

key

When you are asked to "factor" a number or expression, you want to break down the original number or expression into the numbers or expressions that are multiplied together to produce the original number or expression. For instance, to factor the number 100:

So, the factored form of 100 is:  
 $2 \cdot 2 \cdot 5 \cdot 5$

When factoring polynomials, you do the same thing; break the polynomial into expressions that multiply to produce the original polynomial. Each expression is called a factor.

Anytime you are factoring a polynomial, you must first look to see if the terms have any common factors. In fact, you want to look for the greatest common factor (GCF). This is the greatest term that is a factor of each term in the polynomial. GCFs can be a number, a variable or a combination of both numbers and variables. Sometimes, the GCF can even be another polynomial. Think of removing the GCF as **reverse distributive property**

**Model #1:** Determine the greatest common factor for the expression  $3x^2 + 9x$ .

1. Begin with your coefficients, 3 and 9. The GCF for these two numbers is 3 (3 is a factor of 3 and 3 is a factor of 9).
2. Next, look at your variables. The term  $3x^2$  has 2 x-variables multiplied and the term  $9x$  has 1 x-variable.. So, they both at least have 1 x.
3. The GCF of  $3x^2 + 9x$  is  $3x$ .

**Example 1:** Determine the greatest common factor (GCF) for the expression.

a.  $5p^4 - 10p^2 + 25$

GCF: 5

b.  $m^3s^4u^2 + ms^2u^3 - m^2su$

GCF:  $msu$

c.  $4ab - 2a^2b - 6ab^2$

GCF:  $2ab$

d.  $100z^3 - 80z^4 + 120z^2$

GCF:  $20z^2$

When factoring polynomials, first look to see if there is a GCF. If so, rewrite the polynomial as the 2 factors by writing the GCF outside the parentheses and leaving the remaining factor inside the parentheses. To find out what goes inside the parentheses, divide each term by the GCF. This is called "**factoring out the GCF**".

**Model #2:** Factor example "a" from the front:  $5p^4 - 10p^2 + 25$ .

1. We found that the GCF was 5, so this goes outside the parentheses.
2. Divide each term in the original polynomial by 5:

$$5(p^4 - 2p^2 + 5)$$

Every factored polynomial can be checked by multiplying the factors. The product should be the original polynomial.

**Example 2:** Factor the expression by factoring out the GCF.

e.  $m^3s^4u^2 + ms^2u^3 - m^2su$  (refer to example b.)

$$msu(m^2s^3u + su^2 - m)$$

f.  $4ab - 2a^2b - 6ab^2$  (refer to example c.)

$$2ab(2 - a - 3b)$$

g.  $100z^3 - 80z^4 + 120z^2$  (refer to example d.)

$$20z^2(5z - 4z^2 + 6)$$

h.  $7f^5 - 2f^3$

$$f^3(7f^2 - 2)$$

i.  $6m^2n + 15m^2n^3$

$$3m^2n(2 + 5n^2)$$

j.  $36qr^3 + 9q - 27r$

$$9(4qr^3 + q - 3r)$$

k.  $7g^4h^6 + 9g^3h^3 - 3g^4h^2$

$$g^3h^2(7gh^4 + h - 3g)$$

l.  $3(x-1) + y(x-1)$

$$(x-1)[3+y]$$

m.  $x\sqrt{3} + 2\sqrt{3}$

$$\sqrt{3}(x+2)$$

## GCF Factoring

Factor the common factor out of each expression.

1)  $2ab^5 + 4$

$2(ab^5 + 2)$

2)  $15a^5b - 5a^3b^2$

$5a^3b(3a^2 - b)$

3)  $10yx^2 + 5y^2$

$5y(2x^2 + y)$

4)  $6b^5 + 7b^7a^3$

$b^5(6 + 7a^3b^2)$

5)  $8v + 2u$

$2(4v + u)$

6)  $18x^2y^2 + 21$

$3(6x^2y^2 + 7)$

7)  $56v + 48vu^4$

$8v(7 + 6u^4)$

8)  $56y^2 + 16x$

$8(7y^2 + 2x)$

9)  $15y^4x - 3y^3$

$3y^3(5xy - 1)$

10)  $-12x^4y + 6x^3$

$6x^3(-2xy + 1)$

11)  $-5yx + 35y$

$5y(-x + 7)$

12)  $-10x^3y^8 + 5x^3y^6$

$5x^3y^6(-2y^2 + 1)$

13)  $3ab^3 - 12a^2b$

$3ab(b^2 - 4a)$

14)  $6b^2a - 6b$

$6b(ab - 1)$

15)  $32u^4v^4 + 56u^5v^3$

$8u^4v^3(4v + 7u)$

