Key

   Essential Question: How do you use the discriminant to determine the number and type of solutions?

Previously we've used factoring to solve quadratics of the form $ax^2 + bx + c = 0$. Solving

quadratic equations we found different types of solutions. Is there a method to tell what kinds of solutions we will have with certain quadratic equations.   The discriminant: $b^2 - 4ac$

The discriminant determines the number and type of solutions without solving.

If the discriminant is positive the graph has two real solutions	If the discriminant is equal to zero the quadratic has one (repeated) real solution	If the discriminant is negative the quadratic has two complex (imaginary) solutions
$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
$y=0$	$y=0$	$y=0$
2 real solutions	1 real solution	2 complex/imaginary (no real solutions)
2 x-intercepts	1 x-intercept	No x-intercepts
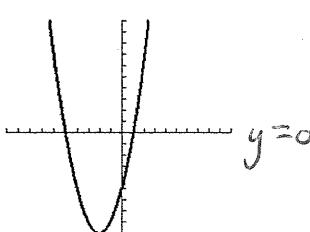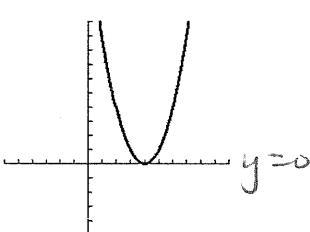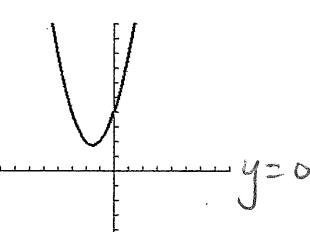
Examples: Find the discriminant for each quadratic, list the number and type of solutions for each. Then factor the quadratic.

1) $x^2 + 6x + 8 = 0$

$a = 1$
$b = 6$
$c = 8$

$6^2 - 4(1)(8)$
$36 - 32$
$= 4, \ 2 \ Real$

2) $x^2 - 10x = -25$

$x^2 - 10x + 25 = 0$

$a = 1$
$b = -10$ $(10)^2 - 4(1)(25)$
$c = 25$ $\boxed{D = 0, \ 1 \ Real}$

3) $5x^2 - 8x = -5$

$5x^2 - 8x + 5 = 0$

$(-8)^2 - 4(5)(5)$ (No Real)
$D = -36 < 0, \ 2 \ Imaginary \ Solutions$

4) $2x^2 = 6x - 8$

$2x^2 - 6x + 8 = 0$

$(-6)^2 - 4(2)(8)$
$36 - 64 = -28$
$D < 0, \ 2 \ Imaginary \ (No \ Reals)$

5) $x^2 + 6x = -2x^2 - 3$

$x^2 + 2x^2 + 6x + 3 = 0$
$3x^2 + 6x + 3 = 0$
$D = 36 - 4(3)(3)$
$D = 36 - 36 = 0$
$1 \ Real$

6) $5x + 7 = 10x^2 + 10x$

$10x^2 + 5x - 7 = 0$
$(5)^2 - 4(10)(-7) = 305$
$D > 0,$
$\boxed{2 \ Real \ Solutions}$

Geometry
Worksheet – Using the Discriminant

Find the discriminant for each quadratic, list the number and type of solutions for each.

1) $2x^2 + 3x + 1 = 0$ $D = 1$

$3^2 - 4(2)(1) > 0$

2 Reals

2) $x^2 + 2x + 5 = 0$ $D = -16$

$2^2 - 4(1)(5) < 0$

2 Imaginary
(No Reals)

3) $x^2 - 4x + 4 = 0$ $D = 0$

$(-4)^2 - 4(1)(4) = 0$

1 Real

4) $x^2 - 5x = 6$ $D = 49$

$x^2 - 5x - 6 = 0$

$(-5)^2 - 4(1)(-6)$

$25 + 24 = 49 > 0$

2 Reals

5) $-2x^2 + x = -4$ $D = 33$

$-2x^2 + x + 4 = 0$

$(1)^2 - 4(-2)(4) > 0$

2 Reals

6) $5x^2 + 7x = -6$

$5x^2 + 7x + 6 = 0$

$(7)^2 - 4(5)(6) = -271 < 0$

2 Imaginary
(No Real)

7) $2x^2 - 1 = 3x + 4$

$2x^2 - 3x - 1 - 4 = 0$

$2x^2 - 3x - 5 = 0$

$(-3)^2 - 4(2)(-5) = 49 > 0$

2 Reals

8) $-x^2 - 4x = 6$

$-x^2 - 4x - 6 = 0$

$(-4)^2 - 4(-1)(-6)$

$16 - 24 = -8 < 0$

2 Imaginary
(No Real)

9) $2x^2 + 4 = -x$

$2x^2 + x + 4 = 0$

$(1)^2 - 4(2)(4) = -31 < 0$

2 Imaginary
(No Real)

10) $x^2 - 4x = -3x + 4$

$x^2 - 4x + 3x - 4 = 0$

$x^2 - 1x - 4 = 0$

$(-1)^2 - 4(1)(-4) = 17 > 0$

2 Reals

11) $-8x^2 = 2x + 5$

$-8x^2 - 2x - 5 = 0$

$(-2)^2 - 4(-8)(-5)$

$4 - 160 = -156 < 0$

2 Imaginary
(No Reals)

12) $3x^2 + 12x = x^2 - 18$

$3x^2 - x^2 + 12x + 18 = 0$

$2x^2 + 12x + 18 = 0$

$(12)^2 - 4(2)(18) = 0$

$\boxed{1 \text{ Reals}}$