

Key

Compare Numerator and Denominator to help determine Integral Rule(s) to apply	Example #1	Example #2
<p>1) Only 1 Term in the Denominator (regardless of degree differences between numerator and denominator)</p> <p><u>Solution:</u> Consider expanding and splitting up the terms into individual fractions and applying integral rule for each term separately.</p>	$\int \frac{x^4 - 5x^3 + 1}{2x^4} dx$ $\int \frac{x^4}{2x^4} - \frac{5x^3}{2x^4} + \int \frac{1}{2x^4} dx$ $\frac{1}{2} - \frac{5}{2}\left(\frac{1}{x}\right) + \frac{1}{2}x^{-4} dx$ $\frac{1}{2}x - \frac{5}{2}\ln x + \frac{1}{2}\left(\frac{x^{-3}}{-3}\right) + C$	$\int \frac{4e^{4x} - e^{2x}}{6e^{3x}} dx$ $\int \frac{4e^{4x}}{6e^{3x}} - \frac{e^{2x}}{6e^{3x}} dx \rightarrow \int \frac{2}{3}e^x - \frac{1}{6}e^{-x} dx$ $\frac{2}{3}e^x + \frac{1}{6}e^{-x} + C$
<p>2) Multiple terms in the denominator and the Denominator has variable exponent degree that is 1 higher than the Numerator</p> <p><u>Solution:</u> Consider U-Substitution</p>	$\int \frac{5x}{7x^2 - 4} dx$ $u = 7x^2 - 4 \quad \int \frac{5x}{u} \cdot \frac{du}{14x}$ $\frac{du}{dx} = 14x \quad \frac{5}{14} \int \frac{1}{u} du$ $14x dx = du \quad \frac{5}{14} \ln 7x^2 - 4 + C$ $dx = \frac{du}{14x}$	$\int \frac{2x^2}{\sqrt[5]{3x^3 - 4}} dx \rightarrow \int \frac{2x^2}{(3x^3 - 4)^{1/5}} dx$ $u = 3x^3 - 4 \quad \int \frac{2x^2}{u^{1/5}} \cdot \frac{du}{9x^2}$ $\frac{du}{dx} = 9x^2 \quad \frac{2}{9} \left(\frac{u^{4/5}}{4/5} \right) + C$ $dx = \frac{du}{9x^2} \quad \frac{2}{9} \int u^{-1/5} du$ $\frac{10}{36} u^{4/5} + C$ $\frac{5}{18} (3x^3 - 4)^{4/5} + C$
<p>3) Multiple terms in the denominator and the Numerator has variable exponent that is Same degree OR Higher than the Denominator.</p> <p><u>Solution:</u> Consider Long Division and/or Synthetic Division</p>	$\int \frac{4x - 3}{x - 5} dx$ <p>Apply long division or synthetic division</p> $\begin{array}{r} x^2 - 2 + \frac{x}{x^2 + 2} \\ x^2 + 2 \overline{)x^4 + x - 4} \\ \underline{-x^4 - 2x^2} \\ \hline -2x^2 + x - 4 \\ \underline{+2x^2} \\ \hline x \end{array}$ $x \leftarrow \text{Remainder}$	$\int \frac{x^4 + x - 4}{x^2 + 2} dx$ <p>u-sub: $u = x^2 + 2$</p> <p>Apply long division (synthetic division does not apply)</p> $\begin{array}{r} x^2 - 2 + \frac{x}{x^2 + 2} \\ x^2 + 2 \overline{)x^4 + x - 4} \\ \underline{-x^4 - 2x^2} \\ \hline -2x^2 + x - 4 \\ \underline{+2x^2} \\ \hline x \end{array}$ $\frac{x^3}{3} - 2x + \frac{1}{2}\ln x^2 + 2 + C$ $\frac{1}{2} \int \frac{1}{u} \cdot \frac{du}{2x}$
<p>4) Multiple terms in the denominator and the Denominator has variable exponent that is higher than the Numerator by 2 or more degrees:</p> <p><u>Solution:</u> Consider ArcTrig Integral Rules</p>	$\int \frac{1}{x^2 - 8x + 4} dx$ <p>Apply Arctan Integral Rule</p> $\int \frac{dx}{(x-4)^2 + (\sqrt{20})^2} \quad u = x-4$ $a = \sqrt{20}$ $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$ $\frac{1}{\sqrt{20}} \arctan\left(\frac{x-4}{\sqrt{20}}\right) + C$	$\int \frac{5x}{\sqrt{1-x^4}} dx$ <p>Apply Arcsin Integral Rule</p> $\int \frac{5x}{\sqrt{(1)^2 - (x^2)^2}} dx \quad dx = \frac{du}{2x}$ $\int \frac{5x}{\sqrt{1-x^2}} \cdot \frac{du}{2x}$ $\frac{5}{2} \int \frac{du}{\sqrt{1-u^2}}$