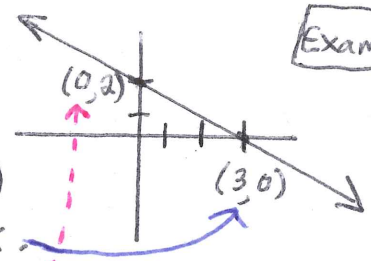
Summer Review Problems:



Example:

1) Find x, y-intercepts

(in numerator)

* x-intercepts: set $y=0$ and solve for x.
* y-intercepts: set $x=0$ and solve for y.

a) $y = x^2 + x - 1$

$0 = x^2 + x - 1$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$

x-ints: $\left(\frac{-1 \pm \sqrt{5}}{2}, 0\right)$

$y = 0^2 + 0 - 1$

$y = -1$

y-int: $(0, -1)$

b) $y = (x-1)\sqrt{9 - x^2}$

$0 = (x-1)\sqrt{(3-x)(3+x)}$

$x = 1, 3, -3$

x-ints: $(1,0), (3,0), (-3,0)$

$y = (0-1)\sqrt{9 - 0^2}$

$= -1\sqrt{9}$

$= -1(3)$

$y = -3$

y-int: $(0, -3)$

c) $y = \frac{x^2 + 3x}{(3x+1)^2}$

$0 = x^2 + 3x$

$0 = x(x+3)$

$x = 0, -3$

x-ints: $(0,0), (-3,0)$

$y = \frac{0^2 + 3(0)}{(3(0)+1)^2} = \frac{0}{1} = 0$

y-int: $(0,0)$

d) $x^2 y - x^2 + 4y = 0$

$x^2(0) - x^2 + 4(0) = 0$

$x^2 = 0$

x-int: $(0,0)$

$0^2 y - (0)^2 + 4y = 0$

$4y = 0$

y-int: $(0,0)$

Find points of intersection:

2) * Use elimination method or substitution method to solve systems of equations

a) $2x-3y=13$
$5x+3y=1$
$7x+0=14$

$7x=14$
$x=2$

$2(2)-3y=13$
$4-3y=13$
$-3y=9$
$y=-3$

$\boxed{(2,-3)}$

b) $x^2+y^2=25$ and $2x+y=10$

$y=10-2x$

$x^2+(10-2x)^2=25$
$x^2+100-40x+4x^2=25$
$5x^2-40x+75=0$

$5(x^2-8x+15)=0$
$5(x-5)(x-3)=0$
$x=3,5$

$2x+y=10$
$2(3)+y=10$
$y=4$

$2(5)+y=10$
$y=0$

$\boxed{(3,4) \text{ and } (5,0)}$

c) $y=x^3-4x$ and $y=-x$

$-x=x^3-4x$
$0=x^3-3x$
$0=x(x^2-3)$
$x=0,\sqrt{3},-\sqrt{3}$

$\boxed{(0,0) \quad (\sqrt{3},-\sqrt{3}) \quad (-\sqrt{3},\sqrt{3})}$

d) $y=x^4-2x^2+1$ and $y=1-x^2$

$1-x^2=x^4-2x^2+1$
$0=x^4-x^2$
$0=x^2(x^2-1)$
$0=x^2(x+1)(x-1)$
$x=0,-1,1$

$y=1-x^2$
$y=1-(0)^2=1$
$y=1-(-1)^2=0$
$y=1-(1)^2=0$

$\boxed{(0,1) \quad (-1,0) \quad (1,0)}$

3) Write equation of line with the following characteristics:

a) passes through $(3,-4)$ and $(5,2)$

Steps:
1) find slope
2) write equation using point-slope $y-y_1=m(x-x_1)$

$m=\frac{-4-2}{3-5}=\frac{-6}{-2}=3$

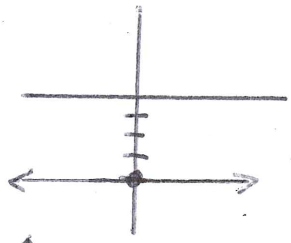
$m=3$

$y-y_1=m(x-x_1)$

$\boxed{y-2=3(x-5)}$

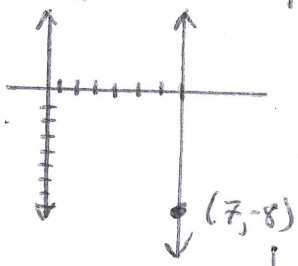
b) is a horizontal line with y-intercept at -4

$\boxed{y=-4}$



c) is a vertical line passing through $(7,-8)$

$\boxed{x=7}$



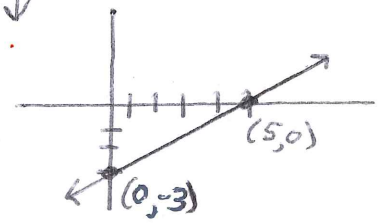
d) has an x-intercept at 5 and y-intercept at -3

* find slope and then write equation

$m=\frac{-3-0}{0-5}=\frac{-3}{-5}=\frac{3}{5}$

$y-y_1=m(x-x_1)$

$\boxed{y-0=\frac{3}{5}(x-5)}$



e) is parallel to $3x+4y=7$, passes thru $(-6,4)$

Steps:
1) find slope of the line
2) write equation of line

$3x+4y=7$

$4y=-3x+7$

$y=\frac{-3}{4}x+\frac{7}{4}$

$m=-3/4$

$y-y_1=m(x-x_1)$

$y-4=-3/4(x+6)$

f) is perpendicular to $5x-3y=0$, passes thru $(\frac{3}{4},\frac{7}{8})$

Steps:
1) find slope of line
2) find new perpendicular slope (opposite reciprocal)
3) write equation of line

$5x-3y=0$

$-3y=-5x$

$y=\frac{5}{3}x$

$m_1=\frac{5}{3}$

$m_2=\frac{-3}{5}$

$\boxed{y-7/8=\frac{-3}{5}(x-3/4)}$

4) For $f(x) = x^2 - 4x$, find the following:

a) $f(4) = (4)^2 - 4(4) = 0$

b) $f(\frac{3}{2}) = (\frac{3}{2})^2 - 4(\frac{3}{2}) = \frac{9}{4} - 6 = \frac{9}{4} - \frac{24}{4} = \frac{-15}{4}$

c) $f(q) = (q)^2 - 4(q) = q^2 - 4q$

d) $f(t+4) = (t+4)^2 - 4(t+4) = t^2 + 8t + 16 - 4t - 16 = t^2 + 4t$

$= (t+4)(t+4) - 4(t+4)$

5) Find value of $\frac{f(x) - f(3)}{x-3}$ for the following:

a) $f(x) = 3x + 7$

$f(3) = 3(3) + 7 = 16$

$\rightarrow \frac{3x + 7 - (16)}{x - 3} = \frac{3x - 9}{x - 3} = \frac{3\cancel{(x-3)}}{\cancel{(x-3)}} = \boxed{3}$

b) $f(x) = 3x^2 - 2x + 1$

$f(3) = 3(3)^2 - 2(3) + 1$

$= 27 - 6 + 1 = 22$

$\rightarrow \frac{3x^2 - 2x + 1 - (22)}{x - 3} = \frac{3x^2 - 2x - 21}{x - 3} = \frac{(3x+7)\cancel{(x-3)}}{\cancel{(x-3)}}$

$= \boxed{3x + 7}$

c) $f(x) = \frac{6}{x}$

$f(3) = \frac{6}{3} = 2$

$\rightarrow \frac{\frac{6}{x} - (2)}{x - 3} = \frac{\left(\frac{6}{x} - 2\right)x}{(x-3)x} = \frac{6 - 2x}{x^2 - 3x} = \frac{2\cancel{(3-x)}^{-1}}{x\cancel{(x-3)}} = \boxed{\frac{-2}{x}}$

6) Find $\frac{f(x+h) - f(x)}{h}$

a) $f(x) = 3x + 7 \rightarrow \frac{3(x+h) + 7 - (3x+7)}{h} = \frac{\cancel{3x} + 3h + \cancel{7} - \cancel{3x} - \cancel{7}}{h} = \frac{3\cancel{h}}{\cancel{h}} = \boxed{3}$

b) $f(x) = 3x^2 - 2x + 1 \rightarrow \frac{3(x+h)^2 - 2(x+h) + 1 - (3x^2 - 2x + 1)}{h} = \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 1 - 3x^2 + 2x - 1}{h}$

$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+ 1} \cancel{- 3x^2} \cancel{+ 2x} \cancel{- 1}}{h} = \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} = \boxed{6x + 3h - 2}$

c) $f(x) = \frac{6}{x} \rightarrow \frac{\frac{6}{x+h} - \frac{6}{x}}{h} = \frac{\frac{6x - 6(x+h)}{x(x+h)}}{h} = \frac{\cancel{6x} - \cancel{6x} - 6h}{x(x+h)(h)} = \frac{-6\cancel{h}}{x(x+h)(\cancel{h})} = \boxed{\frac{-6}{x(x+h)}}$

7) $p(x) = \begin{cases} \sqrt{x+4}, & x \le 5 \\ (x-5)^2, & x > 5 \end{cases}$

a) $p(-3) = \sqrt{-3+4} = \sqrt{1} = 1$

b) $p(0) = \sqrt{0+4} = 2$

c) $p(5) = \sqrt{5+4} = \sqrt{9} = 3$

d) $p(10) = (10-5)^2 = 5^2 = 25$

8) a) $y = |5x+2|$ write as piecewise function:

$5x+2=0$

$x = -2/5$

$y = \begin{cases} 5x+2, & x \ge -2/5 \\ -(5x+2), & x < -2/5 \end{cases}$

b) $y = |3-4x| + 8$

$3-4x = 0$

$x = 3/4$

$y = \begin{cases} 3-4x+8, & x \le 3/4 \\ -(3-4x)+8, & x > 3/4 \end{cases}$

$\boxed{y = \begin{cases} -4x+11, & x \le 3/4 \\ 4x+5, & x > 3/4 \end{cases}}$

9) $f(x) = \frac{3}{x}$ $g(x) = x^2 - 1$

a) $f(g(x)) = \frac{3}{x^2-1} = \boxed{\frac{3}{(x-1)(x+1)}}$ $x \ne 1, -1$ Domain: $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

b) $g(f(x)) = \left(\frac{3}{x}\right)^2 - 1 = \boxed{\frac{9}{x^2} - 1}$ $x \ne 0$ Domain: $(-\infty,0) \cup (0,\infty)$

10) Find domain:

a) $g(x) = \frac{3x^2+2x-8}{2x^2+x-6} = \frac{3x^2+2x-8}{(2x-3)(x+2)}$ $x \ne \frac{3}{2}, -2$

Domain: $(-\infty,-2) \cup (-2,3/2) \cup (3/2,\infty)$

b) $h(x) = \sqrt{x} + \sqrt{1-x}$

$[0,\infty)$ $(-\infty,1]$

Domain: $[0,1]$

c) $k(x) = \sqrt{x^2-3x+2}$

$k(x) = \sqrt{(x-2)(x-1)}$

Domain: $(-\infty,1] \cup [2,\infty)$

Find inverse of the following:

11) a) $f(x) = 4x - 3$

Steps:
1) switch x and y
2) solve for y

$y = 4x - 3$

$x = 4y - 3$

$\frac{x+3}{4} = y$

$\boxed{f^{-1}(x) = \frac{x+3}{4}}$

b) $g(x) = \frac{2x+3}{x-4}$

$y = \frac{2x+3}{x-4}$

$x = \frac{2y+3}{y-4}$

$x(y-4) = 2y+3$

$xy - 4x = 2y + 3$

$xy - 2y = 4x + 3$

$y(x-2) = 4x + 3$

$y = \frac{4x+3}{x-2}$

$\boxed{f^{-1}(x) = \frac{4x+3}{x-2}}$

c) $h(x) = x^3 + 1$

$y = x^3 + 1$

$x = y^3 + 1$

$x - 1 = y^3$

$y = \sqrt[3]{x-1}$

$\boxed{f^{-1}(x) = \sqrt[3]{x-1}}$

12) Determine if function is even, odd, or neither. Find $f(-x)$

a) $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2)$

Since $f(-x) = f(x)$, function is even.

b) $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x}$

Since $f(-x) = -f(x)$ function is odd.

c) $f(x) = x\cos x$

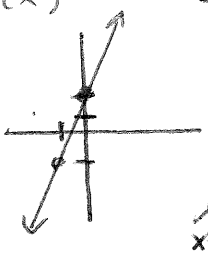
$f(-x) = (-x)\cos(-x)$

$= -x\cos x$

Since $f(-x) = -f(x)$ function is odd.

13) Graph the following:

a) $f(x) = -3x + 2$



Domain: $(-\infty, \infty)$

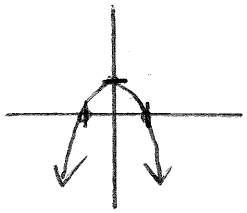
Range: $(-\infty, \infty)$

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

b) $f(x) = 1 - x^2$

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -\infty$

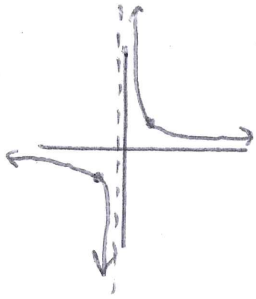


D: $(-\infty, \infty)$

R: $(-\infty, 1]$

13) continued...

c) $f(x) = \frac{1}{x}$



D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

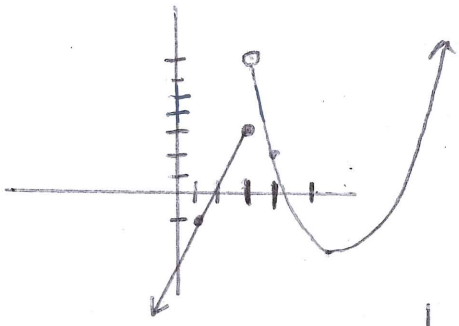
d) $f(x) = \begin{cases} 2x-3 & , x \le 3 \\ (x-6)^2 - 2 & , x > 3 \end{cases}$

$y = 2x - 3$

x	y
3	3
2	1
1	-1

$y = (x-6)^2 - 2$

x	y
3	7
4	2
5	-1



D: $(-\infty, \infty)$

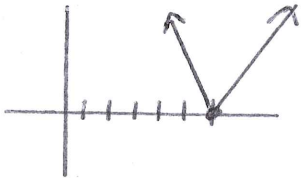
R: $(-\infty, \infty)$

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = +\infty$

e) $f(x) = |6 - x|$

vertex: $(6, 0)$



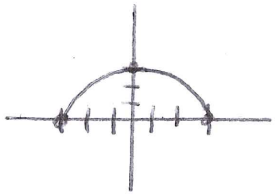
D: $(-\infty, \infty)$

R: $[0, \infty)$

$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to \infty} f(x) = +\infty$

f) $f(x) = \sqrt{9 - x^2}$

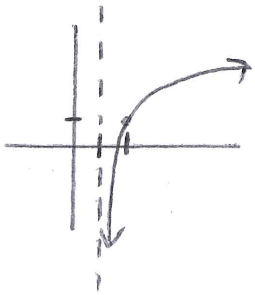


D: $[-3, 3]$

R: $[0, 3]$

g) $f(x) = \ln(x - 1)$

*vertical asymptote: $x = 1$



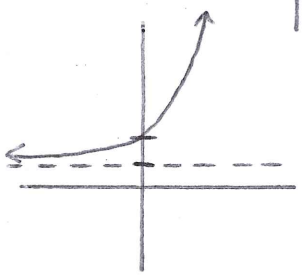
D: $(1, \infty)$

R: $(-\infty, \infty)$

$\lim_{x \to 1^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = +\infty$

h) $f(x) = e^x + 1$



D: $(-\infty, \infty)$

R: $(1, \infty)$

$\lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to \infty} f(x) = +\infty$

16) a) inc: $(-\infty, \infty)$
dec: none

b) inc: $(-\infty, 0)$
dec: $(0, \infty)$

c) inc: none
dec: $(-\infty, 0) \cup (0, \infty)$

d) inc: $(-\infty, 3) \cup (6, \infty)$
dec: $(3, 6)$

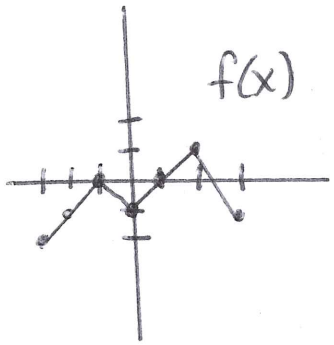
e) inc: $(6, \infty)$
dec: $(-\infty, 6)$

f) inc: $(-3, 0)$
dec: $(0, 3)$

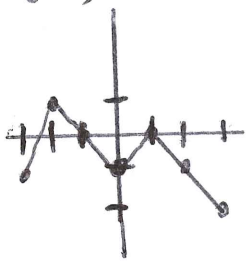
g) inc: $(1, \infty)$
dec: none

h) inc: $(-\infty, \infty)$
dec: none

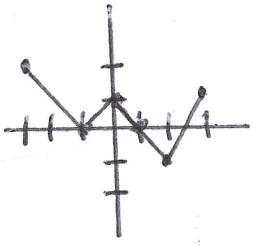
17)



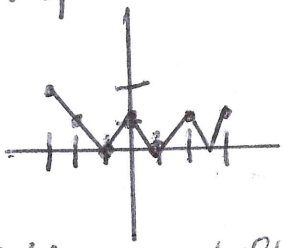
a) $f(-x)$ = reflection over y-axis



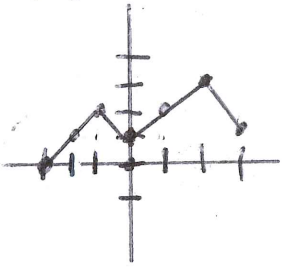
b) $-f(x)$ = reflection over x-axis



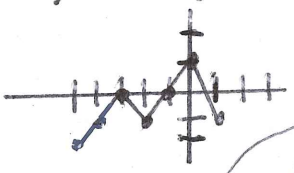
c) $|f(x)|$ = Absolute value of $f(x)$

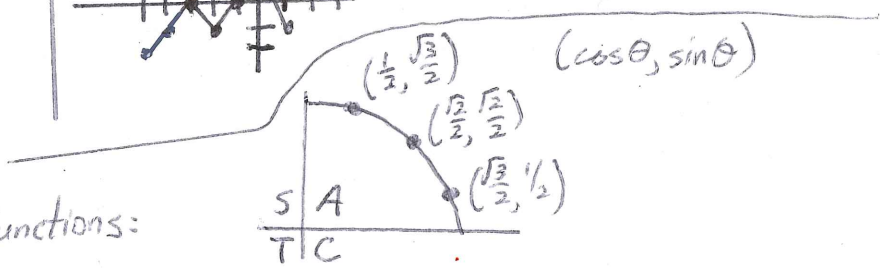


d) $f(x)+2$ = shift up 2 units



e) $f(x+2)$ = shift left 2 units





18) State value of all 6 trig functions:

a) $\sin(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$ $\quad \csc(\frac{5\pi}{4}) = -\sqrt{2}$

$\cos(5\pi/4) = -\frac{\sqrt{2}}{2}$ $\quad \sec(\frac{5\pi}{4}) = -\sqrt{2}$

$\tan(5\pi/4) = 1$ $\quad \cot(\frac{5\pi}{4}) = 1$

b) $\sin(\frac{11\pi}{6}) = -\frac{1}{2}$ $\quad \csc(\frac{11\pi}{6}) = -2$

$\cos(\frac{11\pi}{6}) = \frac{\sqrt{3}}{2}$ $\quad \sec(\frac{11\pi}{6}) = \frac{2\sqrt{3}}{3}$

$\tan(\frac{11\pi}{6}) = -\frac{\sqrt{3}}{3}$ $\quad \cot(\frac{11\pi}{6}) = -\sqrt{3}$

c) $\frac{14\pi}{3} \to \frac{2\pi}{3}$

$\sin(\frac{14\pi}{3}) = \frac{\sqrt{3}}{2}$ $\quad \csc(\frac{14\pi}{3}) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos(\frac{14\pi}{3}) = -\frac{1}{2}$ $\quad \sec(\frac{14\pi}{3}) = -2$

$\tan(\frac{14\pi}{3}) = -\sqrt{3}$ $\quad \cot(\frac{14\pi}{3}) = \frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

d) $\sin(\frac{\pi}{2}) = 1$ $\quad \csc(\pi/2) = 1$

$\cos(\pi/2) = 0$ $\quad \sec(\pi/2) =$ undefined

$\tan(\pi/2) =$ undefined $\quad \cot(\pi/2) = 0$

e) $\sin(\pi) = 0$ $\quad \csc(\pi) =$ undefined

$\cos(\pi) = -1$ $\quad \sec(\pi) = -1$

$\tan(\pi) = 0$ $\quad \cot(\pi) =$ undefined

19) State amplitude, period, phase shift, vertical shift

reflection over x-axis

$$y = -A\,f(Bx + C) + D$$

Amplitude (vertical stretch)

shift left/right

shift up/down

period: $\frac{2\pi}{B}$ or $\frac{\pi}{B}$

phase shift: $\frac{C}{B}$

a) $y = 1 + \cos[3(x - \pi/3)] = 1 + \cos[3x - \pi]$

Amplitude: 1

period: $\frac{2\pi}{3}$

phase shift: $\frac{\pi}{3}$ right

vertical shift: up 1 unit

b) $y = 2 - 3\sin(4x + \pi)$

Amplitude: 3

period: $\frac{2\pi}{4} = \frac{\pi}{2}$

phase shift: left $\frac{\pi}{4}$

vertical shift: up 2 units

Simplify

20) a) $\sin x \cos x \tan x \sec x \csc x$

$$\sin x \cdot \cancel{\cos x} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{1}{\cos x} \cdot \frac{1}{\cancel{\sin x}} = \frac{\sin x}{\cos x} = \boxed{\tan x}$$

b) $\frac{\sin x}{1+\cos x} + \frac{\sin x}{1-\cos x}$

$$\frac{\sin x(1-\cos x) + \sin x(1+\cos x)}{(1+\cos x)(1-\cos x)}$$

$$\frac{\cancel{\sin x} - \cancel{\sin x \cos x} + \cancel{\sin x} + \cancel{\sin x \cos x}}{1 - \cos^2 x}$$

$$\frac{2\sin x}{\sin^2 x}$$

$$\frac{2}{\sin x} = \boxed{2\csc x}$$

c) $\cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x$

$(\underbrace{\cos^2 x + \sin^2 x}_{=1})(\underbrace{\cos^2 x + \sin^2 x}_{=1})$ → * Recall: $\sin^2 x + \cos^2 x = 1$

$(1)(1) = \boxed{1}$

21) Solve the following: $0 \le x < 2\pi$

* recall $\sin^2 x + \cos^2 x = 1$

a) $\cos^2 x - \cos x + 1 = \sin^2 x$ $\quad \sin^2 x = 1 - \cos^2 x$

$\cos^2 x - \cos x + 1 = 1 - \cos^2 x$

$2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$

$x = \pi/2, 3\pi/2$

$2\cos x - 1 = 0$

$\cos x = 1/2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

b) $\sin x \tan x = \sin x$

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$\tan x - 1 = 0$

$\tan x = 1$

$x = \pi/4, 5\pi/4$

c) $\sin x = \cos 2x - 1$ $\quad$ * $\cos 2x = 1 - 2\sin^2 x$

$\sin x = 1 - 2\sin^2 x - 1$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$2\sin x + 1 = 0$

$\sin x = -1/2$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

d) $\sin(4x) = \frac{1}{2}$

$4x = \sin^{-1}(\frac{1}{2})$

$4x = \frac{\pi}{6}$

$x = \frac{\pi}{24}$

$4x = \frac{5\pi}{6}$

$x = \frac{5\pi}{24}$

e) $\cot^2 x - \csc x = 1$

$\cot^2 x = \csc^2 x - 1$

$\csc^2 x - 1 - \csc x = 1$

$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$

$\frac{1}{\sin x} = 2$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\csc x = -1$

$\sin x = -1$

$x = \frac{3\pi}{2}$

f) $\sec^2 x + 2\sec x = 0$

$\sec x(\sec x + 2) = 0$

$\sec x = 0$

none undefined

$\sec x + 2 = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

21) continued...

g) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$* \sin 2x = 2\sin x \cos x$

h) $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

i) $\cot x \cos^2 x = 2\cot x$

$\cot x\cos^2 x - 2\cot x = 0$

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$

$\frac{\cos x}{\sin x} = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos^2 x - 2 = 0$

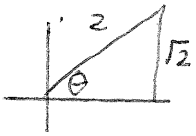
$\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$

$x =$ undefined

22) Simplify, $0 < \theta < \pi$

$\cos\theta = \frac{\sqrt{2}}{2}$



a) $\sin\left(2\arccos\frac{\sqrt{2}}{2}\right)$

$\sin\left(2\left(\frac{\pi}{4}\right)\right) = \sin\left(\frac{\pi}{2}\right) = 1$

b) $\cos\left(\arccos 0 + \arcsin\frac{1}{2}\right) \rightarrow \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos\left(\frac{3\pi}{6} + \frac{\pi}{6}\right)$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

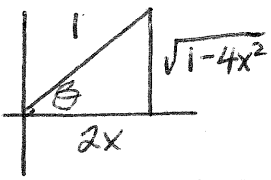
$\sin\theta = \frac{1}{2}$

$\theta = \pi/6$

$= \cos\left(\frac{4\pi}{6}\right)$

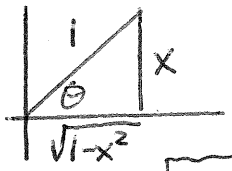
$\cos\left(\frac{2\pi}{3}\right) = \boxed{\frac{-1}{2}}$

23) a) $\sin(\arccos 2x)$



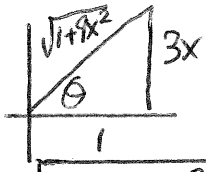
$\sin\theta = \frac{\sqrt{1-4x^2}}{1} = \boxed{\sqrt{1-4x^2}}$

b) $\cot(\arcsin x)$



$\cot\theta = \boxed{\frac{\sqrt{1-x^2}}{x}}$

c) $\sin(\arctan 3x)$



$\boxed{\sin\theta = \frac{3x}{\sqrt{1+9x^2}}}$

24) State horizontal asymptote, vertical asymptote, and holes

a) $y = \dfrac{2x^2-7x-4}{6x^2+7x+2} = \dfrac{\cancel{(2x+1)}(x-4)}{(3x+2)\cancel{(2x+1)}}$

HA: $y = \frac{1}{3}$ ← * take ratio of coefficients $\dfrac{2x^2}{6x^2} \rightarrow \boxed{\frac{1}{3}}$

VA: $x = -2/3$

hole: $(-1/2, -9)$ at $x = \frac{-1}{2}$ $\quad \dfrac{x-4}{3x+2} \rightarrow \dfrac{(\frac{-1}{2})-4}{3(\frac{-1}{2})+2} = \dfrac{\frac{-9}{2}}{1/2} = -9$

b) $y = \dfrac{5x^2+20x}{x^3-3x^2-28x} = \dfrac{5x\cancel{(x+4)}}{x(x-7)\cancel{(x+4)}}$

HA: $y = 0$ ← * denominator has higher degree

VA: $x = 7$

hole: $x = 0,\ x = -4$ $\longrightarrow \dfrac{5(0+4)}{(0-7)(0+4)} = \dfrac{-5}{7}$ $\qquad \dfrac{5}{-4-7} = \dfrac{-5}{11}$

↳ $(0, -5/7), (-4, \frac{-5}{11})$

25) Solve the following:

a) $\dfrac{2x^2-7x-4}{6x^2+7x+2} = 0$ $\quad$ * set numerator $= 0$

$2x^2-7x-4=0$

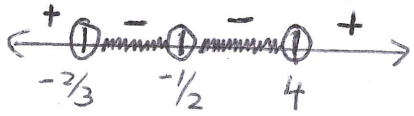
$(2x+1)(x-4)=0$ $\quad \boxed{x = \frac{-1}{2},\ x = 4}$

b) $\dfrac{5x^2+20x}{x^3-3x^2-28x} = 0$ $\quad 5x(x+4)=0$ $\quad \boxed{x = 0, -4}$

c) $\dfrac{2x^2-7x-4}{6x^2+7x+2} < 0$ $\quad$ * find critical points

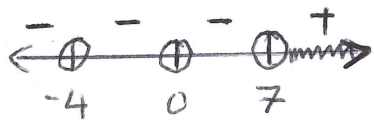
* make sign line, test values in intervals

$\dfrac{(2x+1)(x-4)}{(3x+2)(2x+1)} = 0$ $\quad x = -1/2, 4, -2/3$



$\boxed{(\frac{-2}{3}, \frac{-1}{2}) \cup (\frac{-1}{2}, 4)}$

25) d) $\dfrac{5x^2+20x}{x^3-3x^2-28x} \geq 0$ $\qquad \dfrac{5\cancel{x}\cancel{(x+4)}}{\cancel{x}(x-7)\cancel{(x+4)}} = 0$ $\qquad x = 0, -4, 7$



$\boxed{(7, \infty)}$

e) $\sqrt{3x+1} \leq 4$

$\left(\sqrt{3x+1}\right)^2 = (4)^2$

$3x+1 = 16$

$3x = 15$

$\underline{x = 5}$

* Domain for $\sqrt{3x+1}$ is $[-1/3, \infty)$

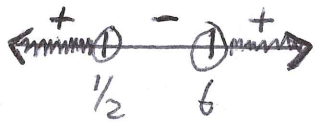
$\boxed{[-1/3, 5]}$

f) $\sqrt{2x^2-13x+6} > 0$

$2x^2-13x+6 = 0$

$(2x-1)(x-6) = 0$

$x = 1/2, 6$



$\boxed{(-\infty, 1/2) \cup (6, \infty)}$

26) Solve the following:

* Recall: If $\log_a b = x$, then $a^x = b$

a) $\log_{81} \sqrt{3} = x$

$81^x = \sqrt{3} \rightarrow (3^4)^x = 3^{1/2}$

$3^{4x} = 3^{1/2}$

$4x = 1/2$

$x = \frac{1}{2} \cdot \frac{1}{4}$

$\boxed{x = 1/8}$

b) $\log_x 64^{1/3} = \frac{1}{2}$

$x^{1/2} = 64^{1/3}$

$x^{1/2} = 4$

$\left(x^{1/2}\right)^2 = (4)^2$

$\boxed{x = 16}$

c) $9 = 4 + \log_2(x+3)$

$5 = \log_2(x+3)$

$2^5 = x+3$

$32 = x+3$

$\boxed{x = 29}$

26) d) $\frac{1}{3}\ln x = \ln 8$

$\ln x^{1/3} = \ln 8$

$x^{1/3} = 8$

$(x^{1/3})^3 = 8^3$

$\boxed{x = 512}$

* Recall power property:

$\log_a b^n = n \log_a b$

e) $\log_b 8 = \log_b x + \log_b (x-2)$

$\log_b 8 = \log_b x(x-2)$

$8 = x^2 - 2x$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, \cancel{x = -2}$

$\boxed{x = 4}$

* Recall product property:

$\log_b m + \log_b n = \log_b mn$

f) $4\ln(x+3) = 12$

$\frac{4\ln(x+3)}{4} = \frac{12}{4}$

$\ln(x+3) = 3$

$\log_e(x+3) = 3$

$e^3 = x+3$

$\boxed{x = e^3 - 3}$

g) $e^{3x} = 6$

$\log_e 6 = 3x$

$\ln 6 = 3x$

$\boxed{x = \frac{\ln 6}{3}}$

h) $7^{x-4} = 100$

$\log_7 100 = x - 4$

$x = \log_7 100 + 4$

$x = \frac{\log 100}{\log 7} + 4 \approx \boxed{6.367}$

27) Write the following using sigma notation

a) $4+8+12+16+20$

$$\boxed{\sum_{i=1}^{5} 4i}$$

b) $1+4+9+16+25+36+49$

$$\boxed{\sum_{i=1}^{7} i^2}$$

c) $7 \overset{-3}{+} 4 \overset{-3}{+} 1 \overset{-3}{+} -2 + \ldots$

$a_n = a_1 + (n-1)d$

$a_1 = 7$

$d = -3$

$a_n = 7 + (n-1)(-3)$

$a_n = 7 - 3n + 3$

$a_n = 10 - 3n$

$$\boxed{\sum_{i=1}^{\infty} 10 - 3i}$$