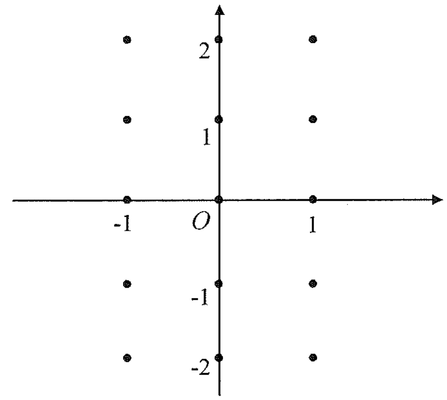
Quiz Review for 6-2, 6-3, and Slope Fields

1. Consider the differential equation $\frac{dy}{dx} = (2x+2)y$.

a) On the axes below, sketch a slope field for the given differential equation at the fifteen points indicated.

x							
y							
y'							

x								
y								
y'								



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 5$.

c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

2. Match the slope field to its differential equation, then sketch a solution that passes through the point (1, -1). Find the particular solution use calculator to confirm the solution sketch below.

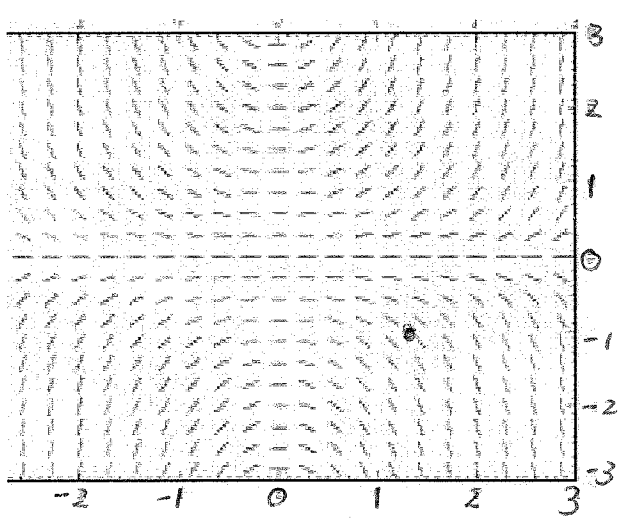
a) $\frac{dy}{dx} = x^3$

b) $\frac{dy}{dx} = x^2$

c) $\frac{dy}{dx} = xy$

d) $\frac{dy}{dx} = \frac{x}{y}$

e) $\frac{dy}{dx} = \ln x$



3. In a certain culture the rate of growth of bacteria is proportional to the amount present. If there are 1000 bacteria present initially and the amount doubles in 12 minutes, how long will it take before there will be 1,000,000 bacteria present?

4. Newton's law of cooling states that the rate at which a body changes temperature is proportional to the difference between its temperature and that of the surrounding medium. If a body is in air of temperature 35° and the body cools from 120° to 60° in 40 minutes, find the temperature of the body after 100 minutes.

5. The rate of increase of a population, P, is directly proportional to the population at any given time, t. If it takes 5 years for the population to double, how long will it take the population to quadruple? If the population is 10,000 after 3 years, what was the initial population?

6. Sugar decomposes in water at a rate proportional to the amount still unchanged. If there were 50 lb of sugar present initially and at the end of 5 hours this is reduced to 20 lb, how long will it take until 90% of the sugar is decomposed?

7. 30% of a radioactive substance disappears in 15 years. Find the half-life of the substance.

8. In a certain culture where the rate of growth of bacteria is proportional to the amount present, the number triples in 3 hours. If at the end of 12 hours there were 10 million bacteria, how many were present initially?

Solution Key

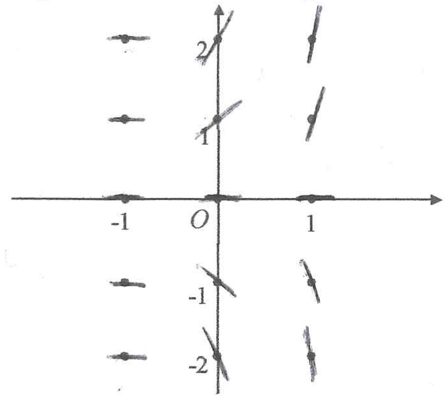
Quiz Review for 6-2, 6-3, and Slope Fields

1. Consider the differential equation $\frac{dy}{dx} = (2x+2)y$.

a) On the axes below, sketch a slope field for the given differential equation at the fifteen points indicated.

x	-1	-1	-1	-1	-1	0	0
y	2	1	0	-1	-2	2	1
y'	0	0	0	0	0	4	2

x	0	0	0	1	1	1	1	1
y	0	-1	-2	2	1	0	-1	-2
y'	0	-2	-4	8	4	0	-4	-8



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 5$.

$\int \frac{dy}{y} = \int 2x+2\,dx$

$\ln|y| = \frac{2x^2}{2} + 2x + C$

$e^{\ln|y|} = e^{x^2+2x+C}$

$y = e^{x^2+2x} \cdot e^{C}$

$y = Ce^{x^2+2x}$

$5 = Ce^{0+0}$

$5 = C$

$y = 5e^{x^2+2x}$

c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

point: $(0, 5)$

slope: $\frac{dy}{dx}_{(0,5)} = (2(0)+2)(5) = 10$

$y - y_1 = m(x - x_1)$

$y - 5 = 10(x - 0)$

$y - 5 = 10x$

$y = 10x + 5$

$y(0.2) = 10(0.2) + 5$

$= 2 + 5 = 7$

$\boxed{f(0.2) \approx 7}$

2. Match the slope field to its differential equation, then sketch a solution that passes through the point (1, -1). Solve equation and graph to confirm sketch (on calculator)

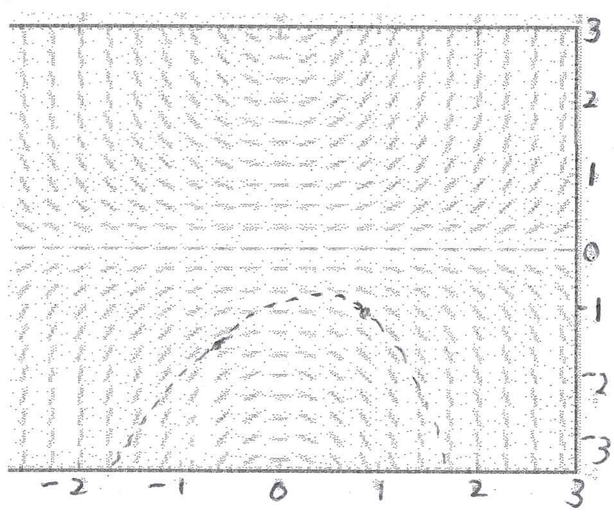
a) ~~$\frac{dy}{dx} = x^3$~~

b) ~~$\frac{dy}{dx} = x^2$~~

c) $\boxed{\frac{dy}{dx} = xy}$

d) ~~$\frac{dy}{dx} = \frac{x}{y}$~~

e) ~~$\frac{dy}{dx} = \ln x$~~



$\frac{dy}{dx} = xy$

$\int \frac{dy}{y} = \int x\,dx$

$e^{\ln|y|} = e^{\frac{x^2}{2} + C}$

$y = Ce^{x^2/2}$

$-1 = Ce^{1/2}$

$\frac{-1}{e^{1/2}} = C$

$C = -e^{-1/2}$

$y = -e^{-1/2} \cdot e^{x^2/2}$

$y = -e^{\frac{-1+x^2}{2}}$

$\boxed{y = -e^{\frac{-1+x^2}{2}}}$

b'

3. In a certain culture the rate of growth of bacteria is proportional to the amount present. If there are 1000 bacteria present initially and the amount doubles in 12 minutes, how long will it take before there will be 1,000,000 bacteria present?

(time, bacteria) (t, b)

$(0, 1000)$

$(12, 2000)$

$(\_\_, 1{,}000{,}000)$

$b' = kb$

$\frac{db}{dt} = kb$

$\int \frac{db}{b} = \int k\,dt$

$\ln|b| = kt + c$

$b = e^{kt} \cdot e^{c}$

$b = Ce^{kt}$

$1000 = Ce^{k(0)}$

$1000 = C$

$b = 1000e^{kt}$

$2000 = 1000e^{k(12)}$

$2 = e^{k12}$

$\ln 2 = \ln e^{k12}$

$\ln 2 = 12k$

$\frac{\ln 2}{12} = k$

$b = 1000e^{\frac{\ln 2}{12}t}$

$1000000 = 1000e^{\frac{\ln 2}{12}t}$

$1000 = e^{\frac{\ln 2}{12}t}$

$\ln 1000 = \ln e^{\frac{\ln 2}{12}t}$

$\ln 1000 = \frac{\ln 2}{12}t$

$\ln 1000 \cdot \frac{12}{\ln 2} = t$

$\boxed{t = 119.589 \text{ mins}}$

4. Newton's law of cooling states that the rate at which a body changes temperature is proportional to the difference between its temperature and that of the surrounding medium. If a body is in air of temperature 35° and the body cools from 120° to 60° in 40 minutes, find the temperature of the body after 100 minutes.

$\frac{dy}{dt} = k(y-35)$

(time, temperature) (t, y)

$(0, 120°)$

$(40, 60°)$

$(100, \_\_)$

$\int \frac{dy}{y-35} = \int k\,dt$

$\ln|y-35| = kt + c$

$y - 35 = e^{kt} \cdot e^{c}$

$y - 35 = Ce^{kt}$

$y = Ce^{kt} + 35$

$120 = Ce^{k(0)} + 35$

$120 - 35 = C$

$85 = C$

$y = 85e^{kt} + 35$

$60 = 85e^{k(40)} + 35$

$25 = 85e^{40k}$

$\frac{5}{17} = e^{40k}$

$\ln 5/17 = \ln e^{40k}$

$\frac{1}{40}\ln 5/17 = k$

$y = 85e^{\frac{1}{40}\ln 5/17\, t} + 35$

$y = 85e^{1/40 \ln(5/17)(100)} + 35$

$y = 3.988 + 35 = \boxed{38.988°}$

5. The rate of increase of a population, P, is directly proportional to the population at any given time, t. If it takes 5 years for the population to double, how long will it take the population to quadruple? If the population is 10,000 after 3 years, what was the initial population?

C = initial population

(time, population) (t, P) solve for k first

$(5, 2C)$

$(10, 4C)$

$(3, 10{,}000)$

$P' = kP$

$\frac{dP}{dt} = kP$

$\int \frac{dP}{P} = \int k\,dt$

$\ln|P| = kt + c$

$P = Ce^{kt}$

$2C = Ce^{k(5)}$

$2 = e^{5k}$

$\ln 2 = \ln e^{5k}$

$\ln 2 = 5k$

$\frac{\ln 2}{5} = k$

$P = Ce^{\frac{\ln 2}{5}t}$

$10{,}000 = Ce^{\frac{\ln 2}{5}(3)}$

$10{,}000 = C(1.5157)$

$\boxed{C = 6597.539}$

6. Sugar decomposes in water at a rate proportional to the amount still unchanged. If there were 50 lb of sugar present initially and at the end of 5 hours this is reduced to 20 lb, how long will it take until 90% of the sugar is decomposed?

$(0.9)(50) = 45\text{ lbs}$ decomposed means 5 lbs remaining

(time, sugar) (t, S)

$(0, 50)$

$(5, 20)$

$(\_\_, 5)$

$S' = kS$

$\frac{dS}{dt} = kS$

$\int \frac{dS}{S} = \int k\,dt$

$\ln|S| = kt + c$

$S = Ce^{kt}$

$50 = Ce^{k(0)}$

$50 = C$

$S = 50e^{kt}$

$20 = 50e^{k(5)}$

$0.4 = e^{5k}$

$\ln 0.4 = \ln e^{5k}$

$\ln 0.4 = 5k$

$\frac{1}{5}\ln 0.4 = k$

$S = 50e^{1/5 \ln 0.4\, t}$

$5 = 50e^{\frac{1}{5}\ln 0.4\, t}$

$0.1 = e^{1/5 \ln 0.4\, t}$

$\ln 0.1 = \ln e^{1/5 \ln 0.4\, t}$

$\ln 0.1 = \frac{1}{5}\ln 0.4\, t$

$\frac{5\ln 0.1}{\ln 0.4} = t$

$12.565\text{ hrs} = t$

$\boxed{t = 12.565 \text{ hrs}}$

7. 30% of a radioactive substance disappears in 15 years. Find the half-life of the substance.

$r = Ce^{kt}$

(time, radioactive substance remaining) (t, r)

$(0, C)$

$(15, 0.7C)$

$(\_\_, 0.5C)$

$0.7C = Ce^{k(15)}$

$0.7 = e^{15k}$

$\ln 0.7 = \ln e^{15k}$

$\ln 0.7 = 15k$

$\frac{\ln 0.7}{15} = k$

$r = Ce^{\frac{\ln 0.7}{15}t}$

$0.5C = Ce^{\frac{\ln 0.7}{15}t}$

$0.5 = e^{\frac{\ln 0.7}{15}t}$

$\ln 0.5 = \cancel{\ln e}^{\frac{\ln 0.7}{15}t}$

$\ln 0.5 = \frac{\ln 0.7}{15}t$

$\frac{15\ln 0.5}{\ln 0.7} = t$

$\boxed{t \approx 29.15 \text{ yrs}}$

8. In a certain culture where the rate of growth of bacteria is proportional to the amount present, the number triples in 3 hours. If at the end of 12 hours there were 10 million bacteria, how many were present initially?

$y' = ky$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + c$

$y = Ce^{kt}$

(time, bacteria) (t, y) millions

$(0, C)$

$(3, 3C)$

$(12, 10)$

$3C = Ce^{k(3)}$

$3 = e^{3k}$

$\ln 3 = \ln e^{3k}$

$\ln 3 = 3k$

$\frac{\ln 3}{3} = k$

$y = Ce^{\frac{\ln 3}{3}t}$

$10 = Ce^{\frac{\ln 3}{3}(12)}$

$10 = C(e^{4\ln 3})$

$10 = C(81)$

$\boxed{0.123 = C}$

$C = 0.123$ million

or

$\boxed{123{,}000 \text{ bacteria}}$