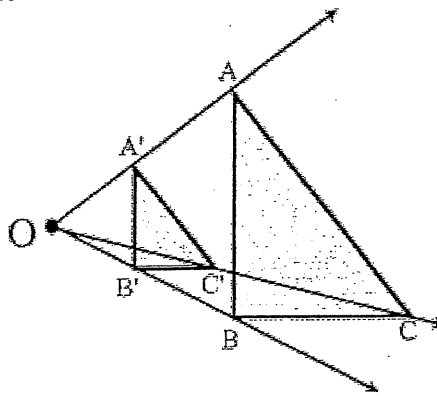
Examples:

1.



$P'$  is the image of  $P$  under a dilation about  $O$  of ratio 2.  
 $OP' = 2OP$  and  $\frac{OP'}{OP} = 2$

2.



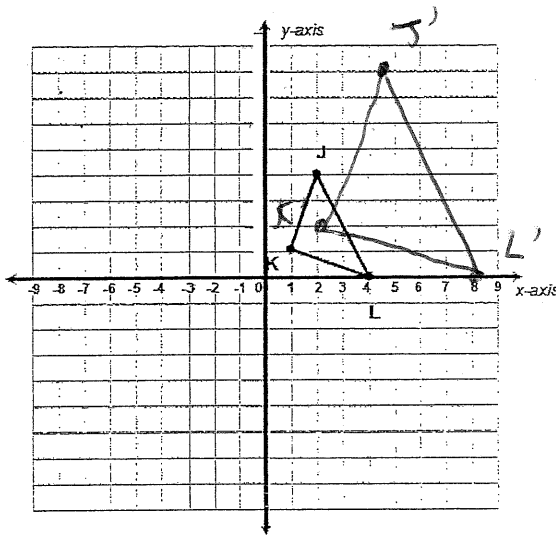
$\triangle A'B'C'$  is the image of  $\triangle ABC$  under a dilation about  $O$  of ratio  $\frac{1}{2}$ .

$$OA' = \frac{1}{2}OA$$

$$OB' = \frac{1}{2}OB$$

$$OC' = \frac{1}{2}OC$$

1.



Graph the dilated image of triangle  $JKL$  using a scale factor of 2 and  $(0,0)$  as the center of dilation.

$$J: (2, 4)$$

$$J': (4, 8)$$

$$K: (1, 1)$$

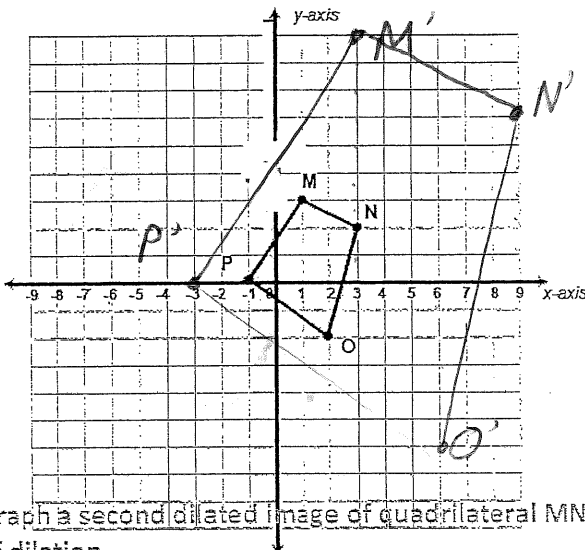
$$K': (2, 2)$$

$$L: (4, 0)$$

$$L': (8, 0)$$

Graph a second dilated image of triangle  $JKL$  (in another color) using a scale factor of 2 and  $(-1, 2)$  as the center of dilation.

2.



Graph the dilated image of quadrilateral  $MNOP$  using a scale factor of 3 and the origin as the center of dilation.

$$M: (1, 3)$$

$$M': (3, 9)$$

$$N: (3, 2)$$

$$N': (9, 6)$$

$$O: (2, -2)$$

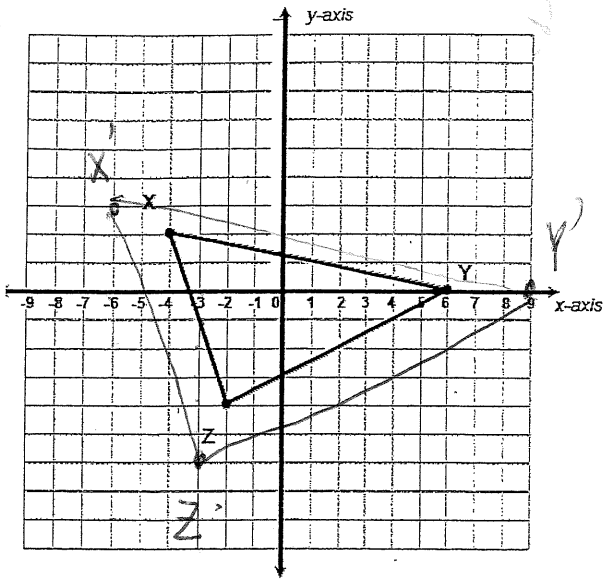
$$O': (6, -6)$$

$$P: (-1, 0)$$

$$P': (-3, 0)$$

Graph a second dilated image of quadrilateral  $MNOP$  (in another color) using a scale factor of 3 and  $(1,1)$  as the center of dilation.

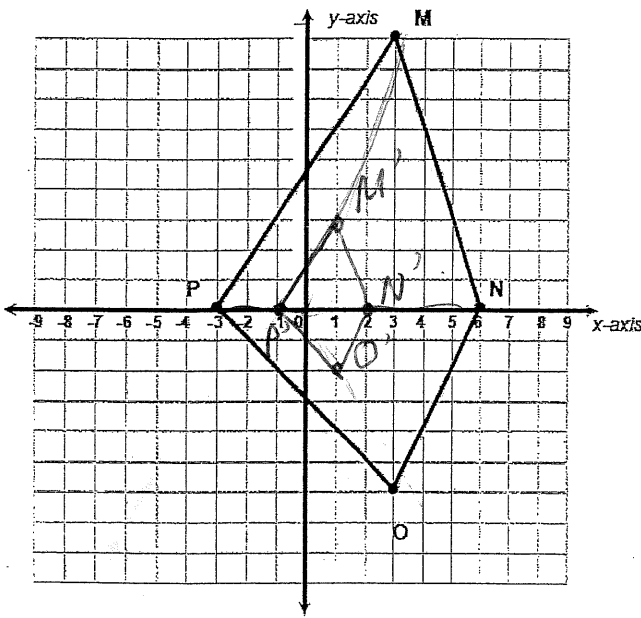
n-3.



Graph the dilated image of triangle XYZ using a scale factor of 1.5 and (0,0) as the center of dilation.

X: <u>(-4, 2)</u>	X': <u>(-6, 3)</u>
Y: <u>(6, 0)</u>	Y': <u>(9, 0)</u>
Z: <u>(-2, -4)</u>	Z': <u>(-3, -6)</u>

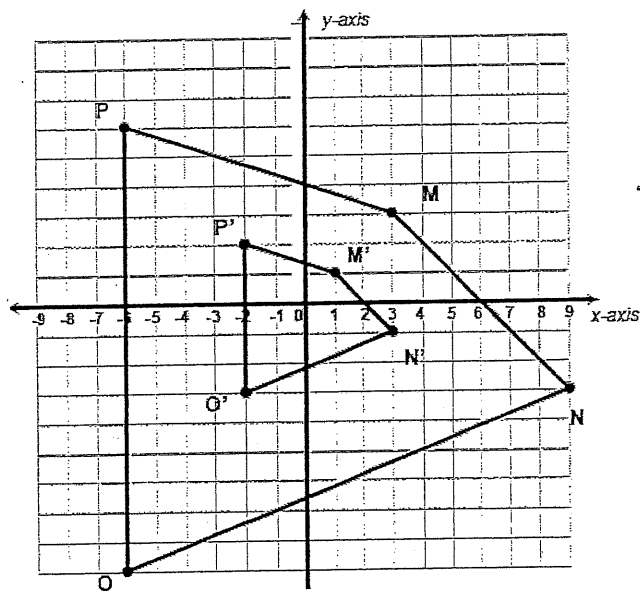
1f.



Graph the dilated image of quadrilateral MNOP using a scale factor of 1/3 and the origin as the center of dilation.

M: <u>(3, 9)</u>	M': <u>(1, 3)</u>
N: <u>(6, 0)</u>	N': <u>(2, 0)</u>
O: <u>(3, -6)</u>	O': <u>(1, -2)</u>
P: <u>(-3, 0)</u>	P': <u>(-1, 0)</u>

5.



Describe the dilation of quadrilateral MNOP, using the origin as the center.

$$k = \frac{1}{3}$$

$$\begin{aligned} M(3, 3) &\rightarrow M'(1, 1) \\ N(9, -3) &\rightarrow N'(3, -1) \\ O(-6, -9) &\rightarrow O'(-2, -3) \\ P(-6, 6) &\rightarrow P'(-2, 2) \end{aligned}$$

6. The table below shows the coordinates of triangle RST and the coordinates of R' in triangle R'S'T'. Triangle R'S'T' is a dilation of triangle RST.

Triangle RST		Triangle R'S'T'	
R	(-2, -3)	R'	(-6, -9)
S	(0, 2)	S'	(0, 6)
T	(2, -3)	T'	(6, -9)

$$k = 3$$