Slope Fields: Additional Notes

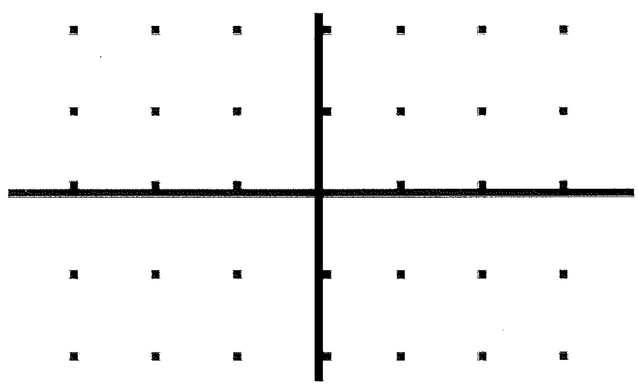
II. Finding Domain for Differential Equations

The domain of the solution to a differential equation is the largest open interval containing the initial value given for which both the differential equation and the solution are defined. This is called the "maximum interval of existence"

1. Solve the differential equation, and find the specific solution
2. Find the initial domain of the specific solution
3. Further restrict domain by:
    a. Finding domain of differential equation
    b. Be sure the initial condition is included in the domain
4. Graph solution and use slope fields to help determine/confirm domain
5. Incorporate restrictions to find the final domain

** Domain can only be ONE open interval that includes the initial value

1. Given $\frac{dy}{dx} = \frac{1}{x^2y}$, and y(-1) = -2   a) Find the particular solution   b) Find the domain.
c) Find the equation of tangent line through (-1,-2)   d) Use it to approximate y(-1.1)



2. Let $P(t)$ represent the number of wolves in a population at time t in years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

a) If P(0) = 500, find P(t) in terms of t and k.
b) If P(2) = 700, find k.
c) Find $\lim\limits_{t \to \infty} P(t)$

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1c) find equation tangent line

point $(-1,-2)$

slope $= \frac{dy}{dx}_{(-1,-2)} = \frac{1}{x^2 y}$

$\frac{dy}{dx} = \frac{1}{(-1)^2(-2)} = \frac{-1}{2}$ $m = \frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y + 2 = \frac{-1}{2}(x+1)$

$\boxed{y = \frac{-1}{2}(x+1) - 2}$

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d) Approximate $y(-1.1)$

$y(-1.1) = \frac{-1}{2}(-1.1+1) - 2$

$\boxed{y(-1.1) = -1.95}$

$\int y\,dy = \int \frac{dx}{x^2} = \int x^{-2}\,dx$

$\frac{y^2}{2} = \frac{x^{-1}}{-1} + C$

$y^2 = 2(-x^{-1} + C)$

$(-2)^2 = 2(-(-1)^{-1} + C)$

$4 = 2(1 + C)$

$2 = 1 + C$

$1 = C$

$y^2 = 2(-x^{-1} + 1)$

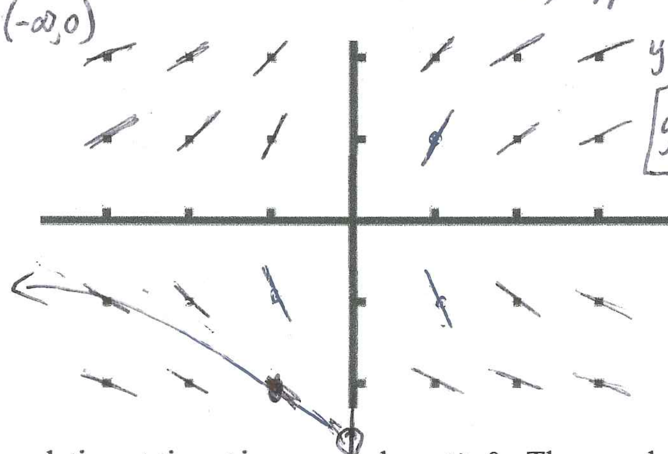
$y^2 = \frac{-2}{x} + 2$

$y = \pm\sqrt{\frac{-2}{x} + 2}$

*choose equation that satisfy $(-1,-2)$

$y = -\sqrt{\frac{-2}{x} + 2}$

Domain: $(-\infty, 0)$



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(time, population)

$(0, 500)$

$(2, 700)$

$P' = k(800 - P)$

$\frac{dP}{dt} = k(800 - P)$

$\int \frac{dP}{800-P} = \int k\,dt$

$u = 800 - P$

$\frac{du}{dP} = -1$ $dP = -du$

$\int \frac{-du}{u} = \int k\,dt$

$-\ln|u| = kt + C$

$-\ln|800 - P| = kt + C$

$\ln|800 - P| = -kt + C$

e e

$800 - P = Ce^{-kt}$

$-P = Ce^{-kt} - 800$

$-500 = Ce^{-k(0)} - 800$

$300 = Ce^{-k(0)}$

$300 = C$

$-P = 300e^{-kt} - 800$

$-700 = 300e^{-kt} - 800$

$100 = 300e^{-k(2)}$

$\frac{1}{3} = e^{-k(2)}$

$\ln \frac{1}{3} = \ln e^{-k(2)}$

$\ln \frac{1}{3} = -2k$

$\boxed{-\frac{1}{2}\ln\frac{1}{3} = k}$

$P(t) = -300e^{\frac{1}{2}\ln(\frac{1}{3})t} + 800$

$\lim_{t\to\infty} -300e^{\frac{1}{2}\ln(\frac{1}{3})(t)} + 800$

$= -300e^{\frac{1}{2}(\ln(\frac{1}{3})(-\infty))} + 800$

$= -300(0) + 800$

$\lim_{t\to\infty} P(t) = \boxed{800}$