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2nd

Key

Solving Differential Equations Task (part 2)

1)

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\boxed{\frac{dB}{dt} = \frac{1}{5}(100 - B).}$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100-B)$$

$$\frac{dB}{dt} = \frac{(100-B)}{5}$$

$$\frac{5dB}{5} = \frac{(100-B)dt}{100-B}$$

$$\int \frac{dB}{100-B} = \int \frac{dt}{5} \rightarrow \frac{1}{5}\int 1 dt$$

$u = 100-B$

$\frac{du}{dB} = -1$

$dB = -1du$

$$\int \frac{-1du}{u}$$

$$-\ln|u| \rightarrow -\ln|100-B|$$

$$\left(-\ln|100-B| = \frac{1}{5}t + C\right)$$

$$\ln|100-B| = \frac{-1}{5}t + C$$

$e \qquad e$

$$|100-B| = e^{\frac{-1}{5}t} \cdot \boxed{e^C} \leftarrow C$$

$$|100-B| = Ce^{\frac{-1}{5}t}$$

$$100-B = \pm Ce^{\frac{-1}{5}t}$$

$$100-B = Ce^{-\frac{1}{5}t}$$

$$100 - Ce^{\frac{-1}{5}t} = B$$

$$B = 100 - Ce^{\frac{-1}{5}t}$$

plug in $(0, 20)$ (t, B)

$$20 = 100 - Ce^{\frac{-1}{5}(0)}$$

$$-80 = -C$$

$$C = 80$$

$$\boxed{B = 100 - 80e^{\frac{-1}{5}t}}$$

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Solve the below differential equation:

2) $y' - xycos(x^2) = 0$ given $y(0) = e$ a) Find general solution b) Find particular solution

$$y' - xy\cos(x^2) = 0$$

$$\frac{dy}{dx} - xy\cos(x^2) = 0$$

$$\frac{dy}{dx} = \frac{xy\cos(x^2)}{1}$$

$$\frac{1dy}{y} = \frac{xy\cos(x^2)dx}{y}$$

$$\frac{dy}{y} = x\cos(x^2)dx$$

$$\int \frac{1}{y}dy = \int x\cos(x^2)dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$2xdx = du$

$dx = \frac{du}{2x}$

$\int \cancel{x}\cos u \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \cos u du$

$$\int \frac{1}{y}dy = \frac{1}{2}\int \cos u du$$

$$\ln|y| = \frac{1}{2}\sin(x^2) + C$$

$e \qquad e$

$a^{m+n} = a^m \cdot a^n$

$$|y| = e^{\frac{1}{2}\sin(x^2)} \cdot \boxed{e^c} \leftarrow C$$

$$y = \pm Ce^{\frac{1}{2}\sin(x^2)} \leftarrow \text{plug in } (0, e)$$

$$e = Ce^{\frac{1}{2}\sin(0)^2}$$

$$e = C(1)$$

$$C = e$$

$$\boxed{y = (e)e^{\frac{1}{2}\sin(x^2)} \text{ or } y = e^{1+\frac{1}{2}\sin(x^2)}}$$

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Solving Differential Equations: Additional Practice Problems

3)Solve the Differential Equation: $y\ln x^4 - xy' = 0$

$$y\ln x^4 - xy' = 0$$

$$y\ln x^4 - x\left(\frac{dy}{dx}\right) = 0$$

$$\frac{y\ln x^4}{1} = \frac{x\,dy}{dx}$$

$$x\,dy = y\ln x^4\,dx$$

$$\int \frac{dy}{y} = \int \frac{\ln x^4}{x}\,dx$$

u-sub

$$\int \frac{1}{y}\,dy = \int \frac{4\ln x}{x}\,dx$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$4\int \frac{u}{\cancel{x}}\cancel{x}\,du$

$$\int \frac{1}{y}\,dy = 4\int u\,du$$

$$\ln|y| = 4\cdot\frac{u^2}{2} + C$$

$$\ln|y| = 2(\ln x)^2 + C$$

e e

$a^{m+n} = a^m \cdot a^n$

$$|y| = e^{2(\ln x)^2}\cdot\boxed{e^c} \quad \pm C$$

$$\boxed{y = \pm C e^{2(\ln x)^2}}$$

$$y = Ce^{2(\ln x)^2}$$

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4) a) Find the general solution b) Find the particular solution

$yy' - 2e^{3x} = 0$ $\quad y(0) = 5$

$$yy' - 2e^{3x} = 0$$

$$y\frac{dy}{dx} - 2e^{3x} = 0$$

$$y\frac{dy}{dx} = \frac{2e^{3x}}{1}$$

$$y\,dy = 2e^{3x}\,dx$$

$$\int y\,dy = \int 2e^{3x}\,dx$$

$u = 3x$ | $3dx = du$

$\frac{du}{dx} = 3$ | $dx = \frac{du}{3}$

$$2\int e^{u}\cdot\frac{du}{3}$$

$$\int y\,dy = 2\cdot\frac{1}{3}\int e^{u}\,du$$

$$\frac{y^2}{2} = \frac{2}{3}e^{3x} + C$$

$$2\left(\frac{y^2}{2} = \frac{2}{3}e^{3x} + C\right)$$

$$y^2 = \frac{4}{3}e^{3x} + C$$

$$5^2 = \frac{4}{3}e^{3(0)} + C$$

$$25 = \frac{4}{3} + C$$

$$25 - \frac{4}{3} = C$$

$$\frac{71}{3} = C$$

$$y^2 = \frac{4}{3}e^{3x} + \frac{71}{3}$$

$$\sqrt{y^2} = \pm\sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}}$$

$$\boxed{y = \sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}}}$$

or

$$\cancel{y = -\sqrt{\frac{4}{3}e^{3x} + \frac{71}{3}}}$$