Similarity Notes #1 Name: ____________________

Ratios and Proportions

The ratio of x to y is the number obtained by dividing x by y.

Example: $\frac{x}{y}$

A ratio can be represented in the following ways:

1. 1 : 2
2. 1/2
3. 1 to 2

Examples:

1. Give each ratio in simplest form:

(a) 8 : 2

4:1

(b) $\frac{24}{36}$ 12 $\boxed{\frac{2}{3}}$

(c) 13xy : 39x

$\frac{\cancel{13}^{1}x\cancel{y}}{\cancel{39}_{3}\cancel{x}}$ $\frac{y}{3}$ or y:3

A proportion is an equation stating that two ratios are equal.

Example: $\frac{1}{2}=\frac{3}{6}$

To Solve for an Unknown in a Proportion:
Cross Multiply.

$\frac{a}{b}=\frac{c}{d}$ then $ad = bc$

CCGPS Analytic Geometry
Similarity

$\frac{2}{x-3} = \frac{6}{x+5}$

2. Solve for x.

(a) $\frac{3}{4} = \frac{6}{x}$

$3x = 4(6)$

$\frac{3x}{3} = \frac{24}{3}$ $\boxed{x=8}$

(b) 3 : 5 = 6 : x

$\frac{3}{5} = \frac{6}{x}$ $\boxed{x=10}$

$\frac{3x}{3} = \frac{30}{3}$

(c) 2 : (x – 3) = 6 : (x + 5)

$2(x+5) = 6(x-3)$

$2x + 10 = 6x - 18$

$-2x + 18 \quad -2x + 18$

$\frac{28}{4} = \frac{4x}{4}$ $\boxed{x=7}$

Now you try!

3. (a) Reduce: $\frac{15}{60} = \frac{3}{12}$

$\boxed{\frac{1}{4}}$

(b) Solve for x.

$\frac{6}{18} = \frac{8}{x}$

$6x = 8(18)$

$\frac{6x}{6} = \frac{144}{6}$

$\boxed{x=24}$

(c) Solve for x.

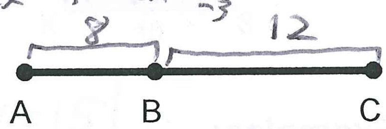
$\frac{3}{(x+3)} = \frac{2}{(x+1)}$

$3(x+1) = 2(x+3)$ $\boxed{x=3}$

$3x + 3 = 2x + 6$

$-2x \ -3 \quad -2x \ -3$

Examples:



4. Given: AB = 8 and BC = 12; State the following ratios.

(a) AB : BC

8:12

$\boxed{2:3}$

(b) AC : BC

$\frac{20}{12} = \boxed{\frac{5}{3}}$

(c) AB : AC

$\frac{8}{20} = \boxed{\frac{2}{5}}$

5. Will the following ratios form a proportion?

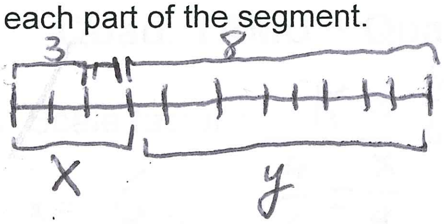
(a) $\frac{6}{24}$ *and* $\frac{4}{16}$ yes

$\frac{1}{4} = \frac{1}{4}$

(b) $\frac{2}{9} = \frac{3}{10}$ no

$20 \neq 27$

6. A segment is divided in the ratio of 3 : 8. If the segment is 44 cm long, find the length of each part of the segment.



$\frac{11}{44} = \frac{3}{x}$

$11x = 132$

$\frac{11}{44} = \frac{8}{y}$

$11y = 352$

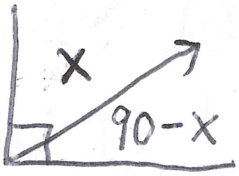
x = 12 cm

y = 32 cm

7. Two complementary angles are in the ratio of 2 : 7. Find the measure of each angle.

*Add to be 90°

$\frac{9}{90} = \frac{2}{x}$



$\frac{2}{7} = \frac{x}{90-x}$

$7x = 2(90-x)$

$7x = 180 - 2x$

$\frac{9x}{9} = \frac{180}{9}$

$x = 20$

20°

70°

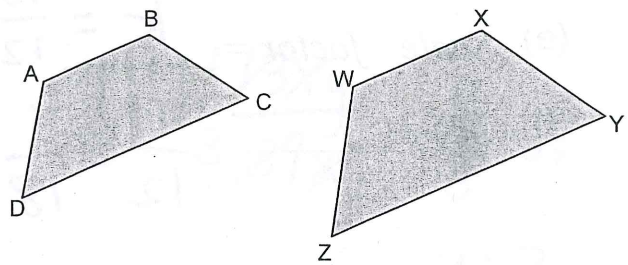
Similarity Notes # 2 Name: ______________________

Similar Polygons

Two polygons are similar **(~) if their vertices can be matches so that:**

- Corresponding angles are congruent.
- Ratios of lengths of corresponding lengths are proportional.

If ABCD ~ WXYZ, then:



1. $\angle A \cong \angle W,\ \angle B \cong \angle X,\ \angle C \cong \angle Y,\ and\ \angle D \cong \angle Z$
2. $\frac{WX}{AB} = \frac{XY}{BC} = \frac{YZ}{CD} = \frac{WZ}{AD}$

Conversely, if parts 1 & 2 are true, then you can conclude that ABCD ~ WXYZ.

The scale factor **of two similar polygons is the ratio that will transform the first polygon to the second.**

To find the scale factor between two figures, write a ratio using the length of one of the sides of the transformed figure (the second figure) over the length of the corresponding sides of the original figure.

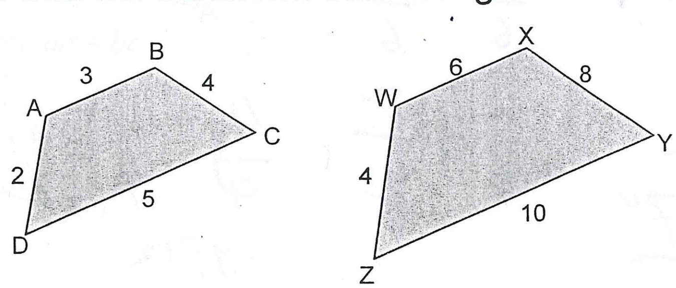
$k = \frac{\text{transformed}}{\text{original}}$

original transformed

Example # 1:

ABCD ~ WXYZ.

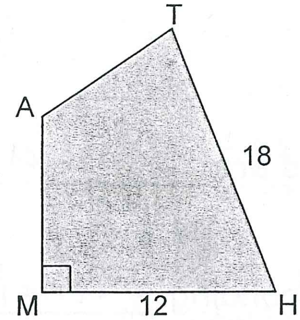
Find the scale factor that will transform the first figure to the second figure.



$k = \frac{6}{3} = \boxed{2}$

original transformed

Example # 2: MATH ~ KIDS

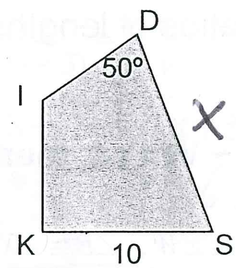


(a) $\angle A \cong$ $\underline{\angle I}$

(b) $m\angle K =$ $\underline{\angle M = 90°}$

(c) $m\angle T =$ $\underline{50°}$

(d) $\dfrac{ID}{AT} = \dfrac{IK}{?}$ $? =$ $\underline{AM}$

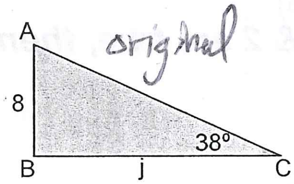


(e) $Scale\ \ factor =$ $\underline{k} = \dfrac{10}{12} = \boxed{\dfrac{5}{6}}$

(f) $DS =$ ______ $\dfrac{10}{12} = \dfrac{x}{18}$ $\dfrac{12x}{12} = \dfrac{180}{12}$ $\boxed{x = 15}$

Now you try!

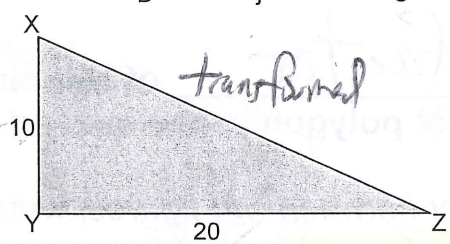
3. ABC ~ XYZ



(a) Scale factor = $\underline{k} = \dfrac{10}{8} = \dfrac{5}{4}$

$\dfrac{5}{4} = \dfrac{20}{j}$

(b) j = $\underline{16}$

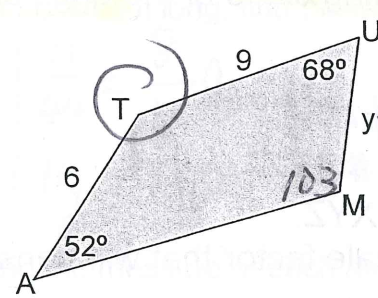


$\dfrac{5j}{5} = \dfrac{80}{5}$

(c) $m\angle Z =$ $\underline{38°}$

$360 - 68 = 52 - 103$

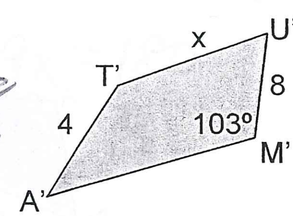
4. Quad. TAMU ~ Quad. T'A'M'U'



(a) Scale factor = $\underline{k} = \dfrac{4}{6} = \dfrac{2}{3}$

(b) x = $\underline{6}$ $\dfrac{4}{6} = \dfrac{x}{9}$ $\dfrac{6x}{6} = \dfrac{36}{6}$

(c) y = $\underline{12}$

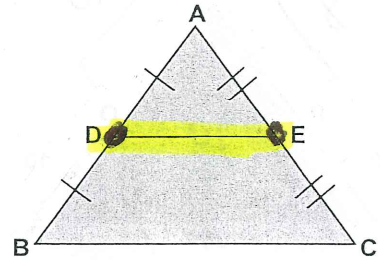


(d) $m\angle U' =$ $\underline{68}$ $\dfrac{4}{6} = \dfrac{8}{y}$ $\dfrac{4y}{4} = \dfrac{48}{4}$

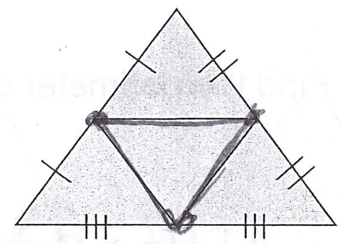
e) $m\angle T =$ $\underline{137°}$ $y = 12$

Similarity Notes # 3 Name: ____________________

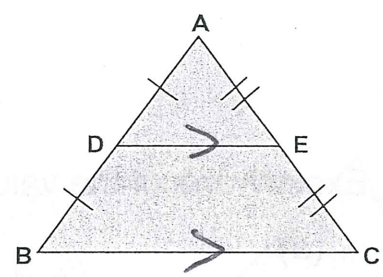
Midsegments of Triangles



Midsegment: The segment connecting the midpoints of two sides of a triangle.



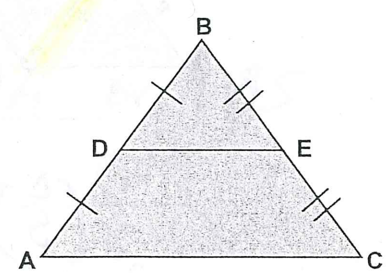
Conjecture: The three midsegments of a triangle divide the triangle into 4 congruent triangles.



Conjecture: A midsegment of a triangle is parallel to the 3rd side and one half the length of the third side.

Notation: $\overline{DE} \parallel \overline{BC}$ and $2(DE) = BC$

Guided Practice:



1. (a) If AC = 20, then DE = 10.

   (b) If DE = 6, then AC = 12.

   (c) If DE = x + 6 and AC = 3x + 4, then x = 8.

$2(DE) = AC$

$2(x+6) = 3x+4$

$2x+12 = 3x+4$

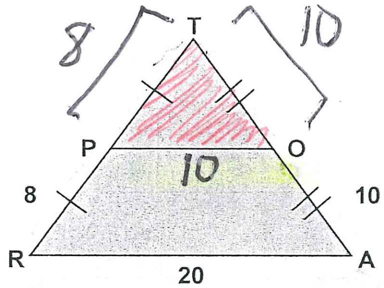
$-2x \quad -2x$

$12 = x+4$

$-4 \quad -4$

$\boxed{8 = x}$

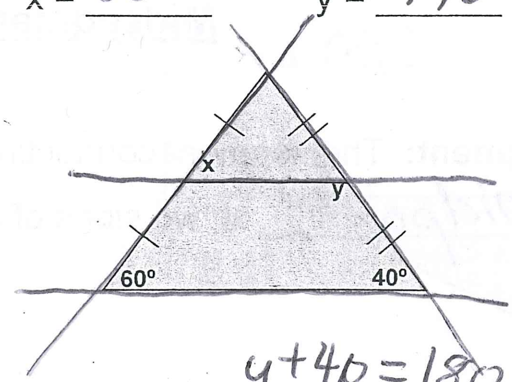
2. Find the perimeter of ΔTOP.



Perimeter = 28 units

3. Find the missing angle measures.

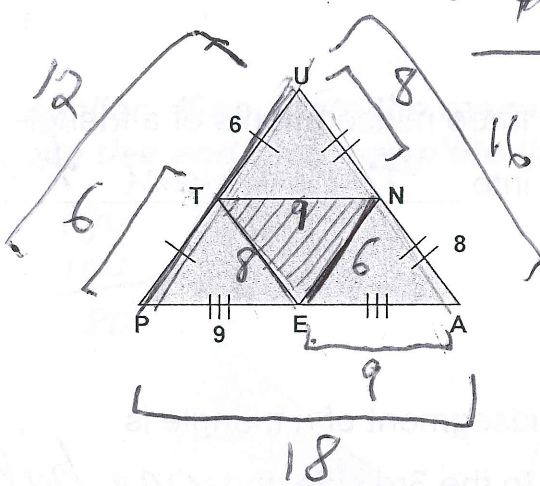
x = 60°   y = 140°

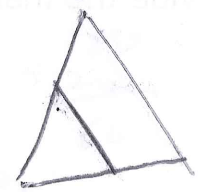


$y + 40 = 180$
$-40 \quad -40$
140

4. Find the perimeter of ΔTEN.

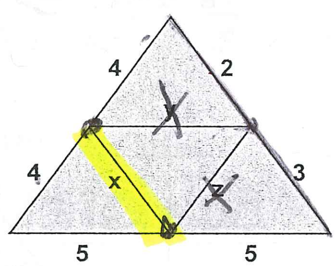
$8+9+6 = 23$ units





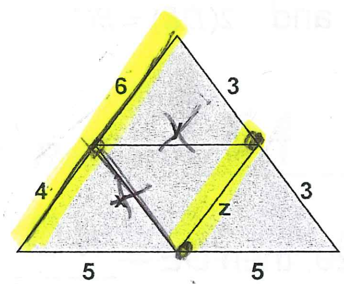
5. Exactly one of the values x, y, or z can be determined. Find it.

(a)



$x = \frac{1}{2}(5) = 2.5$

(b)



$z = \frac{1}{2}(10)$

$z = 5$

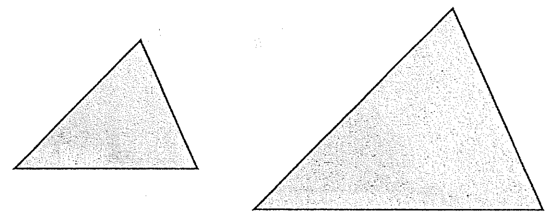
Similarity Notes # 4

Name: ____________________

Triangle Similarity

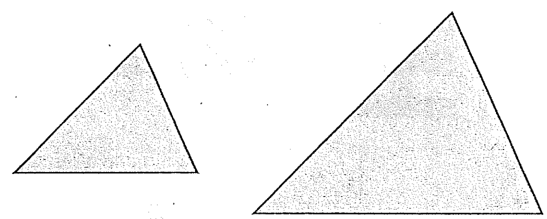
Angle Angle Similarity: **AA~**

If 2 angles of one triangle are congruent to 2 angles of another triangle, then the triangles are similar.



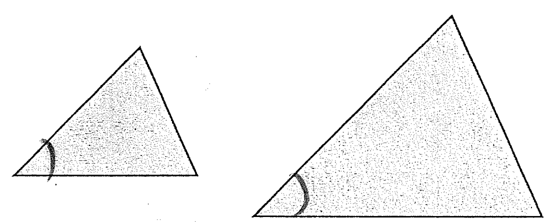
Side Side Side Similarity: **SSS~**

If corresponding sides of two triangles are proportional, then the two triangles are similar.



Side Angle Side Similarity: **SAS~**

If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.



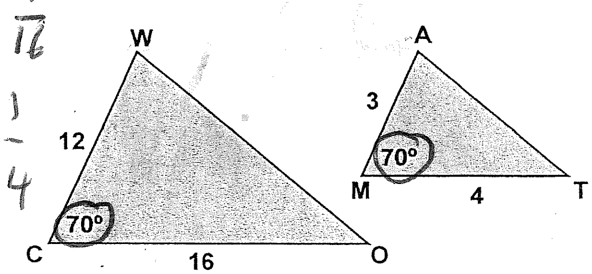
Guided Practice

1. *Reason:* SAS

Δ WCO ~ Δ AMT

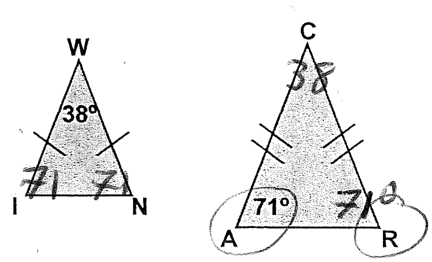
$\frac{3}{12} = \frac{4}{16}$

$\frac{1}{4} = \frac{1}{4}$



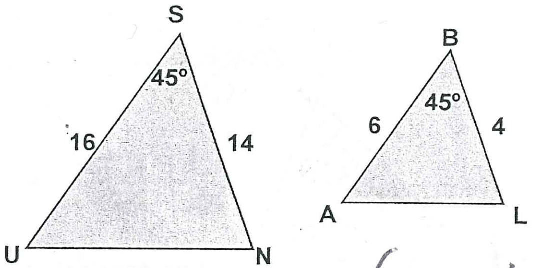
2. *Reason:* AA or SAS

Δ WIN ~ Δ CAR



3. Reason: 2 triangles ~~are~~ not similar

Δ ______ ~ Δ ______

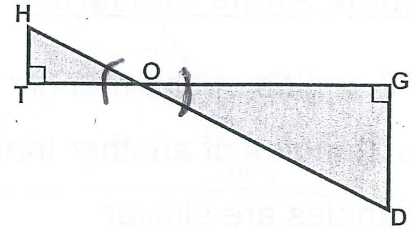


$\frac{6}{16} = \frac{4}{14}$

$\frac{3}{8}$

4. Reason: AA

Δ OTH ~ Δ OGD

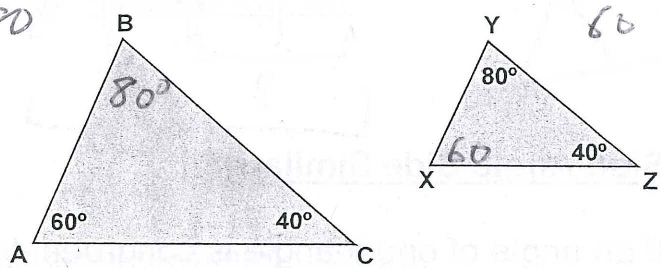


Determine if the triangles are similar. If so, state the reason (AA~, SAS~, or SSS~) that would prove this and then complete the similarity statement.

1. Reason: AA

Δ ABC ~ Δ XYZ

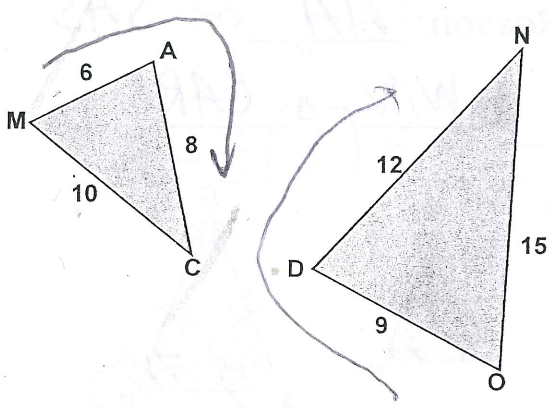
$\begin{array}{r} 180 \\ -100 \\ \hline 80 \end{array}$ $\begin{array}{r} 180 \\ -120 \\ \hline 60 \end{array}$



$\frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{2}{3}$

2. Reason: SSS

Δ MAC ~ Δ ODN



3. Reason: AA

Δ ADE ~ Δ ABC