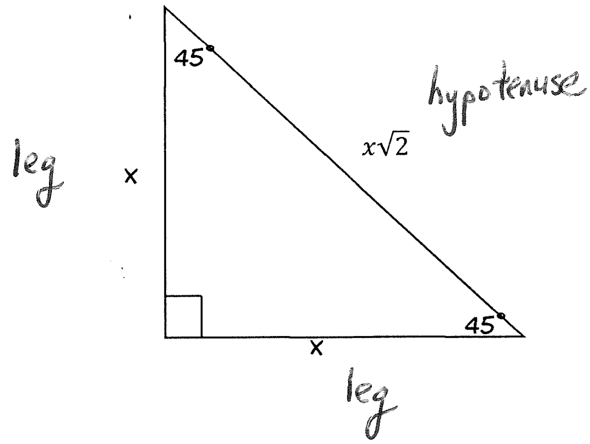
Geometry Fall 2015 Concept Summary and Formula Review

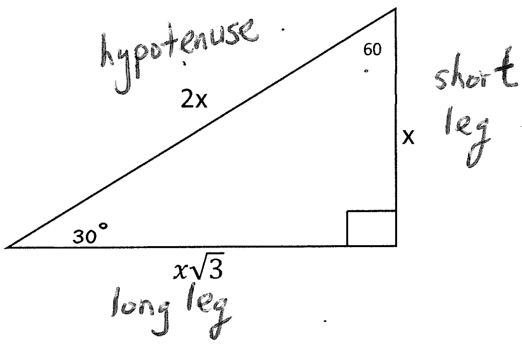
Unit 1: Right Triangle Trig



45-45-90 Triangles

1. To convert leg → hypotenuse, **multiply** leg by $\sqrt{2}$
2. Both legs are congruent to each other
3. To convert hypotenuse → leg, **divide** leg by $\sqrt{2}$

leg = leg $\underset{\div\sqrt{2}}{\overset{\times\sqrt{2}}{\rightleftarrows}}$ hypotenuse



36-60-90 Triangles

1) To convert short leg → hypotenuse, **multiply** short leg by 2
2) To convert hypotenuse → short leg, **divide** hypotenuse by 2

3) To convert short leg → long leg, **multiply** short leg by $\sqrt{3}$
4) To convert long leg → short leg, **divide** long leg by $\sqrt{3}$

long $\underset{\times\sqrt{3}}{\overset{\div\sqrt{3}}{\rightleftarrows}}$ short $\underset{\div 2}{\overset{\times 2}{\rightleftarrows}}$ hypotenuse

Recall the below Trig ratios: SOH – CAH - TOA

$$Sin\ \angle A = \frac{Opp}{Hyp} \qquad Cos\ \angle A = \frac{Adj}{Hyp} \qquad Tan\ \angle A = \frac{Opp}{Adj}$$

Steps for Solving Right Triangle:

A. Find Missing Side

1. Set up trig ratios and use cross product to solve for the variable
2. Use Pythagorean theorem ($a^2 + b^2 = c^2$) if only 1 side is missing

B. Find Missing Angle

1. Set up trig ratio to use inverse trig : Example → cos A = $\frac{12}{13}$ means A = $cos^{-1}\left(\frac{12}{13}\right)$
2. Subtract the angles from 180° if only 1 angle is missing.

Unit 3: Similarity/Transformations/Parallel Lines

Transformation Types: **Translation**(slide), **Reflection**(flip), **Rotation**(turn), **Dilation**(stretch/compress)
Ratio: comparison of 2 quantities using division
Scale Factor: ratio or corresponding sides: new/original
Similar Polygons/Triangles : corresponding angles are congruent and corresponding side lengths are proportional

Triangle Similarity Theorems: 1) **SSS** (If corresponding side lengths of 2 triangles are proportional, then triangles are similar) 2) **SAS** (If 2 corresponding lengths are proportional between 2 triangles and the included angles are congruent, then the triangles are similar:

Transformation Rules:
Line Reflections
$r_{x\text{-axis}}$ (x, y) = (x, –y)
$r_{y\text{-axis}}$ (x, y) = (–x, y)
$r_{y=x}$ (x, y) = (y, x)
$r_{y=-x}$ (x, y) = (–y, –x)
Dilation:
D_k (x, y)= (kx, ky)

Rotations:
(counterclockwise→ 90, 180, 270°)
$R_{90°}$ (x, y) = (–y, x)
$R_{180°}$ (x, y) = (–x, –y)
$R_{270°}$ (x, y) = (y, –x)
(clockwise →– 90°)
$R_{-90°}$ (x, y) = (y, –x)
Translation:
$T_{a,b}$ (x, y) = (x + a, y + b)

Unit 2A: Factoring Unit

Monomial: number,variable,or product of a number and one or more variables with whole number exponents: (ex. $-6a^3b^2$)

Binomial: expression of the sum or the difference of two terms

Polynomial: is an expression consisting of variables and coefficients, that involves the operations of non-negative integer exponents.

Factoring Steps

1) Arrange terms in standard form : $\mathbf{ax^2 + bx + c = 0}$
2) Factor out GCF (greatest common factor)
   ** If equation is missing a "b" value, then add in 0x
3) Find a pair of values that multiply to be $a \times c$ and adds to be **b**
   ___ x ___ = a * c
   ___ + ___ = b
4) Replace "b" term with values from above
5) Pair terms and factor out GCF for each pair
6) Put expression in factored form
7) Solve for each x.

Steps for **Completing the square:**

1) Arrange equation in standard form: $\mathbf{ax^2 + bx + c = 0}$
2) "a" value MUST be equal to 1, so divide each term by the GCF to make a = 1
3) Move the constant to the other side of the equation.
4) Add spaces "+___" to the equation: $\mathbf{x^2 + bx + \_\_\_ = c + \_\_}$
5) Find $\left(\frac{b}{2}\right)^2$ and enter this value into the blank spaces ___ on both sides of the equation
6) Rewrite left side in factored form and add the numbers on the right side
7) take the square root ($\sqrt{\ }$) of both sides (don't forget $\pm$)
8) solve for x

* A quadratic equation is an equation of degree 2, **meaning** that the highest exponent of this function is 2.

* The quadratic formula is used to solve an equation of the form $ax^2 + bx + c = 0$

*This formula can solve any equation that can be solved by factoring and completing the square

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ given $ax^2 + bx + c = 0$

The **Discriminant** is the number (from the expression) inside the square root of the quadratic formula.

Since the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,

the discriminant is the $\mathbf{b^2 - 4ac}$

The discriminant describes the **nature** , or the type, of solutions (or roots)

If the Discriminant is **positive (D>0)**, there are 2 real answers (2 real roots or **solutions**)

If the Discriminant is **negative(D < 0)**, there are 2 imaginary answers (2 imaginary roots)

If the Discriminant is **zero(D = 0)**, there is 1 real answer. (2 real answers with same value,1 real root)

Unit 2B: Graphing Quadratic Functions

1. **Standard form:** $\mathbf{y = ax^2 + bx + c}$

a. The x-coordinate of the vertex is located at $x = \frac{-b}{2a}$

b. The axis of symmetry (AOS) is the vertical line $x = \frac{-b}{2a}$

c. Make a T-table, put the vertex in the middle of the t-table.

d. Fill in the rest of the values (use calculator)

e. Graph the points with a smooth, U-shaped curve

f. **Domain** : always all Real Numbers $(-\infty, \infty)$

g. **Range**: lowest y-value to the highest y-value. *Remember to use bracket on vertex

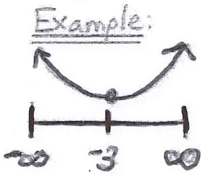
h. Determine **Increasing/Decreasing** Intervals

*Determine the x-value of the vertex

*Create number line with endpoints as $-\infty$ *and* $+\infty$

*Sketch the parabola above number line * determine interval where graph is rising (increasing) and interval where graph is falling (decreasing)

Example:



Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

i. Determine **Positive/Negative** intervals

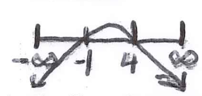
*Determines x-values of the x-intercepts

*Create number line with endpoints as $-\infty$ *and* $+\infty$ and x-values of intercepts

*Sketch the shape of parabola through th x-intercepts

* determine interval where graph is above the number line (positive) and interval where graph is below the number line(negative)

Example:



pos: $(-1, 4)$

neg: $(-\infty, -1) \cup (-1, 4)$

j. Finding **Average Rate of Change**

* Find the 2 ordered pairs using the x-values given in the interval

* Find the slope between the 2 ordered pairs: m $m = \frac{y_2 - y_1}{x_2 - x_1}$

2. **Intercept Form**: $\mathbf{y = a(x - p)(x - q)}$

a) If "a" is positive (> 0) the parabola opens up
   If "a" is negative (< 0) the parabola opens down.
b) The x-intercepts are the points x = p and x = q. Set factors equal to 0 and solve to get p and q.
c) The **x-coordinate of the vertex** is half way between the x-intercepts **(p+q)/2**
d) Make a T-table, put the vertex in the middle of the t-table.
e) Fill in the rest of the values (use calculator)

3. **Vertex Form:** $\mathbf{y = a(x - h)^2 + k}$

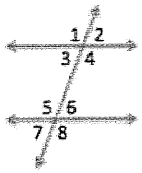
a) If "a" is positive (> 0) the parabola opens up If "a" is negative (< 0) the parabola opens down.
b) The **vertex** is the point (h, k)
c) Make a T-table, put the vertex in the middle of the t-table.
d) Fill in the rest of the values (use calculator)

Quadratic Function Transformations in Vertex Form $y = a(x - h)^2 + k$

1. If **a** is negative, there is a vertical reflection and the parabola will open downwards.
2. $|a|$ is the vertical stretch factor.
   - If $|a| > 1$, vertical stretch
   - $|a| < 1$, vertical compress
3. h is the horizontal translation (shift)
   " **–h**" means shift **right** h units
   " **+h**" means shift **left** h units
4. k is the vertical translation (shift)
   " **–k**" means shift **down** k units
5. " **+k**" means shift **up** k units

Unit 3(continued): Similarity/Transformations/Parallel Lines

Parallel Lines and Transversal ***Helpful Reminder: All acute angles are congruent with each other; All obtuse angles are congruent with each other; Acute + Obtuse = 180°***



Corresponding Angles are Congruent:

$\angle 1 \cong \angle 5$ $\angle 2 \cong \angle 6$
$\angle 3 \cong \angle 7$ $\angle 4 \cong \angle 8$

Alt. External Angles are Congruent:

$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$

Linear pairs are supplementary

$\angle 1 + \angle 3 = 180^\circ$ $\angle 2 + \angle 4 = 180^\circ$
$\angle 5 + \angle 6 = 180^\circ$ $\angle 6 + \angle 8 = 180^\circ$
$\angle 7 + \angle 8 = 180^\circ$ $\angle 4 + \angle 3 = 180^\circ$

Consecutive Angles are supplementary:

$\angle 3 + \angle 5 = 180^\circ$ $\angle 4 + \angle 6 = 180^\circ$

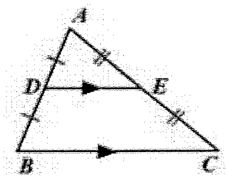
Vertical Angles are Congruent:

$\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$

Alt. Interior Angles are Congruent

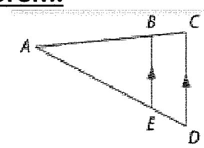
$\angle 3 \cong \angle 6$ $\angle 4 \cong \angle 5$

Midsegment of triangle: joins the midpoints of two sides of a triangle such that its length is **half the length of the third side** of the triangle.
2*(Midsegment) = length of parallel base
Example: 2(DE) = BC



Triangle Proportionality Theorem:

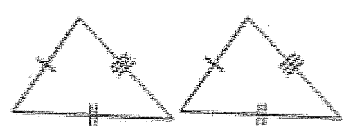
If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$



Unit 4 Triangle Congruence: **Triangle Congruence Postulates/Theorems** : SSS, SAS, HL, ASA, AAS

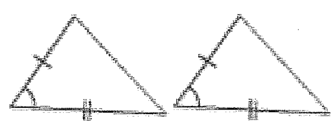
NOT triangle theorems/postulates: 1) SSA (NOT a triangle congruence theorem, only if Right Triangle) 2) AAA

SSS



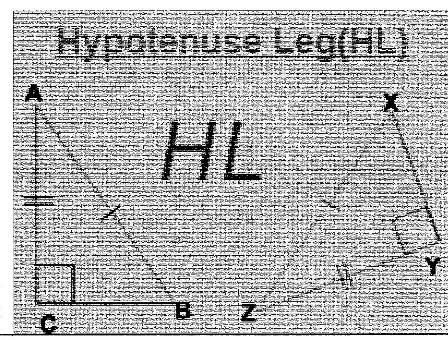
Three pairs of corresponding sides are congruent.

SAS

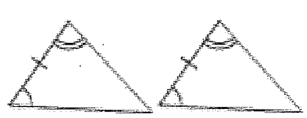


Two pairs of corresponding sides and their included angles are congruent.

Side-Side-Angle(SSA)for Right Triangles Only!

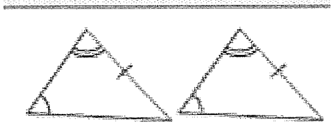


ASA



Two pairs of corresponding angles and their included sides are congruent.

AAS



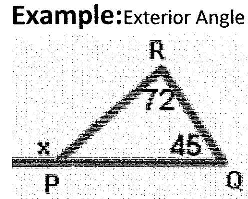
Two pairs of corresponding angles and the corresponding nonincluded sides are congruent

Steps for Proofs:
1) Write the given 2) Mark the figure
3) Use definitions from the given
4) Check for symbols (perpendicular / parallel)
5) List the obvious (vertical angles/reflexive)
6) Definition of Angle bisector
7) Definition of Midpoint
8) Parallel Lines: Alt. Interior Angles ≅
9) Triangles congruent (SSS, SAS, HL, ASA, AAS)
10) CPCTC

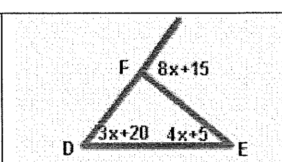
Classify Triangles: By angles: **acute**(3 acute angles), **equiangular** (3 congruent acute angles) , **obtuse**(1 obtuse angle) , **right** (1 right angle) By sides: **equilateral**(3 congruent sides), **isosceles**(at least 2 congruent sides), **scalene** (no congruent sides)

The **Exterior Angle Theorem:** states that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

Example: Exterior Angle Thm



$x = 45 + 72$
$x = 117$



$8x + 15 = (4x + 5) + (3x + 20)$
$8x + 15 = 7x + 25$
$8x = 7x + 10$
$x = 10$