

Analytic Geometry Quadratic Formula and Discriminant

Key

\* A **quadratic equation** is as an equation of degree 2, meaning that the highest exponent of this function is 2.

\* The quadratic formula is used to solve an equation of the form  $ax^2 + bx + c = 0$

\* This formula can solve any equation that can be solved by factoring and completing the square

**Quadratic Formula:**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  given  $ax^2 + bx + c = 0$

Solve for x below using quadratic formula

1.  $x^2 - 5x + 6 = 0$

$a=1$   
 $b=-5$   
 $c=6$   

$$\frac{5 \pm \sqrt{25 - 4(1)(6)}}{2(1)}$$

$$\frac{5 \pm \sqrt{1}}{2} = \frac{5+1}{2}, \frac{5-1}{2}$$

$x = \frac{6}{2}, \frac{4}{2}$

$x = 3, 2$

2.  $x^2 + 24x = 4$

$x^2 + 24x - 4 = 0$   
 $a=1$   
 $b=24$   
 $c=-4$

$$\frac{-24 \pm \sqrt{24^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{-24 \pm \sqrt{592}}{2}$$

$x = \frac{-24 \pm 4\sqrt{37}}{2}$ 
 $x = \frac{-24}{2} \pm \frac{4\sqrt{37}}{2} = -12 \pm 2\sqrt{37}$

3.  $2x^2 + 10 = x$

$2x^2 - x + 10 = 0$

$$\frac{1 \pm \sqrt{1^2 - 4(2)(10)}}{2(2)} = \frac{1 \pm \sqrt{-79}}{4}$$

$\frac{1 \pm i\sqrt{79}}{4}$

4.  $2x^2 - 9 = 0$

$a=2$   
 $b=0$   
 $c=-9$

$$\frac{0 \pm \sqrt{0 - 4(2)(-9)}}{2(2)}$$

$$\frac{0 \pm \sqrt{72}}{4} = \frac{6\sqrt{2}}{4} = \frac{3\sqrt{2}}{2}$$

$x = \pm \frac{3\sqrt{2}}{2}$

$x^2 = \frac{9}{2}$   
 $x = \pm \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$$x^2 - 6x + 9 \quad 6 \pm \sqrt{\quad}$$

The Discriminant is number (from the expression) inside the square root of the quadratic formula.

Since the quadratic formula is  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , the discriminant is the  $b^2 - 4ac$

The discriminant describes the **nature**, or the type, of solutions

If the Discriminant is **positive**, there are 2 real answers (2 real roots)

If the Discriminant is **negative**, there are 2 imaginary answers (2 imaginary roots)

If the Discriminant is **zero**, there is 1 real answer. (2 real answers being the same value) (1 real root)

Find the discriminant for the below and describe the type of roots for each equation:

|                       |                        |                        |                   |
|-----------------------|------------------------|------------------------|-------------------|
| 5. $x^2 - 5x + 6 = 0$ | 6. $x^2 + 24x - 4 = 0$ | 7. $2x^2 - x + 10 = 0$ | 8. $2x^2 - 9 = 0$ |
| $b^2 - 4ac$           | $24^2 - 4(1)(-4)$      | $1^2 - 4(2)(10)$       | $0 - 4(2)(-9)$    |
| $5^2 - 4(1)(6)$       | $592 > 0$              | $-79 < 0$              | $72 > 0$          |
| $25 - 24 = 1 > 0$     | 2 Real solutions       | 2 imaginary roots      | 2 Real Roots      |
| 2 Real solutions      |                        |                        |                   |

a) Solve equation using quadratic formula    b) find discriminant    c) describe the nature of the roots

9.  $2x^2 - 3x - 5 = 0$

$$\frac{3 \pm \sqrt{9 - 4(2)(-5)}}{2(2)} = \frac{3 \pm \sqrt{49}}{4}$$

$3 \pm 7$      $\frac{70}{4}$      $\frac{-4}{4}$      $D = 49 > 0$   
 $\frac{4}{4}$      $\frac{2}{4}$      $\frac{1}{4}$     2 Reals

$x = \frac{5}{2}, -1$

10.  $x^2 + 12x = 3$

$$x^2 + 12x - 3 = 0$$

$D = 156 > 0$   
2 Real Roots

$$\frac{-12 \pm \sqrt{144 - 4(1)(-3)}}{2(1)}$$

$$\frac{-12 \pm \sqrt{156}}{2}$$

$$\frac{-12 \pm 2\sqrt{39}}{2} = -6 \pm \sqrt{39}$$

$-6 \pm \sqrt{39}$

11.  $2x^2 + 9 = 3x$

$$2x^2 - 3x + 9 = 0$$

$D = -63 < 0$   
2 Imaginary Roots

$$\frac{3 \pm \sqrt{9 - 4(2)(9)}}{2(2)}$$

$$\frac{3 \pm \sqrt{-63}}{4}$$

$x = \frac{3 \pm 3i\sqrt{7}}{4}$

12.  $2x^2 - 7 = 0$

$$0 \pm \sqrt{b^2 - 4(2)(-7)}$$

$D = 56 > 0$   
2 Reals

$$\frac{0 \pm \sqrt{56}}{2(2)}$$

$$\pm \frac{\sqrt{56}}{4}$$

$$\frac{2\sqrt{14}}{4} = \frac{\sqrt{14}}{2}$$

$x = \pm \frac{\sqrt{14}}{2}$

# Quadratic Formula and Discriminant Day 1 Homework

a) Solve equation using quadratic formula    b) find discriminant    c) describe the nature of the roots

1.  $4x^2 - 11x = 3$

$$4x^2 - 11x - 3 = 0$$

$$\frac{11 \pm \sqrt{121 - 4(4)(-3)}}{2(4)} = \frac{11 \pm \sqrt{169}}{8}$$

$$\frac{11+13}{8} \quad \frac{11+13}{8}, \quad \frac{11-13}{8}$$

$$\frac{24}{8}, \quad \frac{-2}{8}$$

$$x = 3, \frac{-1}{4}$$

2.  $x^2 + 7x = 5$

$$x^2 + 7x - 5 = 0$$

$$\frac{-7 \pm \sqrt{49 - 4(1)(-5)}}{2(1)}$$

$$\frac{-7 \pm \sqrt{69}}{2}$$

$$x = \frac{-7 \pm \sqrt{69}}{2}$$

3.  $5x^2 + 1 = 3x$

$$5x^2 - 3x + 1 = 0$$

$$\frac{3 \pm \sqrt{9 - 4(5)(1)}}{2(5)}$$

$$\frac{3 \pm \sqrt{-11}}{10}$$

$$x = \frac{3 \pm i\sqrt{11}}{10}$$

4.  $5x^2 - 9 = 0$

$$\frac{0 \pm \sqrt{0 - 4(5)(-9)}}{2(5)}$$

$$\frac{\pm \sqrt{180}}{10}$$

$$\frac{6\sqrt{5}}{10}$$

$$x = \pm \frac{3\sqrt{5}}{5}$$

a) Solve equation using quadratic formula b) find discriminant c) describe the nature of the roots

5.  $-7 + 3x^2 = 4x$

$$3x^2 - 4x - 7 = 0$$

$$\frac{4 \pm \sqrt{16 - 4(3)(-7)}}{2(3)}$$

$$\frac{4 \pm \sqrt{100}}{6}$$

$$\frac{4 \pm 10}{6} \quad x = \frac{14}{6}, \frac{-6}{6}$$

$$x = \frac{7}{3}, -1$$

6.  $x^2 + 11x = -4$

$$x^2 + 11x + 4 = 0$$

$$\frac{-11 \pm \sqrt{121 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-11 \pm \sqrt{105}}{2}$$

7.  $2x^2 + 13 = 5x$

$$2x^2 - 5x + 13 = 0$$

$$\frac{5 \pm \sqrt{25 - 4(2)(13)}}{2(2)}$$

$$\frac{5 \pm \sqrt{-79}}{4}$$

$$x = \frac{5 \pm i\sqrt{79}}{4}$$

8.  $3x^2 - 17 = 0$

$$\frac{0 \pm \sqrt{0^2 - 4(3)(-17)}}{2(3)} = \frac{\sqrt{204}}{6}$$

$$x = \pm \frac{2\sqrt{51}}{6}$$

$$x = \pm \frac{\sqrt{51}}{3}$$