Geometry   Factoring Quadratics   Sept 2, 2015 (Wed)   *Key*

In the previous lesson, we factored polynomials by identifying the GCF among the terms of the polynomial. Sometimes a polynomial has other factors besides a GCF that multiply to make it. Therefore, we need to learn other methods for factoring.

Follow these steps to factor a quadratic polynomial: $ax^2 + bx + c$

1. **Factor out the GCF.** (There may or may not be a GCF, but Always look for it **FIRST!**)

2. Write the terms in standard form: $ax^2 + bx + c$

3. Find values that MULTIPLY to be **a*c** and ADDS to be **b**

4. List factor pairs for the product **a*c**

5. Fill in the spaces: _____ x _____ = a*c   and _____ + _____ = b

6. Write the the polynomial as four terms using the factors of the middle term:

   $(ax^2 + $___$ + $___$ + c)$ with the two factors from step 3 filling in the blanks.

7. Group the first two terms together and the last two terms together.

8. Factor out the common factor.

9. CHECK your factored form by distributing to see if this matches your standard form from #2 above

10. Solve:

$$a = 1 \qquad \underline{2} \times \underline{5} = 10 \qquad 1,10$$
$$b = 7 \qquad \underline{2} + \underline{5} = 7 \qquad 2,5$$
$$c = 10$$

Example 1 : Factor $x^2 + 7x + 10$ using the steps above.

$$1x^2 + 2x + 5x + 10$$

$$x(x+2) + 5(x+2)$$

$$\boxed{(x+2)(x+5)}$$

$$a = 1 \qquad \underline{-2} \times \underline{10} = -20 \qquad 1,20$$
$$b = 8 \qquad \underline{-2} + \underline{10} = 8 \qquad 2,10$$
$$c = -20 \qquad\qquad\qquad\qquad 4,5$$

Example 2: Factor $x^2 + 8x - 20$ using the steps above.

$$x^2 - 2x + 10x - 20$$

$$x(x-2) + 10(x-2)$$

$$\boxed{(x-2)(x+10)}$$

1. **Factor out the GCF.** (There may or may not be a GCF, but Always look for it **FIRST!**)
2. Write the terms in standard form: $ax^2 + bx + c$
3. Find values that MULTIPLY to be **a*c** and ADDS to be **b**
4. List factor pairs for the product **a*c**
5. Fill in the spaces: _____ x _____ = a*c   and _____ + _____ = b
6. Write the the polynomial as four terms using the factors of the middle term:

   $(ax^2 + \underline{\quad} + \underline{\quad} + c)$ with the two factors from step 3 filling in the blanks.
7. Group the first two terms together and the last two terms together.
8. Factor out the common factor.   9. Distribute to check your factored form
9. Solve.

$a = 2 \qquad 1, 20$
$b = -1 \qquad 2, 10$
$c = -10 \qquad 4, 5$

3. Factor $x^2 - 36$

$a = 1$
$b = 0$
$c = -36$

$\underline{6} \times \underline{-6} = -36$

$\underline{6} + \underline{-6} = 0$

$x^2 + 6x - 6x - 36$

$x(x+6) - 6(x+6)$

$\boxed{(x+6)(x-6)}$

4. Factor $2x^2 - x - 10$

$\underline{4} \times \underline{-5} = -20$

$\underline{4} + \underline{-5} = -1$

$2x^2 + 4x - 5x - 10$

$2x(x+2) - 5(x+2)$

$\boxed{(x+2)(2x-5)}$

5. Factor $6x^4 - 16x^3 + 10x^2$

$2x^2(3x^2 - 8x + 5)$

$a = 3 \qquad \underline{-3} \times \underline{-5} = 15$

$b = -8 \qquad \underline{-3} + \underline{-5} = -8$

$c = 5$

$1, 15$
$3, 5$

$3x^2 - 3x - 5x + 5$

$3x(x-1) - 5(x-1)$

$\boxed{2x^2(x-1)(3x-5)}$

6. Factor $3x^5 - 300x^3$

$3x^3(x^2 - 100)$

$a = 1$
$b = 0$
$c = -100$

$\underline{10} \times \underline{-10} = -100$

$\underline{10} + \underline{-10} = 0$

$x^2 + 10x - 10x - 100$

$x(x+10) - 10(x+10)$

$\boxed{3x^3(x+10)(x-10)}$

Examples: Factor each polynomial

a. $11x^2 - 12x + 1$

$a = 11$
$b = -12$
$c = 1$

$\dfrac{-11}{} \times \dfrac{-1}{} = 11$

$\dfrac{-11}{} + \dfrac{-1}{} = -12$

$11x^2 - 11x - 1x + 1$

$11x(x-1) - 1(x-1)$

$(x-1)(11x-1)$

b. $5x^2 - 2x - 7$

$-7 \times +5 = -35$
$-7 + +5 = -2$

$5x^2 - 7x + 5x - 7$

$x(5x-7) + 1(5x-7)$

$\boxed{(5x-7)(x+1)}$

GCF: 6

c) $6x^2 - 48x - 120$

$6(x^2 - 8x - 20)$

$-10 \times 2 = -20$
$-10 + 2 = -8$

$6(x^2 - 10x + 2x - 20)$

$x(x-10) + 2(x-10)$

$\boxed{6(x-10)(x+2)}$

GCF: 2

d) $4x^2 - 2x - 56$

$2(2x^2 - x - 28)$

$8 \times 7 = -56$
$-8 + 7 = -1$

$2x^2 - 8x + 7x - 28$

$2x(x-4) + 7(x-4)$

$\boxed{2(2x+7)(x-4)}$

GCF: x

5 16
2 40
4 20
10, 8

e) $4x^3 - 15x^2 - 20x$

$\boxed{x(4x^2 - 15x - 20)}$

$-x- = -80$
$-+- = -15$

GCF: x

f) $14x^3 - 2x + 3x^2$

$14x^3 + 3x^2 - 2x$

$x(14x^2 + 3x - 2)$

$+7 \times -4 = -28$
$+7 + -4 = 3$

$14x^2 + 7x - 4x - 2$

$7x(2x+1) - 2(2x+1)$

$\boxed{x(2x+1)(7x-2)}$