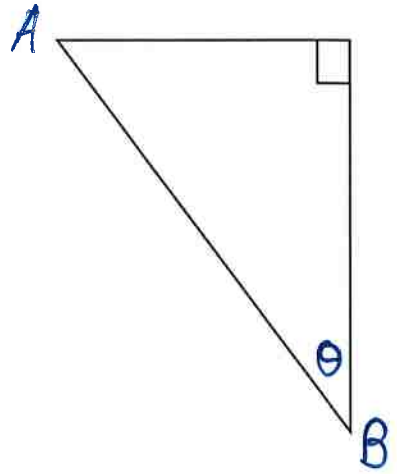


3)

- a) Two cyclists are traveling on perpendicular roads. Cyclist A travels East at 15 mph. Cyclist B travels South at 20 mph. Find the rate at which the distance between the 2 cyclists are changing when $x = 8$ miles and $y = 15$ miles.
- b) What is the rate of change of θ at that moment?



4) Find $\frac{dy}{dx}$ $x \cot 2y - y = \sqrt{\tan^5(5y^2)} + 3x - 7$

5) Find $\frac{dy}{dx}$ $y = x^3 \cos^{-1}(\pi x) - \sec(3x^2) + 5\pi e^2$

6) Solve for t: $2 \cos\left(\frac{5\pi t}{2}\right) = 1$ on $[0, 2]$

Key

Trig Unit Morning Test Review WS 4

1. Practice writing out the 6 trig derivative rules, 6 arc trig derivative rules, rule for $\ln u$ & e^u

$\frac{d}{dx} \sin u = \cos u \cdot u'$	$\frac{d}{dx} \cos u = -\sin u \cdot u'$	$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx} \arccos u = \frac{-u'}{\sqrt{1-u^2}}$
$\frac{d}{dx} \tan u = \sec^2 u \cdot u'$	$\frac{d}{dx} \cot u = -\csc^2 u \cdot u'$	$\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$	$\frac{d}{dx} \text{arccot} u = \frac{-u'}{1+u^2}$
$\frac{d}{dx} \sec u = \sec u \tan u \cdot u'$	$\frac{d}{dx} \csc u = -\csc u \cot u \cdot u'$	$\frac{d}{dx} \text{arcsec} u = \frac{u'}{ u \sqrt{u^2-1}}$	$\frac{d}{dx} \text{arccsc} u = \frac{-u'}{ u \sqrt{u^2-1}}$
$\frac{d}{dx} \ln u = \frac{u'}{u}$	$\frac{d}{dx} e^u = e^u \cdot u'$		

2. An object's position is given by $F(t) = 2\sin^2(3t)$, continuous and differentiable on $0 \leq t \leq \pi$ seconds. $F(t)$ is given in meters $F(t) = 2[\sin(3t)]^2$

a) Find the average velocity (rate of change) from $0 \leq t \leq \pi/4$

$F(0) = 2[\sin(0)]^2 = 0$
 $F(\pi/4) = 2[\sin(3\pi/4)]^2 = 2[\frac{\sqrt{2}}{2}]^2 = 2 \cdot \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$

Avg. velocity = $\frac{\frac{\sqrt{2}}{2} - 0}{\pi/4 - 0} = \frac{\sqrt{2}}{2} \cdot \frac{4}{\pi} = \frac{2\sqrt{2}}{\pi}$ m/s

b) At what point in time does the instantaneous velocity equal the average velocity from part (a)? Set up equation but do not solve.

$F(t) = 2[\sin(3t)]^2$ $F'(t) = 4[\sin(3t)] \cdot \cos(3t) \cdot 3$

out: $2[\]^2$
 in: $\sin(3t)$

$12 \sin(3t) \cos(3t) = \frac{2\sqrt{2}}{\pi}$

Instantaneous ROC Avg. ROC

*MVT
 $F'(c) = \frac{F(b)-F(a)}{b-a}$

c) When on the interval $0 \leq t \leq \pi$, does $F(t) = 1$? (Show work leading to answer)

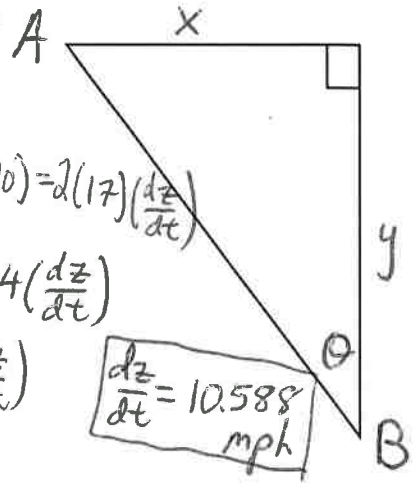
$2[\sin(3t)]^2 = 1$
 $\sqrt{[\sin(3t)]^2} = \sqrt{\frac{1}{2}}$
 $\sin(3t) = \pm \frac{\sqrt{2}}{2}$

$3t = \sin^{-1}(\pm \frac{\sqrt{2}}{2})$
 $3t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4, 9\pi/4, 11\pi/4, 15\pi/4$
 $t = \pi/12, \pi/4, 5\pi/12, 7\pi/12, 9\pi/12, 11\pi/12, 15\pi/12$

3)

a) Two cyclists are traveling on perpendicular roads. Cyclist A travels East at 15 mph. Cyclist B travels South at 20 mph. Find the rate at which the distance between the 2 cyclists are changing when $x = 8$ miles and $y = 15$ miles.

b) What is the rate of change of θ at that moment?



$$a) x^2 + y^2 = z^2$$

$$2x \left(\frac{dx}{dt} \right) + 2y \left(\frac{dy}{dt} \right) = 2z \left(\frac{dz}{dt} \right)$$

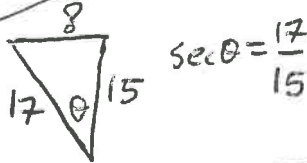
$$x = 8 \quad \frac{dx}{dt} = -15$$

$$y = 15 \quad \frac{dy}{dt} = 20$$

$$z = \quad \frac{dz}{dt} = \quad$$

$$z^2 = 8^2 + 15^2$$

$$z = 17$$



$$\sec \theta = \frac{17}{15}$$

$$2(8)(-15) + 2(15)(20) = 2(17) \left(\frac{dz}{dt} \right)$$

$$-240 + 600 = 34 \left(\frac{dz}{dt} \right)$$

$$360 = 34 \left(\frac{dz}{dt} \right)$$

$$\frac{dz}{dt} = 10.588 \text{ mph}$$

$$b) \tan \theta = \frac{x}{y}$$

$$\sec^2 \theta \left(\frac{d\theta}{dt} \right) = \frac{\frac{f'}{g} - f \cdot \frac{g'}{g^2}}{\frac{y^2}{g^2}}$$

4) Find $\frac{dy}{dx}$ $x \cot 2y - y = \sqrt{\tan^5(5y^2)} + 3x - 7$

$$\left(\frac{17}{15} \right)^2 \frac{d\theta}{dt} = \frac{(-15)(15) - 8(20)}{15^2}$$

$$\left(\frac{17}{15} \right)^2 \left(\frac{d\theta}{dt} \right) = \frac{-385}{225}$$

$$\frac{d\theta}{dt} = \frac{-385}{225} \cdot \frac{15^2}{17^2} = \frac{-385}{289} \text{ or}$$

$$-1.332 \text{ rad/hr}$$

$$x \cot y - y = \left[\tan(5y^2) \right]^{5/2} + 3x - 7$$

*chain Rule
out: $[]^{5/2}$
in: $\tan(5y^2)$

$$1 \cdot \cot(y) + x \cdot -\csc^2 y \left(\frac{dy}{dx} \right) - 1 \left(\frac{dy}{dx} \right) = \frac{5}{2} \left[\tan(5y^2) \right]^{3/2} \cdot \sec^2(5y^2) \cdot 10y \left(\frac{dy}{dx} \right) + 3 - 0$$

$$-x \csc^2 y \left(\frac{dy}{dx} \right) - 1 \left(\frac{dy}{dx} \right) - 25y \left[\tan(5y^2) \right]^{3/2} \sec^2(5y^2) \left(\frac{dy}{dx} \right) = -\cot(y) + 3$$

$$\frac{dy}{dx} \left(-x \csc^2 y - 1 - 25y \left[\tan(5y^2) \right]^{3/2} \sec^2(5y^2) \right) = -\cot(y) + 3$$

$$\frac{dy}{dx} = \frac{-\cot(y) + 3}{-x \csc^2 y - 1 - 25y \left[\tan(5y^2) \right]^{3/2} \sec^2(5y^2)}$$

5) Find $\frac{dy}{dx}$ $y = x^3 \cos^{-1}(\pi x) - \sec(3x^2) + 5\pi e^2$

$$\frac{dy}{dx} = \overbrace{3x^2}^{f'} \cdot \overbrace{\cos^{-1}(\pi x)}^g + \overbrace{x^3}^f \cdot \overbrace{\frac{-\pi}{\sqrt{1-(\pi x)^2}}}_{g'}} - \sec(3x^2) \tan(3x^2) \cdot 6x + 0$$

$$\frac{dy}{dx} = 3x^2 \cos^{-1}(\pi x) - \frac{\pi x^3}{\sqrt{1-(\pi x)^2}} - 6x \sec(3x^2) \tan(3x^2)$$

$$2 \cos\left(\frac{5\pi t}{2}\right) = 1$$

6) Solve for t: $2 \cos\left(\frac{5\pi t}{2}\right) = 1$ on $[0, 2]$

$$\cos\left(\frac{5\pi t}{2}\right) = \frac{1}{2}$$

$$\frac{5\pi t}{2} = \cos^{-1}\left(\frac{1}{2}\right)$$

Added $6\pi/3$

$$\frac{2}{5\pi} \left[\frac{5\pi t}{2} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3} \right]$$

$$t = \frac{2}{15}, \frac{2}{3}, \frac{14}{15}, \frac{22}{15}, \frac{26}{15}, \frac{34}{15}$$

7)

Sketch the graph of function $f(x) = \frac{1}{2}x - \cos x$ on interval $[-\pi, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

$$f'(x) = \frac{1}{2} - (-\sin x)$$

$$f'(x) = \frac{1}{2} + \sin x$$

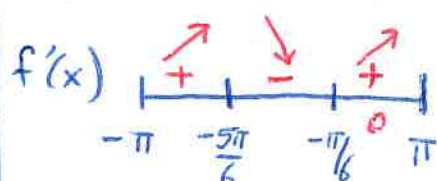
$$0 = \frac{1}{2} + \sin x$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$$

$$\frac{11\pi}{6} - \frac{12\pi}{6} = -\frac{\pi}{6}$$



$$f\left(-\frac{5\pi}{6}\right) = \frac{1}{2}\left(-\frac{5\pi}{6}\right) - \cos\left(-\frac{5\pi}{6}\right)$$

$$= -\frac{5\pi}{12} - \left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{5\pi}{12} + \frac{\sqrt{3}}{2}}$$

$$f\left(-\frac{\pi}{6}\right) = \frac{1}{2}\left(-\frac{\pi}{6}\right) - \cos\left(-\frac{\pi}{6}\right)$$

$$= \boxed{-\frac{\pi}{12} - \frac{\sqrt{3}}{2}}$$

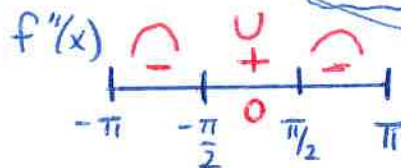
$$f''(x) = \cos x$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{3\pi}{2} - \frac{4\pi}{2} = -\frac{\pi}{2}$$

$$x = \frac{\pi}{2}, -\frac{\pi}{2}$$



$$f\left(-\frac{\pi}{2}\right) = \frac{1}{2}\left(-\frac{\pi}{2}\right) - \cos\left(-\frac{\pi}{2}\right) = -\frac{\pi}{4} - 0 = \boxed{-\frac{\pi}{4}}$$

$$f\left(\frac{\pi}{2}\right) = \frac{1}{2}\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right) = \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

Include "Because" statements for parts a,b, & c

a) the relative extrema min: $\left(-\frac{\pi}{6}, -\frac{\pi}{12} - \frac{\sqrt{3}}{2}\right)$ b/c $f'(x)$ changes from - to + max: $\left(-\frac{5\pi}{6}, -\frac{5\pi}{12} + \frac{\sqrt{3}}{2}\right)$ b/c $f'(x)$ changes from + to -

b) the point(s) of inflection: $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$ b/c $f''(x)$ change signs

c) concavity intervals up: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ b/c $f''(x) > 0$

down: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: $-\frac{\pi}{12} - \frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{6}$ Absolute max: $\frac{\pi}{2} + 1$ at $x = \pi$

d) Sketch the graph endpoints $\left\{ \begin{array}{l} f(-\pi) = \frac{1}{2}(-\pi) - \cos(-\pi) = -\frac{\pi}{2} + 1 \\ f(\pi) = \frac{1}{2}(\pi) - \cos(\pi) = \frac{\pi}{2} + 1 \end{array} \right.$

