CCGPS Analytic Geometry Notes: Graphing Quadratics in Intercept Form

key Sept 28 (Mon)

Essential Question: How do you identify characteristics of a function on a table or graph?

There are three different forms in which quadratics can be presented: yesterday, we explored the **standard form** of the quadratic function: $y = ax^2 + bx + c$ Using this form, it was not too hard to graph the parabola after identifying the vertex and using a table to find points. But, the X-intercepts were not always easy to name. Today we are going to look at another form of the quadratic function. Identifying the x-intercepts is easier with the form called the .....

***Intercept Form:** $y = a(x-p)(x-q)$

- If "a" is positive, the parabola opens up. If "a" is negative, the parabola opens down.
- The x-intercepts are the points $x=p$ and $x=q$. Set factors equal to 0 and solve to get p and q.
- The x-coordinate of the vertex is $\frac{p+q}{2}$; it is half way between the x-intercepts, so find the AVERAGE of the x-intercepts. To find the y-coordinate of the vertex, substitute this value in for x in the function and solve for y.
- The axis of symmetry is the vertical line $\boxed{x = \frac{p+q}{2}}$; the AOS passes through the vertex.
- To graph, plot the x-intercepts, vertex, and axis of symmetry. Then connect with a smooth curve. You may want to use substitute in another value for x to get 4th and 5th point (using symmetry) for the graph.

Example 1: Find the x-intercepts of the quadratic and the x-value of vertex. Hint: you may need to factor!

a) $y = 2(x-3)(x+5)$

$x=3, x=-5$

x-intercepts: $x=3, x=-5$

$(3,0)\ (-5,0)$

$x = \frac{3-5}{2} = \frac{-2}{2} = -1$

Vertex: $(-1, \_)$

b) $y = x^2 + 5x + 6$

$(x+2)(x+3)$

$x=-2, x=-3$

x-intercepts: $x=-2$

$(-2,0)\ (-3,0)$

$\frac{-2+-3}{2} = \frac{-5}{2}$

Vertex: $(-5/2, \_)$

c) $y = 3x^2 - 12x - 15$

$3(x^2-4x-5)$

$3(x-5)(x+1)$

$x=5, x=-1$

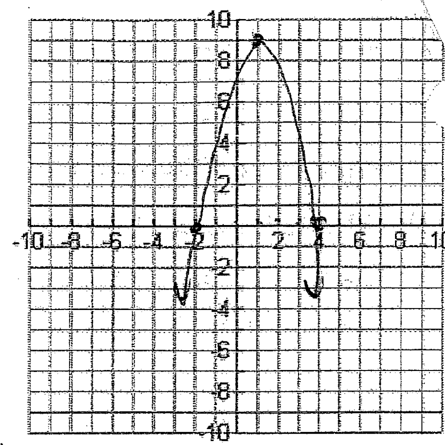
x-intercepts: $(5,0)\ (-1,0)$

$x = \frac{5-1}{2} = \frac{4}{2} = 2$

Vertex: $(2, \_)$

Intercept form: a(x – p)(x – q)   vertex x-value: $\frac{p+q}{2}$

Example 2: Graph y = -(x + 2) (x – 4) Opens: down p: -2 q: 4



$x = \frac{4-2}{2} = \frac{2}{2} = 1$

Vertex: 1,   a = -1 Max / Min (Circle one)

AOS: x=1 x – intercept(s): (-2,0) (4,0) y – intercept: (0,8)

Domain: (-∞,∞) Range: (-∞,9]

Avg. Rate of Change [-2, 1]: 3

(-2,0) (1,9) $\frac{9-0}{1+2} = \frac{9}{3} = 3$

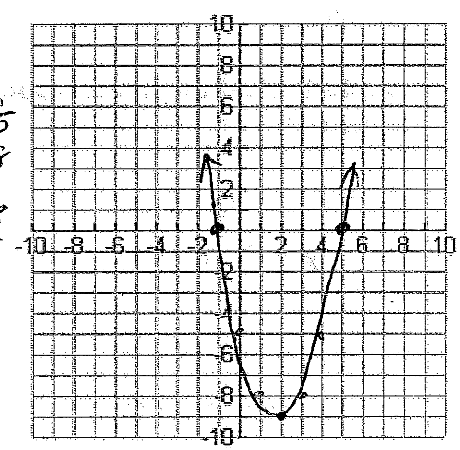
x	y
-1	
0	
1	9
2	
3	

End Behavior:
As x→ ∞, $f(x) \to$ -∞ Increasing: (-∞,1) Positive: (-2,4)

As x→ -∞, $f(x) \to$ -∞ Decreasing: (1,∞) Negative: (-∞,-2)U(4,∞)

Example 3: Graph y = (x – 5) (x + 1) Opens: up p: 5 q: -1

x	y
-1	0
0	-5
1	-8
2	-9
3	-8
4	-5
5	0



$\frac{5-1}{2} = \frac{4}{2} = 2$

Vertex: (2,-9) a = -1 Max / Min (Circle one)

AOS: x= 2 x – intercept(s): (5,0) (-1,0) y – intercept: (0,-5)

Domain: (-∞,∞) Range: [-9, ∞)

Avg. Rate of Change [3, 5]: 4

(3,-8) $m = \frac{-8-0}{3-5} = \frac{-8}{-2}$

(5,0)

End Behavior:
As x→ ∞, $f(x) \to$ +∞ Increasing: (2,∞) Positive: (-∞,-1)U(5,∞)

As x→ -∞, $f(x) \to$ +∞ Decreasing: (-∞,2) Negative: (-1,5)

Homework: Graphing Quadratics in INTERCEPT Form Sept 28 (Mon)

Intercept form: $a(x-p)(x-q)$ vertex x-value: $\frac{p+q}{2}$

1. Graph $y = -2x(x-4)$ Opens: down p: 0 q: 4

$-2(x-0)(x-4)$

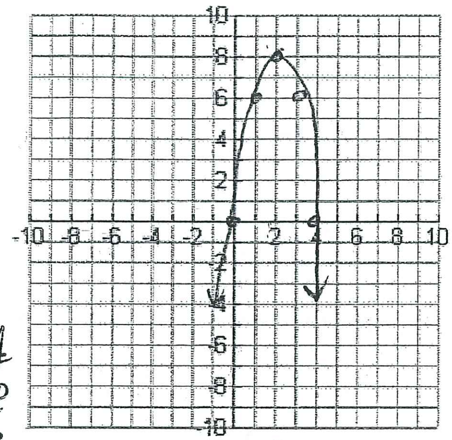
Vertex: (2,8) a = -2 Max / Min (Circle one)

AOS: $x=2$ x – intercept(s): (0,0)(4,0) y – intercept: (0,0)

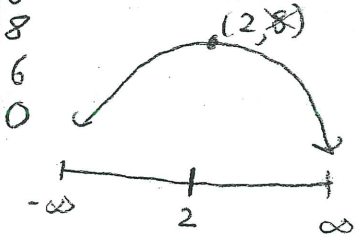
Domain: $(-\infty,\infty)$ Range: $(-\infty, 8]$

Avg. Rate of Change [0, 2]: 4

(0, 0)
(2, 8) $m = \frac{8-0}{2-0} = \frac{8}{2} = 4$

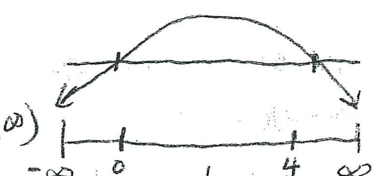


x	y
0	0
1	6
2	8
3	6
4	0



End Behavior:
As $x \to \infty, f(x) \to$ $-\infty$ Increasing: (x-value) $(-\infty, 2)$ Positive: (0,4)

As $x \to -\infty, f(x) \to$ $-\infty$ Decreasing: $(2,\infty)$ Negative: $(-\infty,0)\cup(4,\infty)$



2. $y = 4(x+1)(x-1)$ Opens: up p: -1 q: 1

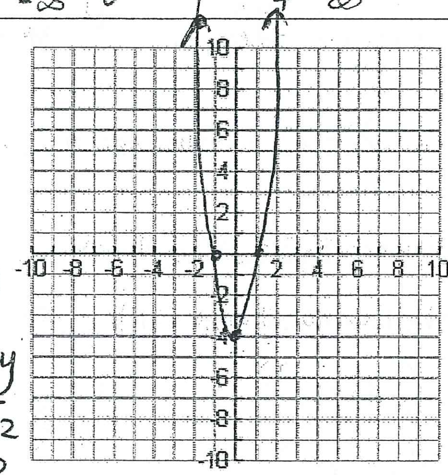
Vertex: (0, -4) a = 4 Max / Min (Circle one)

AOS: $x=0$ x – intercept(s): (-1,0)(1,0) y – intercept: (0,-4)

Domain: $(-\infty,\infty)$ Range: $[-4,\infty)$

Avg. Rate of Change [-1, 0]: -4

(-1, 0)
(0, -4) $\frac{-4-0}{0--1} = \frac{-4}{1}$

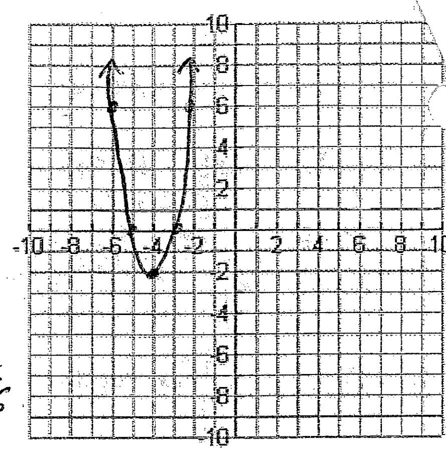


x	y
-2	12
-1	0
0	-4
1	0
2	12

End Behavior:
As $x \to \infty, f(x) \to$ $+\infty$ Increasing: $(0,\infty)$ Positive: $(-\infty,-1)\cup(1,\infty)$

As $x \to -\infty, f(x) \to$ $+\infty$ Decreasing: $(-\infty,0)$ Negative: (-1,1)

3. 2(x + 3) (x + 5) Opens: up p: -3 q: -5



Vertex: (-4, _) a = 2 Max / Min (Circle one)

AOS: x=-4 x – intercept(s): (-5,0)(-3,0) y – intercept: (0,30)

Domain: (-∞, ∞) Range: [-2, ∞)

x	y
-6	6
-5	0
-4	-2
-3	0
-2	6
0	30

Avg. Rate of Change [-3, -2]: 6

(-3, 0)
(-2, 6)

$\frac{6-0}{-2+3}$

End Behavior:
As $x \to \infty, f(x) \to$ +∞ Increasing: (-4, ∞) Positive: (-∞, -5) U (-3, ∞)

As $x \to -\infty, f(x) \to$ +∞ Decreasing: (-∞, 4) Negative: (-5, -3)

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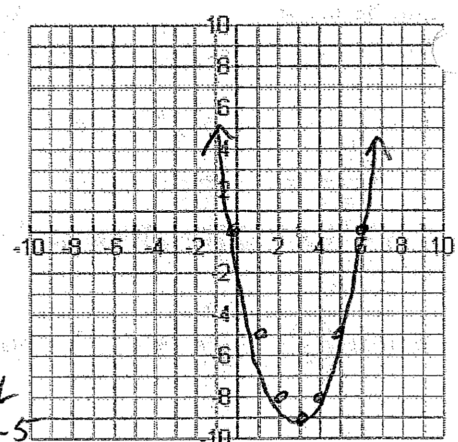
4. y = x(x – 6) Opens: up p: 0 q: 6

y=(x-0)(x-6)



Vertex: (3, -9) a = 1 Max / Min (Circle one)

AOS: x=3 x – intercept(s): (0,0)(6,0) y – intercept: (0,0)

Domain: (-∞, ∞) Range: [-9, ∞)

x	y
1	-5
2	-8
3	-9
4	-8
5	-5

Avg. Rate of Change [3, 6]: -3

(3, -9)
(6, 0)

$\frac{-9-0}{3-6} = \frac{-9}{3}$

End Behavior:
As $x \to \infty, f(x) \to$ +∞ Increasing: (3, ∞) Positive: (-∞, 0) U (6, ∞)

As $x \to -\infty, f(x) \to$ +∞ Decreasing: (-∞, 3) Negative: (0, 6)