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CCGPS Analytic Geometry
Notes: Compound Probability & Independent Events

In the previous lesson, we calculated the probability that one or the other of 2 events would happen. Today we will consider the probability of two events happening, one after the other. This calculation is done one of two ways, determined by whether or not one event occurring influences the chance that the other event will happen. Today, we will consider events that do not influence each other. These are called **independent events.**

Some examples of independent events:

- Landing on heads from a coin toss **AND** rolling a 1 on a die roll.
- Rolling a 3 on a die **AND** then rolling a 4 on a second roll of the die.
- Choosing a King from a deck of cards, replacing it, **AND** then choosing an Ace as the second card.

To find the probability of two independent events occurring in a sequence, find the probability of each event occurring separately, and then multiply the probabilities.

P (A and B) = _________ * _________

Example 1: A jar contains 5 red, 2 pink, and 3 white marbles. If a marble is drawn at random, returned to the jar, and then a second marble is drawn at random, find the indicated probabilities.

a. P (red and red)

b. P (white and pink)

Does the selection of the first marble influence the selection of the second marble? Explain.

Example 2: A card is selected from a standard deck of 52 cards, replaced, and then a second card is selected. Find the indicated probabilities.

a. P (King and Ace)

b. P (diamond and Queen)

Does the selection of the first card influence the selection of the second card? Explain.

This formula may be expanded for more than 2 events.

Example 3: The school surveyed the student body and determined that 8 out of 10 students like pizza. If three students are selected at random with replacement, what is the probability that they will all like pizza?

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Example 4: Four cards are chosen at random from a standard deck of 52 cards, with replacement. Find the indicated probabilities.

a. What is the probability of choosing 4 spades in a row?

b. What is the probability of choosing 4 Aces in a row?

Example 5: A jar contains 5 red marbles, 3 green marbles, 2 yellow marbles, and 1 blue marble. Find the indicated probabilities.

a. P (Drawing a blue marble)

b. P (Drawing a blue <u>or</u> red marble)

c. P (Drawing a marble that is not red or yellow)

d. P (Drawing a purple marble)

e. P (Drawing a blue marble, replacing it, and then a yellow marble)

f. P (Drawing two red marbles, with replacement of the first marble)

g. P (Drawing a yellow, then a blue, and finally a green marble, all with replacement)

Example 6: A nationwide survey showed that 75% of teenagers in the United States own an iPod. If 5 teenagers are chosen at random with replacement, what is the probability that all 5 own an iPod?

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GPS Analytic Geometry
otes: More on Probability and Independence
Homework: Steelers vs. Packers Worksheet

Essential Question: How can we show that two events are independent in a contingency table?

In review of simple probability, answer the following.

1 . A bag contains 16 marbles. Four are red, ten are blue, and 2 are green. What is the probability of randomly drawing a blue marble from the bag?

2. Each of the letters of the word "MATHEMATICS" are on separate cards face down on the table. If you pick a card at random, what is the probability that its letter will be "T"?

In review of Mutually Exclusive events versus Overlapping events, answer the following and use knowledge of vocabulary in each answer with an explanation.

3. Can flipping a heads and tails happen at the same time (with one coin)?

4. In a standard deck of 52 cards, can you draw a 5 and a heart at the same time?

In review of Independent events versus Dependent events, answer the following and use knowledge of vocabulary in each answer.

5. In a standard deck of 52 cards, is drawing a 5 and then drawing a heart without replacement of the first card independent events?

6. There are 15 different colored marbles in a bag. Assume we draw one marble, replace it, and then draw another. Are the events independent?

Often, we can determine if two events are independent by simply asking, "Does one probability affect the other?" or "Does the second probability depend on the first?" Other cases, however, are not as easy to see. When you cannot determine independence easily, you must show that

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$

Let's use this knowledge in the following contingency table.

Example 1: Complete the table to answer the following questions.

	Male	Female	Subtotal
Born in GA	7	5	
Not Born in GA	16	13	
Subtotal			

a. P (Female) =

b. P (Born in GA) =

c. P(Female | Born in GA) =

d. P(Born in GA | Female) =

e. Are the events Female and Born in GA independent?

Example 2: Use the definition of independent events to determine whether the events from rolling a die (6-sided) are independent or dependent.

1. The event odd and the event 3 or 6

a. P(odd) =

c. P(odd | 3 or 6) =

b. P(3 or 6) =

d. P(3 or 6 | odd) =

These events are ____________________ because

2. The event less than 3 and the event more than 5

3. The event even and the event prime number

Name: ______________________ Date: ______________ Period: ______ Key

CCGPS Analytic Geometry
Notes: Compound Probability & Independent Events

In the previous lesson, we calculated the probability that one or the other of 2 events would happen. Today we will consider the probability of two events happening, one after the other. This calculation is done one of two ways, determined by whether or not one event occurring influences the chance that the other event will happen. Today, we will consider events that do not influence each other. These are called **independent events.**

Some examples of independent events:

- Landing on heads from a coin toss **AND** rolling a 1 on a die roll.
- Rolling a 3 on a die **AND** then rolling a 4 on a second roll of the die.
- Choosing a King from a deck of cards, replacing it, **AND** then choosing an Ace as the second card.

To find the probability of two independent events occurring in a sequence, find the probability of each event occurring separately, and then multiply the probabilities.

P (A and B) = P(A) * P(B)

10 total

Example 1: A jar contains 5 red, 2 pink, and 3 white marbles. If a marble is drawn at random, returned to the jar, and then a second marble is drawn at random, find the indicated probabilities.

a. P (red and red)

$\frac{5}{10} \cdot \frac{5}{10} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = \boxed{0.25}$

b. P (white and pink)

$\frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100} = \boxed{0.06}$

Does the selection of the first marble influence the selection of the second marble? Explain.

No, independent events

Example 2: A card is selected from a standard deck of 52 cards, replaced, and then a second card is selected. Find the indicated probabilities.

a. P (King and Ace)

$\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169} = 0.006$

b. P (diamond and Queen)

$\frac{13}{52} \cdot \frac{4}{52} = \frac{1}{52} = \boxed{0.019}$

Does the selection of the first card influence the selection of the second card? Explain.

No, card was replaced before 2nd card is selected.

This formula may be expanded for more than 2 events.

$8/10 = 4/5$

Example 3: The school surveyed the student body and determined that 8 out of 10 students like pizza. If three students are selected at random with replacement, what is the probability that they will all like pizza?

$\frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \boxed{0.512}$

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Example 4: Four cards are chosen at random from a standard deck of 52 cards, with replacement. Find the indicated probabilities.

a. What is the probability of choosing 4 spades in a row?

$\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \left(\frac{1}{4}\right)^4 = \boxed{0.0039}$

b. What is the probability of choosing 4 Aces in a row?

$\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$ $\left(\frac{1}{13}\right)^4$ $\boxed{0.000035}$

Example 5: A jar contains 5 red marbles, 3 green marbles, 2 yellow marbles, and 1 blue marble. Find the indicated probabilities. 11 total

a. P (Drawing a blue marble) $= \frac{1}{11}$

b. P (Drawing a blue or red marble) $\frac{1}{11} + \frac{5}{11} = \frac{6}{11}$

c. P (Drawing a marble that is not red or yellow) $\frac{4}{11}$

d. P (Drawing a purple marble) $\frac{0}{11}$

e. P (Drawing a blue marble, replacing it, and then a yellow marble) $\frac{1}{11} \cdot \frac{2}{11} = \frac{2}{121} = \boxed{0.0165}$

f. P (Drawing two red marbles, with replacement of the first marble)

$\frac{5}{11} \cdot \frac{5}{11}$

g. P (Drawing a yellow, then a blue, and finally a green marble, all with replacement)

$\frac{2}{11} \cdot \frac{1}{11} \cdot \frac{3}{11} = \frac{6}{11^3} = 0.004$

$3/4$

Example 6: A nationwide survey showed that 75% of teenagers in the United States own an iPod. If 5 teenagers are chosen at random with replacement, what is the probability that all 5 own an iPod?

$\left(\frac{3}{4}\right)^5 = 0.2373$

Name ______________________ Date __________ Period __________

Integrated Advanced Algebra
Notes: More on Probability and Independence
Homework: Steelers vs. Packers Worksheet

Essential Question: How can we show that two events are independent in a contingency table?

In review of simple probability, answer the following.

1. A bag contains 16 marbles. Four are red, ten are blue, and 2 are green. What is the probability of randomly drawing a blue marble from the bag?

> 10/16 = 5/8 = .625 = 62.5%

2. Each of the letters of the word "MATHEMATICS" are on separate cards face down on the table. If you pick a card at random, what is the probability that its letter will be "T"?

> P(T) = 2/11 = .182 = 18.2%

In review of Mutually Exclusive events versus Overlapping events, answer the following and use knowledge of vocabulary in each answer.

3. Can flipping a heads and tails happen at the same time (with one coin)?

> No, the events are Mutually Exclusive.

4. In a standard deck of 52 cards, can you draw a 5 and a heart at the same time?

> Yes, the events are Overlapping.

In review of Independent events versus Dependent events, answer the following and use knowledge of vocabulary in each answer.

5. In a standard deck of 52 cards, is drawing a 5 and then drawing a heart without replacement of the first card independent events?

> No, drawing the 5 and keeping it affects the probability of drawing the heart.

6. There are 10 red an 5 blue marbles in a bag. Assume we draw a blue marble, replace it, and then draw another. Are the events independent?

> Yes, drawing the first marble and replacing it does not affect the probability of drawing the next one.

Often, we can determine if two events are independent by simply asking, "Does one probability affect the other?" or "Does the second probability depend on the first?" Other cases, however, are not as easy to see. When you cannot determine independence easily, you must show that

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$

Let's use this knowledge in the following contingency table.

Example 1: Complete the table to answer the following questions.

	Male	Female	Subtotal
Born in GA	7	5	
Not Born in GA	16	13	
Subtotal			

a. P (Female) = 18/41

b. P (Born in GA) = 12/41

c. P(Female | Born in GA) = 5/12

d. P(Born in GA | Female) = 5/18

e. Are the events Female and Born in GA independent?

No, P(Female) $\neq$ P(Female | Born in GA) and P(Born in GA) $\neq$ P(Born in GA | Female).
NOTE: Only one of these has to be unequal to be dependent.

Example 2: Use the definition of independent events to determine whether the events from rolling a die (6-sided) are independent or dependent.

1. The event odd and the event 3 or 6

a. P(odd) = 3/6 = 1/2

b. P(3 or 6) = 2/6 = 1/3

c. P(odd | 3 or 6) = 1/2

d. P(3 or 6 | odd) = 1/3

These events are independent because P(odd) = P(odd | 3 or 6) and P(3 or 6) = P(3 or 6 | odd)

2. The event less than 3 and the event more than 5

a. P(<3) = 2/6 = 1/3 c. P(<3 | >5) = 0
b. P(<5) = 1/6 d. P(>5 | < 3) = 0
Dependent because P(<3) $\neq$ P(<3 | >5) and P(>5) $\neq$ P(>5 | <3).
NOTE: Only one of the above has to be unequal.

3. The event even and the event prime number

a. P(even) = 3/6 = 1/2 c. P(even | prime) = 1/3
b. P(prime) = 3/6 = 1/2 d. P(prime | even) = 1/3
Dependent because P(even) $\neq$ P(even | prime) and P(prime) $\neq$ P(prime | even).
NOTE: Only one of the above has to be unequal.