

AP Calculus AB

Morning Quiz Review: 6-2, 6-3, and Slope Fields

1. Consider the differential equation: $\frac{dy}{dt} = t \cos(t^2)$

a. Find the particular solution if $y\left(\sqrt{\frac{\pi}{2}}\right) = 1$

b. Write the equation of the line tangent to $y(t)$ at $(\sqrt{\pi}, \frac{1}{2})$

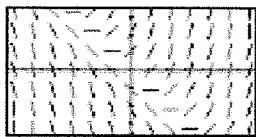
c. Use the tangent line to approximate $y(1.3)$

2. The rate of depreciation of a property value, y , is directly proportional to the value at any given time, t .

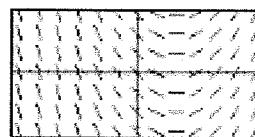
a. Write the differential equation and find the general solution

b. Given that 1960 represents $t = 0$ and that it takes 40 years for the property value to decrease by half. Find the property value in 1960 if in 1987 the property value is estimated to be \$125,000. (Round to the nearest dollar)

(A)



(B)



(C)



(D)



3. Match the differential equations to correct slope fields above:

i. $\frac{dy}{dx} = -\frac{x}{y}$ _____

ii. $\frac{dy}{dx} = .5y$ _____

4. Match the general equations to the correct slope field above:

i. $y = x^2 - x + C$ _____

ii. $y = Ce^{-x^2}$ _____

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Solution Key

1. Consider the differential equation: $\frac{dy}{dt} = t \cos(t^2)$

- a. Find the particular solution if $y\left(\frac{\pi}{2}\right) = 1$

$$\begin{aligned} dy &= t \cos(t^2) dt \\ u &= t^2 \\ du &= 2t dt \\ \frac{du}{dt} &= 2t \\ dt &= \frac{du}{2t} \end{aligned}$$

$$\begin{aligned} y &= \frac{1}{2} \sin(t^2) + C \\ 1 &= \frac{1}{2} \sin\left(\frac{\pi^2}{4}\right) + C \\ 1 &= \frac{1}{2}(1) + C \\ \frac{1}{2} &= C \end{aligned}$$

- b. Write the equation of the line tangent to $y(t)$ at $(\sqrt{\pi}, \frac{1}{2})$

$$\begin{aligned} y'(\sqrt{\pi}) &= \sqrt{\pi} \cos(\sqrt{\pi})^2 = \sqrt{\pi}(-1) = -\sqrt{\pi} \\ \text{point: } (\sqrt{\pi}, \frac{1}{2}) &| y - y_1 = m(x - x_1) \\ m = -\sqrt{\pi} &| y - \frac{1}{2} = -\sqrt{\pi}(x - \sqrt{\pi}) \end{aligned}$$

- c. Use the tangent line to approximate $y(1.3)$

$$y = -\sqrt{\pi}(x - \sqrt{\pi}) + \frac{1}{2} \quad y(1.3) = -\sqrt{\pi}(1.3 - \sqrt{\pi}) + \frac{1}{2} \approx 1.337$$

2. The rate of depreciation of a property value, y , is directly proportional to the value at any given time t .

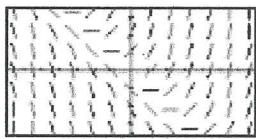
- a. Write the differential equation and find the general solution

$$y' = ky \quad \frac{dy}{dt} = ky \quad \int \frac{dy}{y} = k \int dt \quad \ln|y| = kt + C \quad y = Ce^{kt}$$

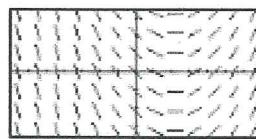
- b. Given that 1960 represents $t = 0$ and that it takes 40 years for the property value to decrease by half. Find the property value in 1960 if in 1987 the property value is estimated at \$125,000. (Round to the nearest dollar)

$$\begin{array}{ll} \text{(time, value)} & y = Ce^{kt} \\ (0, C) & \frac{1}{2}C = Ce^{k(40)} \\ (40, \frac{1}{2}C) & \ln 0.5 = \ln e^{40k} \\ (27, 125,000) & \ln 0.5 = 40k \\ & \frac{1}{40} \ln 0.5 = k \end{array} \quad \begin{array}{l} y = Ce^{\frac{1}{40} \ln 0.5 t} \\ 125000 = Ce^{\frac{1}{40} \ln 0.5 (27)} \\ \frac{125000}{e^{\frac{27}{40} \ln 0.5}} = C \\ C = 199574.597 \\ C \approx \$199,575 \end{array}$$

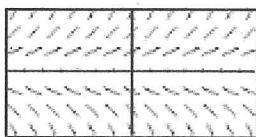
(A)



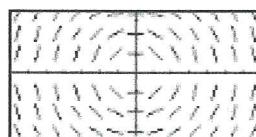
(B)



(C)



(D)



3. Match the differential equations to correct slope fields above:

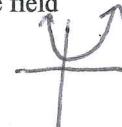
i. $\frac{dy}{dx} = -\frac{x}{y}$
 • slope does not exist on x-axis
• slope = 0 on y-axis

ii. $\frac{dy}{dx} = .5y$
 • slope = 0 on x-axis

- positive slope for positive y -values
 • negative slope for negative y -values

4. Match the general equations to the correct slope field above:

i. $y = x^2 - x + C$



ii. $y = Ce^{-\frac{x^2}{2}}$

