

## Differential Equations Unit

## Slope Fields (Direction Fields) Notes

Slope Fields: a graphical approach to look at all the solutions of a differential equation. Slope fields consists of short line segments representing slope (steepness) sketched at lots of different points

These line segments are the tangents to a family of solution curves for the differential equation at various points. The tangents show the direction in which the solution curves will follow. Slope fields are useful in sketching solution curves without having to solve a differential equation algebraically.

Steps:

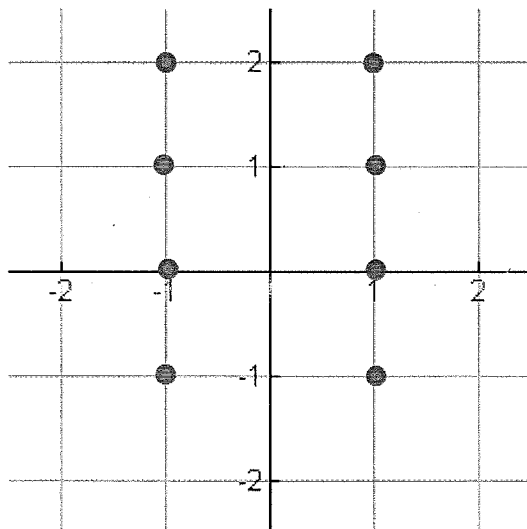
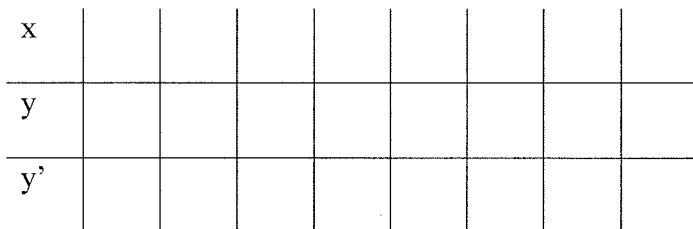
- 1) Identify the ordered pairs indicated on the graph.
- 2) Plug in the ordered pairs in the differential equation to find slope
- 3) Sketch a short line segment representing the slope through the given point
- 4) Repeat this for all remaining ordered pairs.

\*Use the differential equation to find the individual slope segments, creating the slope field (ex:  $\frac{dy}{dx} = x$ )

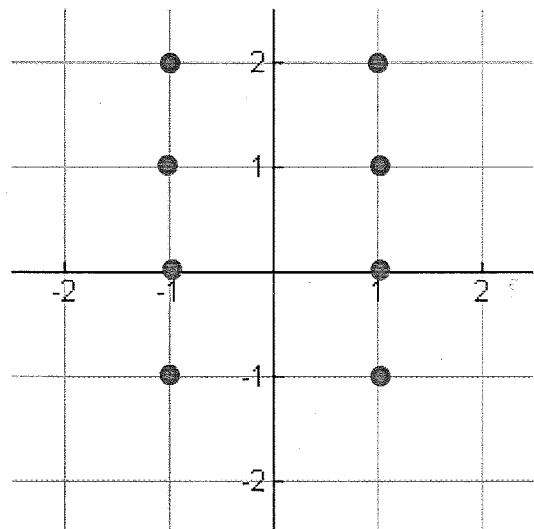
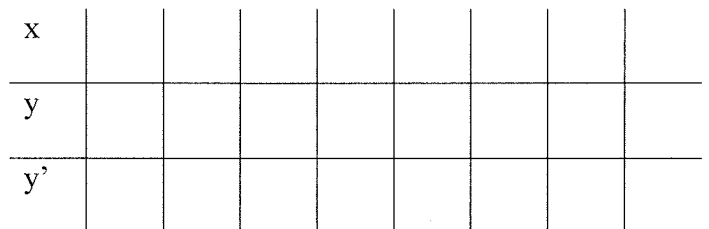
\*Use the solution of the differential equation to match with the shape of the slope field: (ex:  $y = \frac{1}{2}x^2 + C$ )

**Example 1:** Sketch a slope field for the given differential equation at the indicated eight points.

a)  $\frac{dy}{dx} = x - 2y$

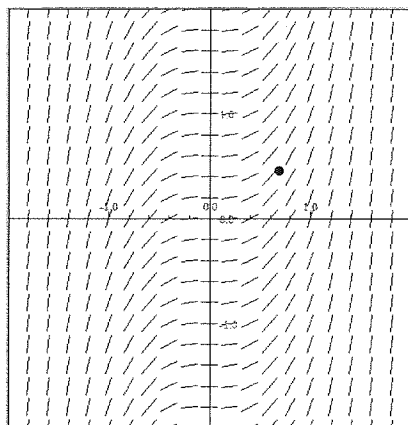


b)  $\frac{dy}{dx} = \frac{2-x}{y}$

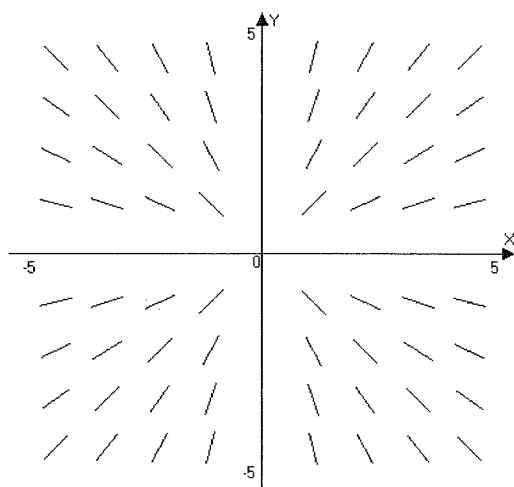


Determine the differential equation being graphed by each of the slope fields below. Then sketch a solution that passes through the indicated point.

2. a)  $\frac{dy}{dx} = x^3$   
 b)  $\frac{dy}{dx} = 3x^2$   
 c)  $\frac{dy}{dx} = 2x + y$   
 d)  $\frac{dy}{dx} = \frac{x}{y}$   
 e)  $\frac{dy}{dx} = \ln x$



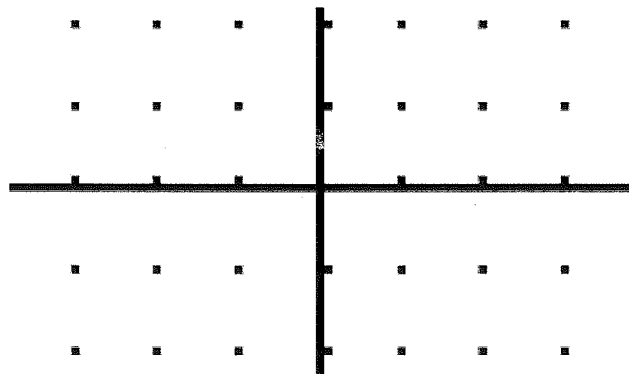
3. a)  $\frac{dy}{dx} = x - 2$   
 b)  $\frac{dy}{dx} = x^3$   
 c)  $\frac{dy}{dx} = x - y$   
 d)  $\frac{dy}{dx} = \frac{y}{x}$   
 e)  $\frac{dy}{dx} = e^y$



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Sketch slope fields for the following differential equation. Then find the general solution analytically

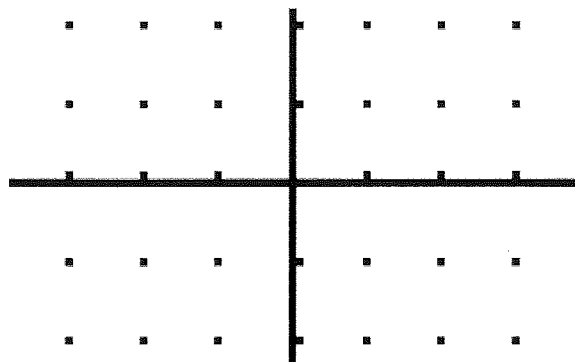
4.  $\frac{dy}{dx} = \frac{-x}{y}$



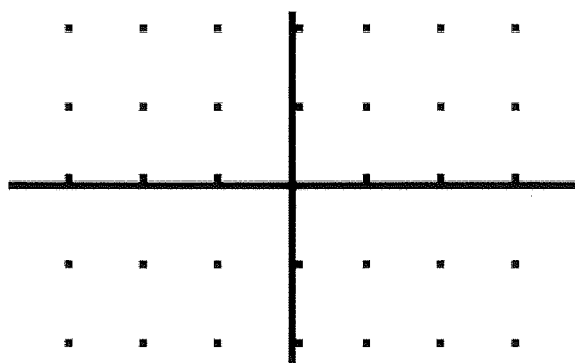
### Slope Fields Homework

Sketch slope fields for each of the following differential equations. Then find the general solution analytically. For problems 4 through 6, find the general solution first, then the specific solution that passes through the given point and state the domain of that solution

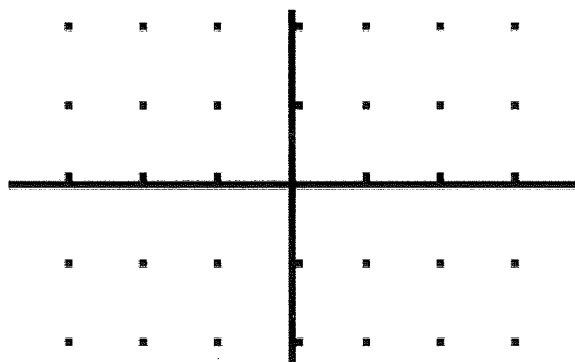
1.  $\frac{dy}{dx} = 4 - y$



2.  $\frac{dy}{dx} = 3x^2$

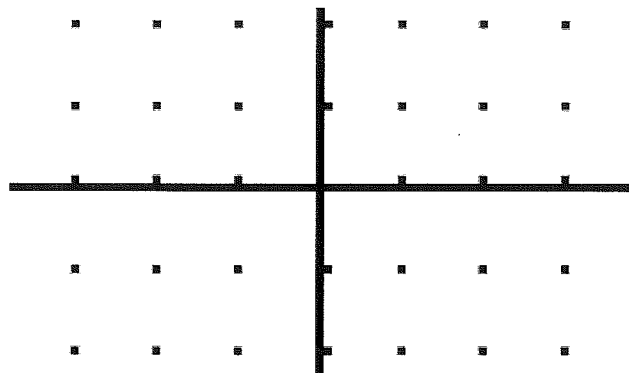


3.  $\frac{dy}{dx} = \frac{-x}{y}$



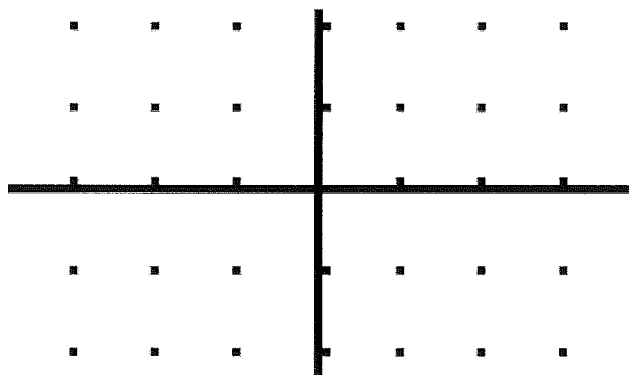
4.  $\frac{dy}{dx} = \frac{1}{y}$

point (1, 3)



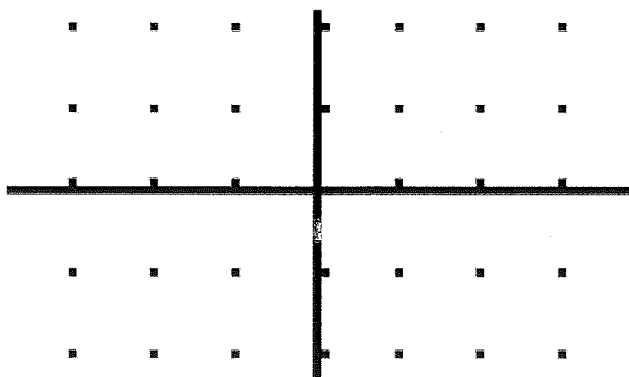
5.  $\frac{dy}{dx} = \frac{xy}{8}$

point (0, -2)



6.  $\frac{dy}{dx} = \frac{y}{x}$

point (3, -6)



Slope Fields: a graphical approach to look at all the solutions of a differential equation. Slope fields consists of short line segments representing slope (steepness) sketched at lots of different points

It consists of a set of short line segments drawn on a pair of axes. These line segments are the tangents to a family of solution curves for the differential equation at various points. The tangents show the direction in which the solution curves will follow. Slope fields are useful in sketching solution curves without having to solve a differential equation algebraically.

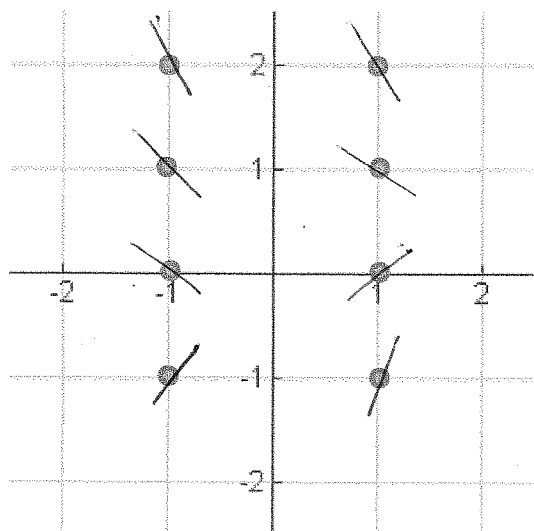
Steps:

- 1) Identify the ordered pairs indicated on the graph.
- 2) Plug in the ordered pairs in the differential equation to find slope
- 3) Sketch a short line segment representing the slope through the given point
- 4) Repeat this for all remaining ordered pairs.

**Example 1:** Sketch a slope field for the given differential equation at the indicated eight points.

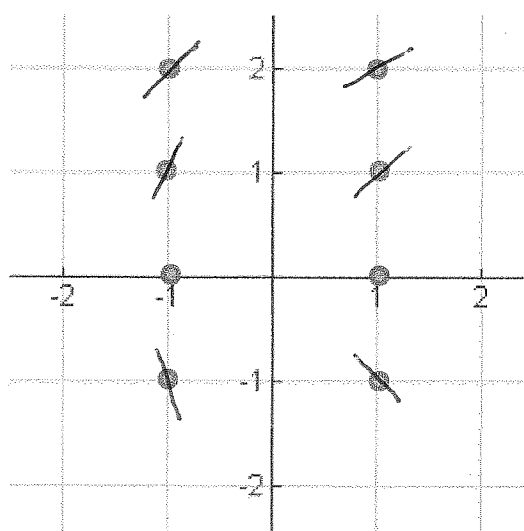
a)  $\frac{dy}{dx} = x - 2y$

x	-1	-1	-1	-1	1	1	1	1
y	2	1	0	-1	2	1	0	-1
y'	-5	-3	-1	1	-3	-1	1	3



b)  $\frac{dy}{dx} = \frac{2-x}{y}$

x	-1	-1	-1	-1	1	1	1	1
y	2	1	0	-1	2	1	0	-1
y'	3/2	3	und.	-3	1/2	1	und.	-1



Determine the differential equation being graphed by each of the slope fields below. Then sketch a solution that passes through the indicated point.

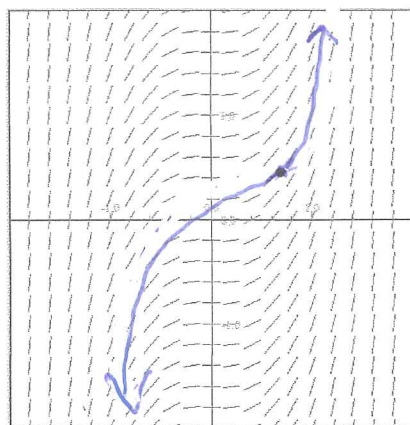
a)  $\frac{dy}{dx} = x^3$

b)  $\frac{dy}{dx} = 3x^2$  *All slopes are positive except y-axis*

c)  $\frac{dy}{dx} = 2x + y$

d)  $\frac{dy}{dx} = \frac{x}{y}$

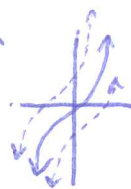
e)  $\frac{dy}{dx} = \ln x$



$$\frac{dy}{dx} = 3x^2 \quad \int dy = \int 3x^2 dx$$

$$y = \frac{3x^3}{3} + C$$

$$y = x^3 + C$$



\*The shape of the slope field graph resembles the family of functions of the general solution above,  $y = x^3 + C$

$x=0$

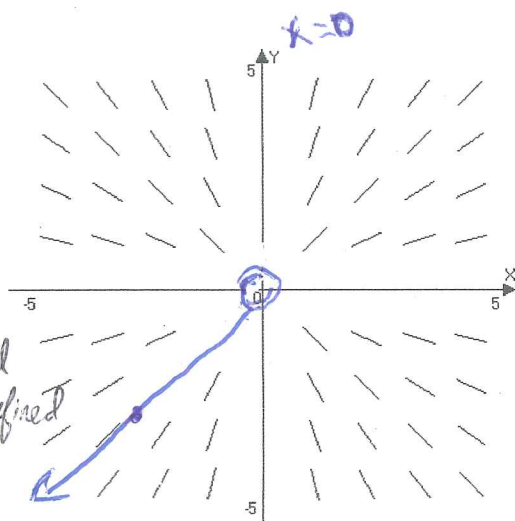
3. a)  $\frac{dy}{dx} = x - 2$

b)  $\frac{dy}{dx} = x^3$

c)  $\frac{dy}{dx} = x - y$

d)  $\frac{dy}{dx} = \frac{y}{x}$  *y-axis ordered pairs are undefined and x-axis has slope = 0*

e)  $\frac{dy}{dx} = e^y$



$$\frac{dy}{dx} = \frac{y}{x} \quad \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + C = e^{\ln x} \cdot e^C$$

$$y = Cx$$

\*The slope field resembles the linear equation of the general solution  $y = Cx$

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Sketch slope fields for the following differential equation. Then find the general solution analytically

4.  $\frac{dy}{dx} = \frac{-x}{y}$

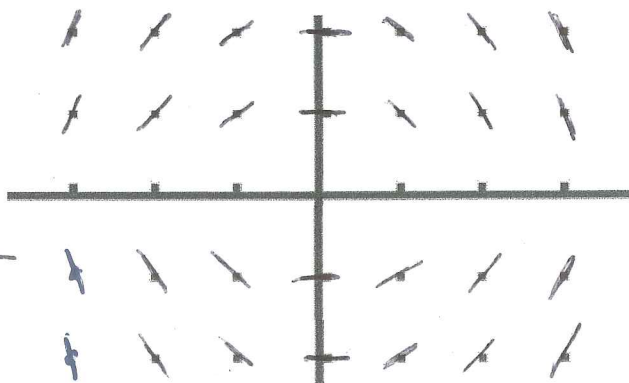
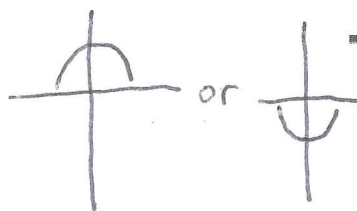
$$\int y dy = \int -x dx$$

$$y = \pm \sqrt{-x^2 + C_1}$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$y^2 = 2\left(\frac{-x^2}{2}\right) + 2C$$

$$y^2 = -x^2 + 2C$$



## Slope Fields Homework

Sketch slope fields for each of the following differential equations. Then find the general solution analytically.

For problems 4 through 6, find the general solution first, then the specific solution that passes through the given point and state the domain of that solution

1.  $\frac{dy}{dx} = 4 - y$

$$\int \frac{dy}{4-y} = \int dx$$

$$u = 4 - y$$

$$\frac{du}{dy} = -1$$

$$dy = -1 du$$

$$\int -\frac{du}{u} = \int dx$$

$$-\ln|u| = x + C$$

$$-\ln|4-y| = x + C$$

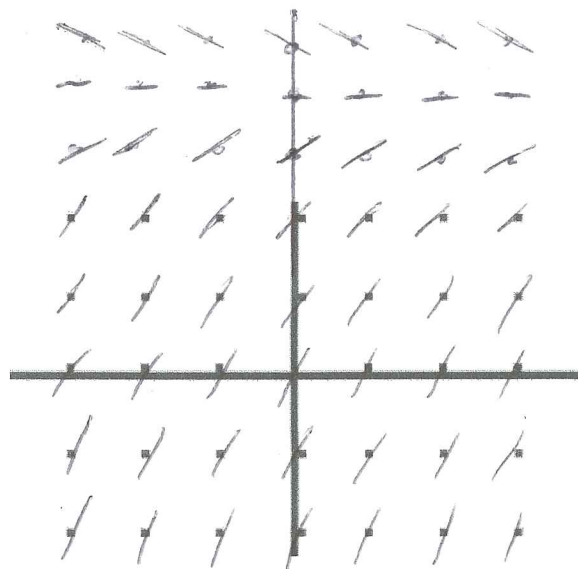
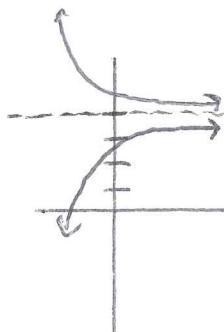
$$\ln|4-y| = -x - C$$

$$e^{\ln|4-y|} = e^{-x-C}$$

$$4-y = Ce^{-x}$$

$$-y = Ce^{-x} - 4$$

$$y = -Ce^{-x} + 4$$

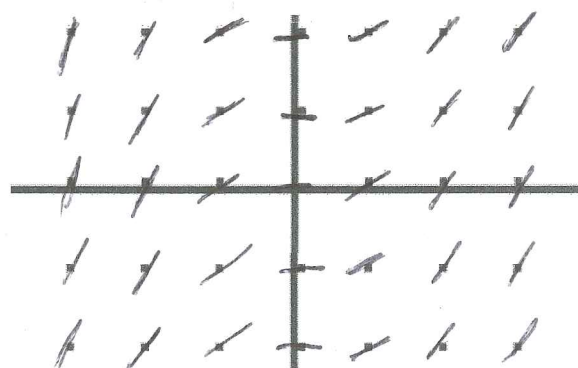
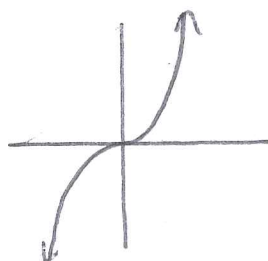


2.  $\frac{dy}{dx} = 3x^2$

$$\int dy = \int 3x^2 dx$$

$$y = \frac{3x^3}{3} + C$$

$$y = x^3 + C$$



3.  $\frac{dy}{dx} = \frac{-x}{y}$

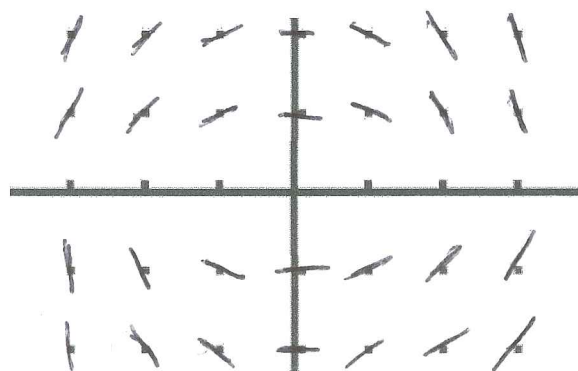
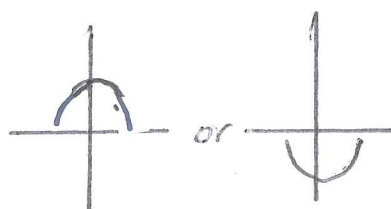
$$\int y dy = \int -x dx$$

$$y = \pm \sqrt{-x^2 + C}$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$y^2 = -x^2 + 2C$$



4.  $\frac{dy}{dx} = \frac{1}{y}$

point (1, 3)

$$\int y dy = \int dx$$

$$\frac{y^2}{2} = x + c$$

$$\frac{9}{2} = 1 + c$$

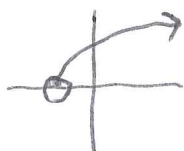
$$\frac{7}{2} = c$$

$$\frac{y^2}{2} = x + \frac{7}{2}$$

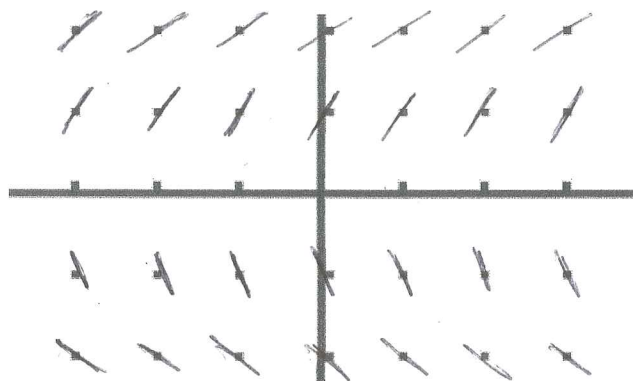
$$y^2 = 2x + 7$$

$$y = \pm \sqrt{2x + 7}$$

$$y = \sqrt{2x + 7}$$



Domain:  $(-\frac{7}{2}, \infty)$



5.  $\frac{dy}{dx} = \frac{xy}{8}$

point (0, -2)

$$\int \frac{dy}{y} = \int \frac{1}{8} x dx$$

$$-2 = C e^{9/16} \quad -2 = C$$

$$y = -2 e^{x^2/16}$$

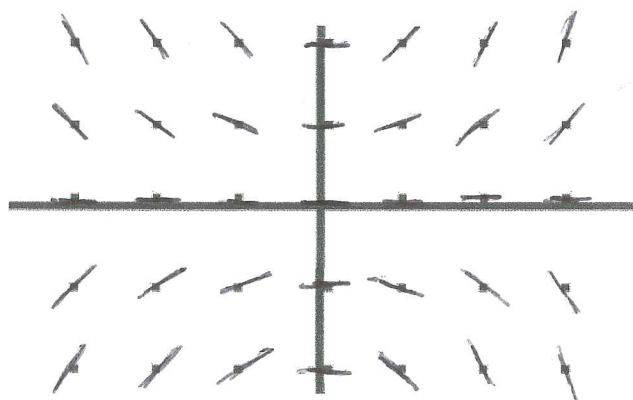
$$\ln|y| = \frac{1}{8} \left( \frac{x^2}{2} \right) + c$$

$$\ln|y| = \frac{x^2}{16} + c$$

$$y = C e^{x^2/16}$$



Domain:  $(-\infty, \infty)$



6.  $\frac{dy}{dx} = \frac{y}{x}$

point (3, -6)

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + c \quad = e^{\ln|x|} \cdot e^c$$

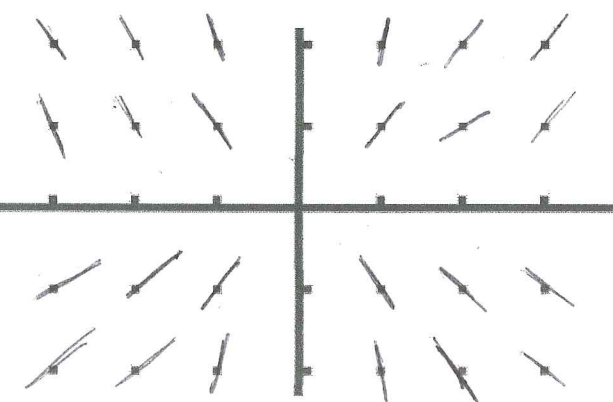
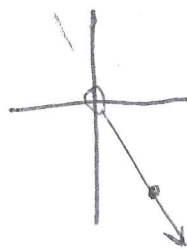
$$y = C e^{\ln x}$$

$$y = Cx$$

$$-6 = C(3)$$

$$-2 = C$$

$$y = -2x$$



Domain:  $(0, \infty)$