KEY

1

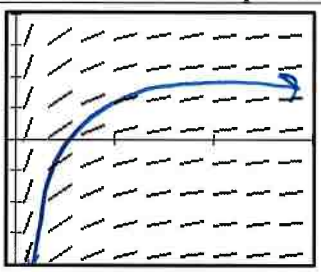
7.4 Reasoning Using Slope Fields

Calculus

Practice

The slope field from a certain differential equation is shown for each problem. The multiple choice answers are either differential equations OR a specific solution to that differential equation.

1.



(A) $y = \ln x$

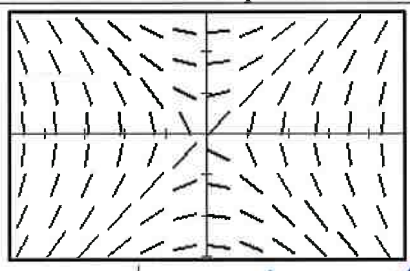
(B) $y = e^x$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = x^2$

2.



(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = \frac{x}{y}$

(C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = (x-1)y$

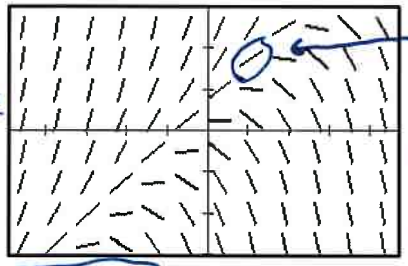
(E) $\frac{dy}{dx} = x(y-1)$

when $x=1$ slope should be 0

when $y=1$, slope should be 0

As x increases, the slope becomes steeper.

3.



All Q2 are positive slopes

(1, 2) has positive slope B, C

(A) $\frac{dy}{dx} = y - x$

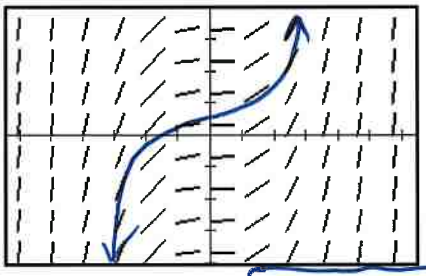
(B) $\frac{dy}{dx} = -\frac{x}{y}$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = y(x-1)$

(E) $\frac{dy}{dx} = x(y-1)$

4.



(A) $y = \sin x$

(B) $y = \cos x$

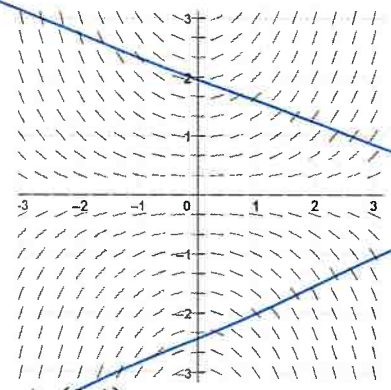
(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{4}x^4$

For each slope field, plot and label the points A and B and sketch the particular solution that passes through each of those points. (Two separate solutions for each slope field.)

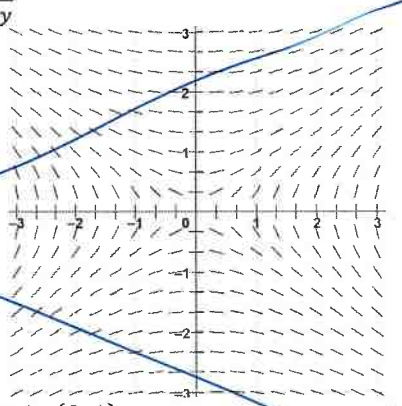
5. $\frac{dy}{dx} = \frac{xy}{2}$



Point A: $(0, 1)$

Point B: $(-2, -1)$

6. $\frac{dy}{dx} = \frac{x}{2y}$



Point A: $(0, 1)$

Point B: $(-2, 0)$

2

Calculus

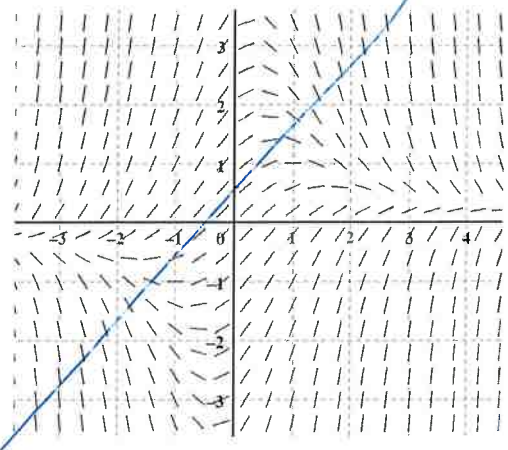
7.4 Reasoning Using Slope Fields

Notes

Write your questions and thoughts here!

Identify the particular solution that goes through a point.

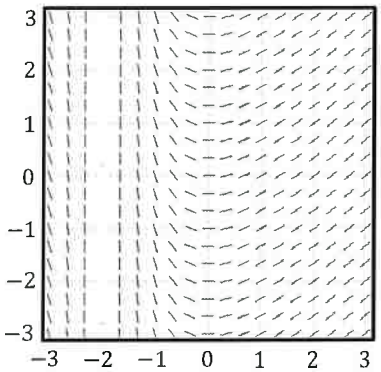
1. The figure to the right shows the slope for the differential equation $\frac{dy}{dx} = 1 - xy$.
   a. Sketch the graph of a particular solution that contains $(0, 2)$. Label this point as Point A.
   b. Sketch the graph of a particular solution that contains $(-1, -2)$. Label this point as Point B.



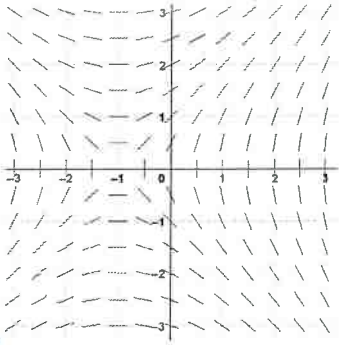
2. The slope field for a certain differential equation is shown to the right. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 0$?

   (A) $y = \frac{x}{x^2-4}$ (C) $y = e^{x+2}$

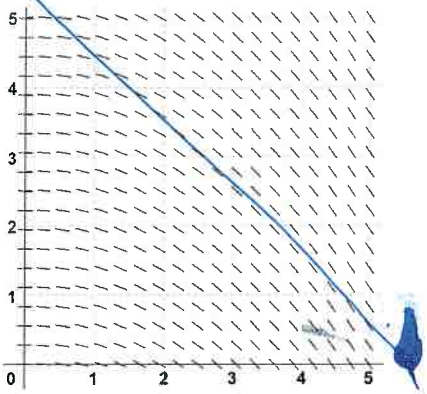
   (B) $y = \frac{\tan x}{2+x}$ (D) $y = \frac{x^2}{2+x}$



3. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ and its slope field shown. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.



4. Explain why the following could not be a slope field for the differential equation $\frac{dy}{dt} = -0.3y$

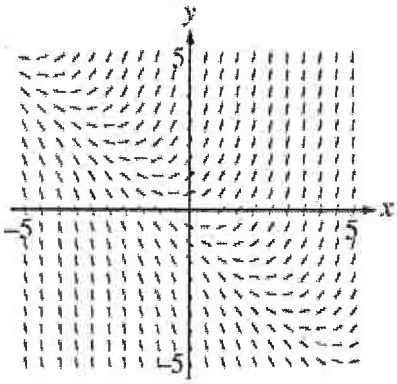


3

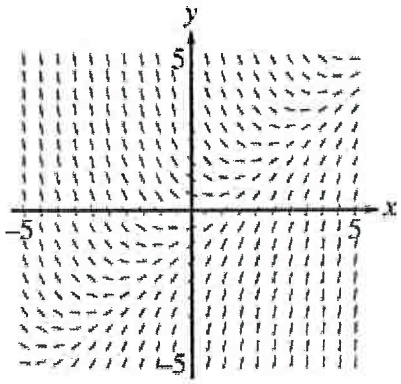
3. (calculator not allowed)

Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

(A)

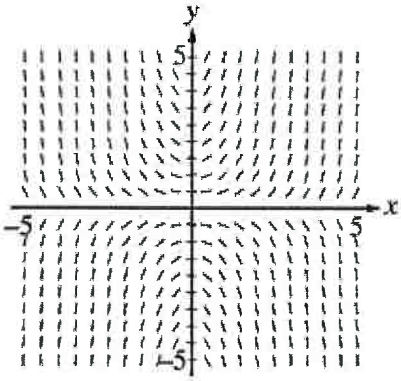


(B)

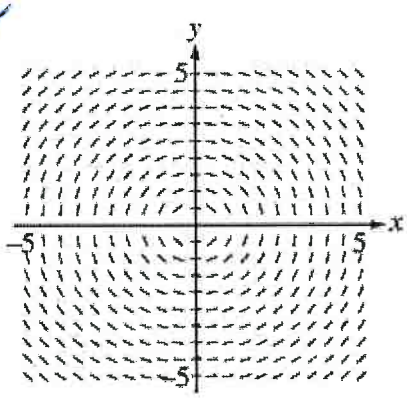


*Q1 should have all positive slopes

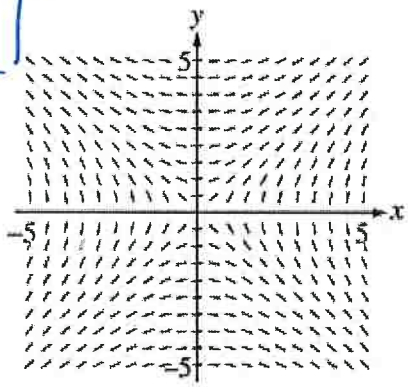
(C)



(D)



(E)



* when x and y are the same values, the slope is always 1

Ex: $(1,1) \rightarrow \frac{dy}{dx} = 1$

$(2,2) \rightarrow \frac{dy}{dx} = 1$

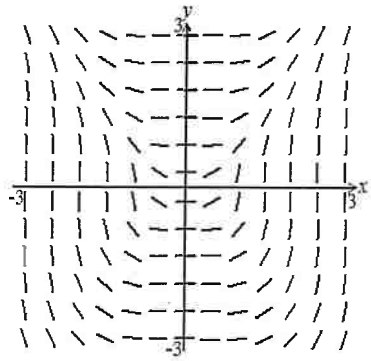
$(3,3) \rightarrow \frac{dy}{dx} = 1$

$(-1,-1) \rightarrow \frac{dy}{dx} = 1$

$(-2,-2) \rightarrow$

4

4. (calculator not allowed)



Shown above is a slope field for which of the following differential equations?

*Q1, Q4 positive slopes (4, 4), (+4, -4)
Q2, Q3 negative slopes (-4, 4)

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

5. (calculator not allowed)

Which of the following is the solution to the differential equation $\frac{dy}{dx} = 3\cos(2x)$ with the initial condition $y\left(\frac{\pi}{2}\right) = 2$?

(A) $y = -\frac{3}{2}\sin(2x) + 2$

(B) $y = \frac{3}{2}\sin(2x) + 2$

(C) $y = \frac{3}{2}\sin(2x) - 2$

(D) $y = 3\sin(2x) + 5$

$\frac{dy}{dx} = \frac{3\cos(2x)}{1}$

$dy = 3\cos(2x)\,dx$

$\int 1\,dy = \int 3\cos(2x)\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int 3\cos u \cdot \frac{du}{2}$

$\frac{3}{2}\int \cos u\,du$

$y = \frac{3}{2}\sin u + C$

$y = \frac{3}{2}\sin(2x) + C$ $\left(\frac{\pi}{2}, 2\right)$

$2 = \frac{3}{2}\sin\left(2 \cdot \frac{\pi}{2}\right) + C$

$2 = 0 + C$

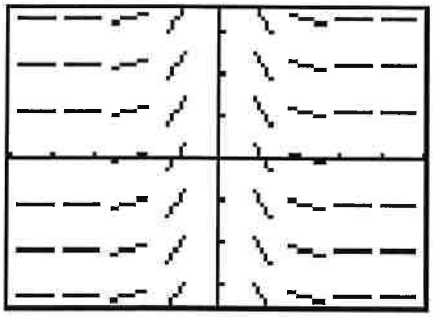
$2 = C$

$y = \frac{3}{2}\sin(2x) + 2$

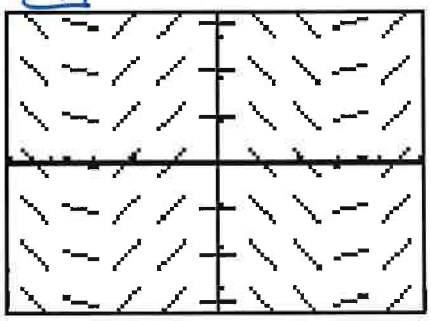
5

For 7 – 12, match each slope field with the **equation** that the slope field could represent.

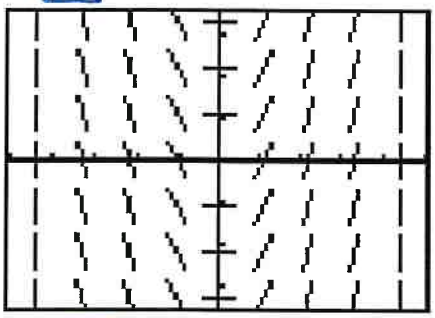
7. E



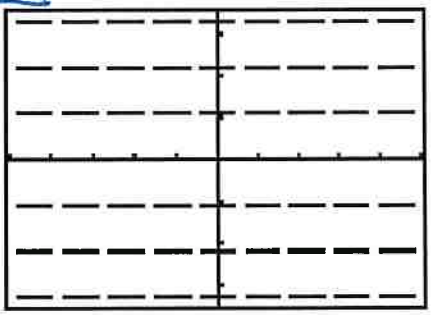
8. G



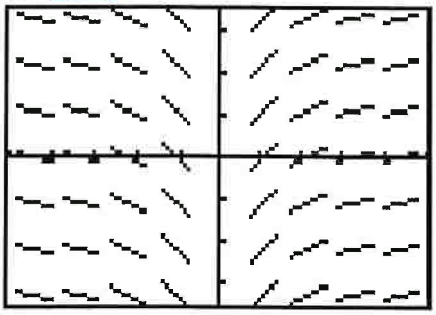
9. C



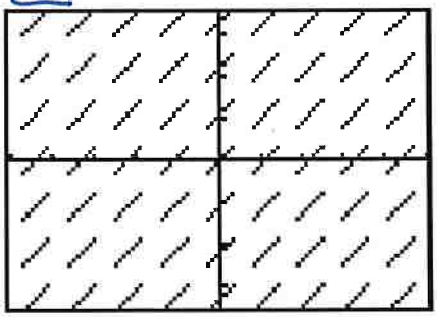
10. A



11. H



12. B



(A) $y = 1$

(B) $y = x$

(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$ (Not used)

(E) $y = \frac{1}{x^2}$

(F) $y = \sin x$ (Not used)

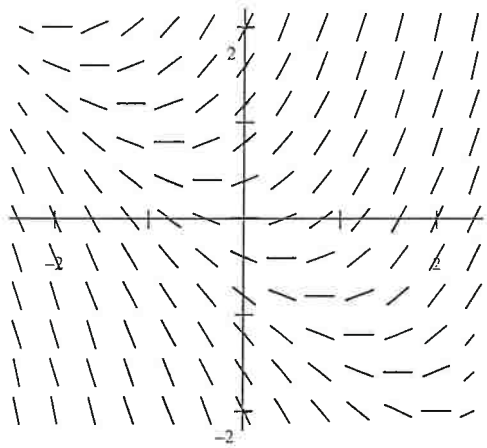
(G) $y = \cos x$

(H) $y = \ln|x|$

6

10.
1998 BC 24

Q1 positive slope
Q3 negative slope



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ $\boxed{\text{(C) } \frac{dy}{dx} = x + y}$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

when $x = -1$, the slope should all be $\frac{dy}{dx} = 0$

* when x and y have equal but opposite signs, slope = 0

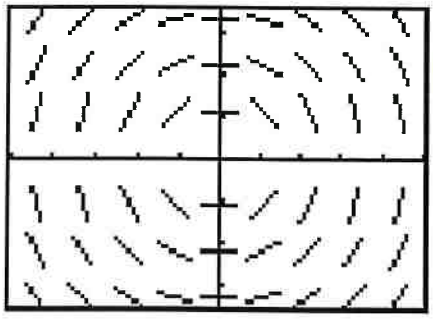
$(-1, 1)$
$(2, -2)$
$(3, -3)$
$(5, -5)$
$(-4, 4)$
$\frac{dy}{dx} = 0$

when $y = 0$ (x-axis), slope should be undefined if this is the correct differential equation

7

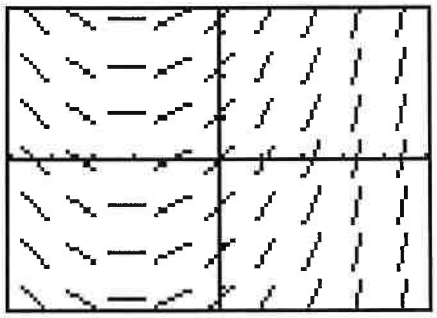
For 13 – 16, match the slope fields with their differential equations.

13.



D

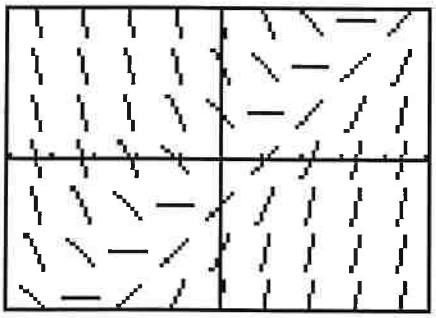
14.



A

slope 0 at $x=-2$

15.

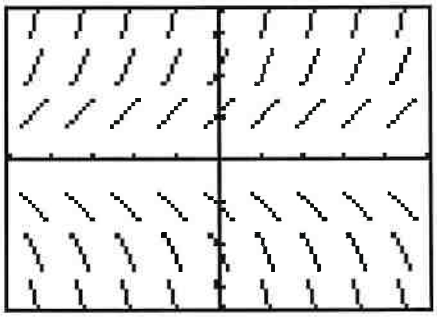


B

zero slope when $x=y$

Ex: $(1,1)$ $(-3,-3)$ $(2,2)$ $(3,3)$ $(-1,-1)$

16.



C

positive slopes when $y>0$. neg. slopes when $y<0$

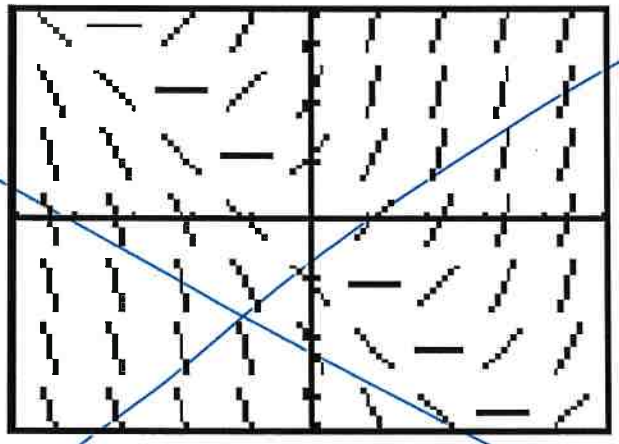
(A) $\frac{dy}{dx} = \frac{1}{2}x + 1$ → Graph #14

(B) $\frac{dy}{dx} = x - y$ → Graph #15

(C) $\frac{dy}{dx} = y$ → Graph #16

(D) $\frac{dy}{dx} = -\frac{x}{y}$ → Graph #13

17. The calculator-drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.



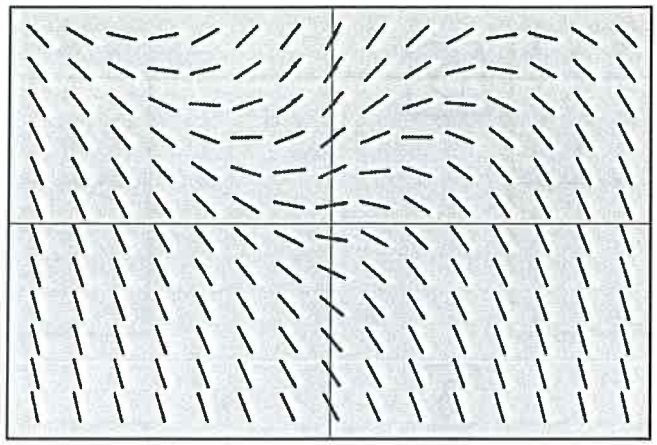
(a) Sketch the solution curve through the point $(0,1)$.

(b) Sketch the solution curve through the point $(-3,0)$.

(c) Approximate $y(-3.1)$ using the equation of the tangent line to $y = f(x)$ at the point $(-3,0)$.

8

22. Which of the following differential equations would produce the slope field shown below?



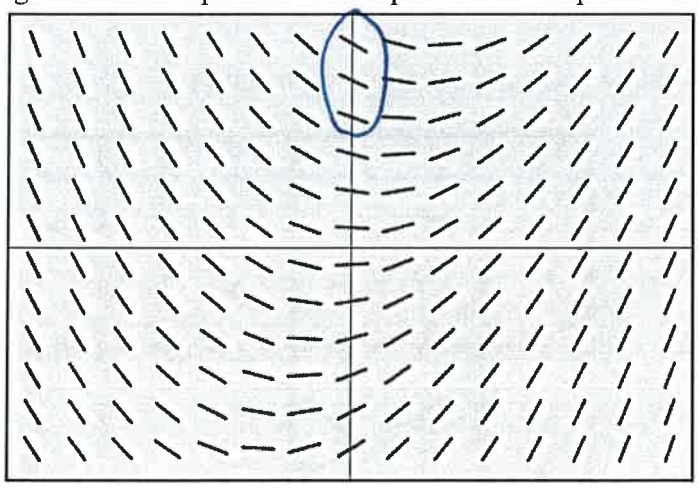
[–3, 3] by [–1.98, 1.98]

(A) $\frac{dy}{dx} = y - |x|$ (B) $\frac{dy}{dx} = |y| - x$ (C) $\frac{dy}{dx} = |y - x|$ (D) $\frac{dy}{dx} = |y + x|$ (E) $\frac{dy}{dx} = |y| - |x|$

$\frac{dy}{dx} < 0$ when $y < 0$

(Q3 and Q4 always negative slopes)

23. Which of the following differential equations would produce the slope field shown below?



*At points (0,4), (0,5), (0,6) they are clearly negative slopes

(A) $\frac{dy}{dx} = y - 3x$ (B) $\frac{dy}{dx} = y - \frac{x}{3}$ (C) $\frac{dy}{dx} = y + \frac{x}{3}$ (D) $\frac{dy}{dx} = x + \frac{y}{3}$ (E) $\frac{dy}{dx} = x - \frac{y}{3}$