Trigonometric Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities:

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Even/Odd Identities:

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Sum & Difference Identities:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double-Angle Identities:

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\quad$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Half-Angle Identities:

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\quad$ $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\quad$ $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Accelerated Precalculus

Unit Circle Trigonometry

Date: Key 11.01 HW

Simplify the following using what you know about trigonometry and the unit circle.

1. $\cos\frac{3\pi}{4}$ Q2 $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\boxed{-\frac{\sqrt{2}}{2}}$

2. $\cos 0$ $(1, 0)$

$\boxed{1}$

3. $\sin\frac{2\pi}{3}$ Q2 $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{\sqrt{3}}{2}}$

4. $\sin\frac{11\pi}{6}$ Q4 $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\boxed{-\frac{1}{2}}$

5. $\cos\frac{7\pi}{6}$ Q3 $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\boxed{-\frac{\sqrt{3}}{2}}$

6. $\cos\frac{5\pi}{3}$ Q4 $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{1}{2}}$

7. $\sin\pi$ $(-1, 0)$

$\boxed{0}$

8. $\sin\frac{5\pi}{4}$ Q3 $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\boxed{-\frac{\sqrt{2}}{2}}$

9. $\sec\frac{7\pi}{4}$ Q4 $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ recip

$\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$

10. $\sec\frac{\pi}{2}$ $(0, 1)$ recip

$\frac{1}{0} = \boxed{\text{undefined}}$

11. $\csc\frac{4\pi}{3}$ Q2 $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ recip

$\frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

12. $\csc\frac{5\pi}{6}$ Q2 $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ recip

$\frac{2}{1} = \boxed{2}$

13. $\sec\frac{\pi}{6}$ Q1 $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ recip

$\frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

14. $\sec\frac{2\pi}{3}$ Q2 $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ recip

$-\frac{2}{1} = \boxed{-2}$

15. $\csc\frac{3\pi}{2}$ $(0, -1)$ recip

$\frac{1}{-1} = \boxed{-1}$

16. $\csc\frac{3\pi}{4}$ Q2 $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ recip

$\frac{2}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$

17. $\tan\frac{5\pi}{4}$ Q3 $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ denom num

$\frac{-\sqrt{2}/\cancel{2}}{-\sqrt{2}/\cancel{2}} = \boxed{1}$

18. $\tan\frac{2\pi}{3}$ Q2 $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ denom num

$\frac{\sqrt{3}/\cancel{2}}{-1/\cancel{2}} = \boxed{-\sqrt{3}}$

19. $\cot\frac{11\pi}{6}$ Q4 $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ num denom

$\frac{\sqrt{3}/\cancel{2}}{-1/\cancel{2}} = \boxed{-\sqrt{3}}$

20. $\cot 0$ $(1, 0)$ num denom

$\frac{1}{0} = \boxed{\text{undefined}}$

21. $\tan\frac{5\pi}{6}$ Q2 $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ denom num

$\frac{1/\cancel{2}}{-\sqrt{3}/\cancel{2}} = \frac{1}{-\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

22. $\tan\pi$ $(-1, 0)$ denom num

$\frac{0}{-1} = \boxed{0}$

23. $\cot\frac{3\pi}{4}$ Q2 $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ num denom

$\frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$

24. $\cot\frac{4\pi}{3}$ Q3 $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ num denom

$\frac{-1/\cancel{2}}{-\sqrt{3}/\cancel{2}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

25. $\csc\frac{5\pi}{3}$ Q4 $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ recip

$\frac{2}{-\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

26. $\tan\frac{3\pi}{2}$ $(0, -1)$ denom num

$\frac{-1}{0} = \boxed{\text{undefined}}$

27. $\cot\frac{2\pi}{3}$ Q2 $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ num denom

$\frac{-1/\cancel{2}}{\sqrt{3}/\cancel{2}} = -\frac{1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

28. $\sec\frac{5\pi}{4}$ Q3 $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ recip

$-\frac{2}{\sqrt{2}} = \frac{-\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{-\sqrt{2}}$

State all angles in the interval $[0, 2\pi)$ that solve each of the following equations.

29. $\cos\theta = \frac{1}{2}$ $\left(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$

$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}}$

Q1 Q4

30. $\cos\theta = -\frac{\sqrt{2}}{2}$ $\left(-\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$

Q2, Q3

$\boxed{\theta = \frac{3\pi}{4} \text{ or } \frac{5\pi}{4}}$

31. $\sin\theta = -\frac{1}{2}$ $\left(\pm\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Q3, Q4

$\boxed{\theta = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$

32. $\sin\theta = 1$ $(0, 1)$

$\boxed{\theta = \frac{\pi}{2}}$

33. $\cos\theta = 0$ $(0, \pm 1)$

$\boxed{\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$

34. $\cos\theta = -\frac{\sqrt{3}}{2}$ $\left(-\frac{\sqrt{3}}{2}, \pm\frac{1}{2}\right)$

Q2, Q3

$\boxed{\theta = \frac{5\pi}{6} \text{ or } \frac{7\pi}{6}}$

35. $\sin\theta = \frac{\sqrt{2}}{2}$ $\left(\pm\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Q1, Q2

$\boxed{\theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}}$

36. $\sin\theta = \frac{\sqrt{3}}{2}$ $\left(\pm\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Q1, Q2

$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}}$

37. $\sec\theta = -1$

$\cos\theta = -1$ $(-1, 0)$

$\boxed{\theta = \pi}$

38. $\sec\theta = -2$

$\cos\theta = \frac{1}{-2}$ $\left(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$

Q2, Q3

$\boxed{\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}}$

39. $\csc\theta = -\frac{2\sqrt{3}}{3}$

$\sin\theta = -\frac{3}{2\sqrt{3}} = \frac{-3\sqrt{3}}{2\cdot 3} = \frac{-\sqrt{3}}{2}$

$\left(\pm\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

Q3, Q4

$\boxed{\theta = \frac{4\pi}{3} \text{ or } \frac{5\pi}{3}}$

40. $\csc\theta = -\sqrt{2}$

$\sin\theta = \frac{1}{-\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\left(\pm\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

Q3, Q4

$\boxed{\theta = \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}}$

41. $\tan\theta = 1 = \frac{1}{1} = \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ or $\left(-\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

Q1, Q3

$\boxed{\theta = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}}$

42. $\tan\theta = -\sqrt{3} = \frac{-\sqrt{3}}{1} = \frac{-\sqrt{3}/2}{1/2}$

$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ or $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Q2, Q4

$\boxed{\theta = \frac{2\pi}{3} \text{ or } \frac{5\pi}{3}}$

43. $\cot\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

Q1, Q3

$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{4\pi}{3}}$

44. $\cot\theta = \frac{0}{1}$

$(0, 1)$ or $(0, -1)$

$\boxed{\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$

45: $\sin\theta = 5$

$\boxed{\text{Not Possible}}$

sin has max value of 1

46. $\csc\theta = 0$

$\boxed{\text{Not Possible}}$

Range of csc is $(-\infty, -1] \cup [1, \infty)$

Pre-Calculus Date Key

1.10 Building the Unit Circle Using Special Right Triangles

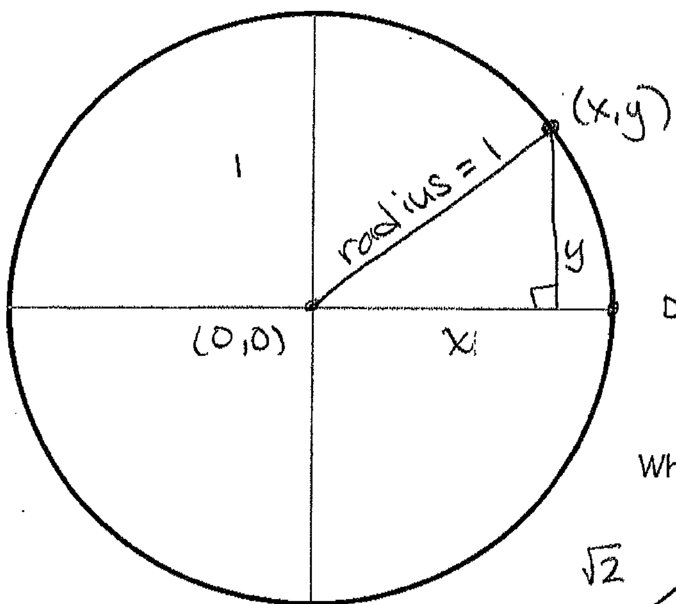
1) Draw a radius from the origin to a point on the unit circle in QI.

What is the length of the radius? 1 unit Why? unit circle

2) Draw a line from that point perpendicular to the x-axis. What is the result? Right △

What are the coordinates of the point (and every point) on the circle? (x, y)

3) Label the reference angle, θ.



Define the following in terms of x and y:

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$ $\csc\theta = \frac{r}{y} = \frac{1}{y}$

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$ $\sec\theta = \frac{r}{x} = \frac{1}{x}$

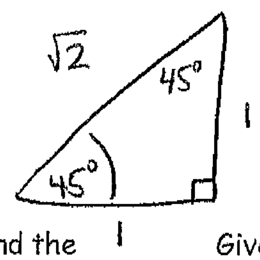
$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

reciprocals

Define tan and cot in terms of sin and cos:

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$ quotients

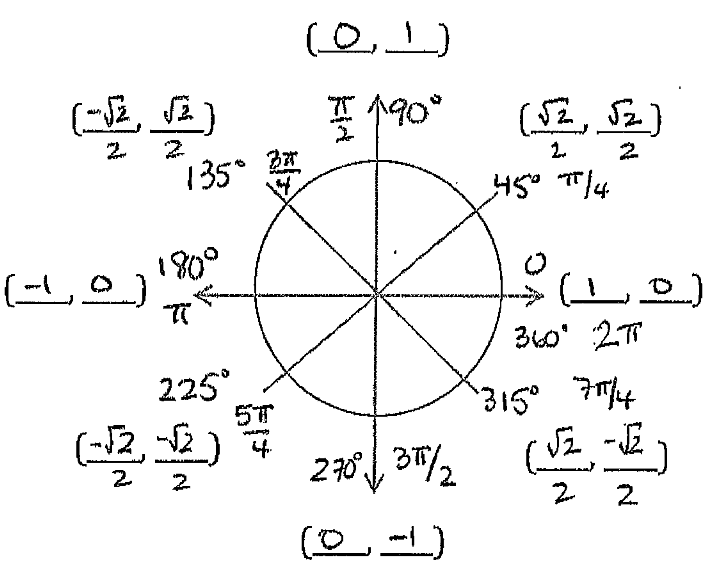
What if your triangle was 45-45-90? (Draw one here)



$\sin 45° = \frac{O}{H} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{A}{H} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Use what you know about quadrantals and the 45-45-90 triangle to complete the figure below:



Given that (x, y) represents (cosθ, sinθ) use the coordinates to answer the following:

$\sin 90° = y = 1$ $\tan\frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} = \text{und}$

$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$ $\sec 2\pi = \frac{1}{x} = \frac{1}{1} = 1$

$\tan 135° = \frac{y}{x} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$ $\csc\frac{3\pi}{2} = \frac{1}{y} = \frac{1}{-1} = -1$

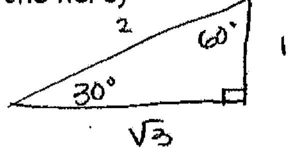
$\cot\pi = \frac{x}{y} = \frac{1}{0} = \text{und}$ $\cos\frac{3\pi}{4} = x = -\sqrt{2}/2$

$\cot 270° = \frac{x}{y} = \frac{0}{-1} = 0$ $\csc\frac{\pi}{4} = \frac{1}{y} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

$\sec\frac{5\pi}{4} = \frac{1}{x} = \frac{1}{-\sqrt{2}/2} = \frac{-2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\cancel{2}\sqrt{2}}{\cancel{2}} = -\sqrt{2}$ $\tan\frac{7\pi}{4} = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = 1$

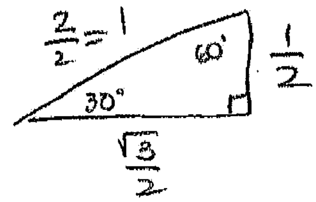
Let's take a look at the unit circle and another very special triangle, 30 - 60 - 90.

(Draw one here)

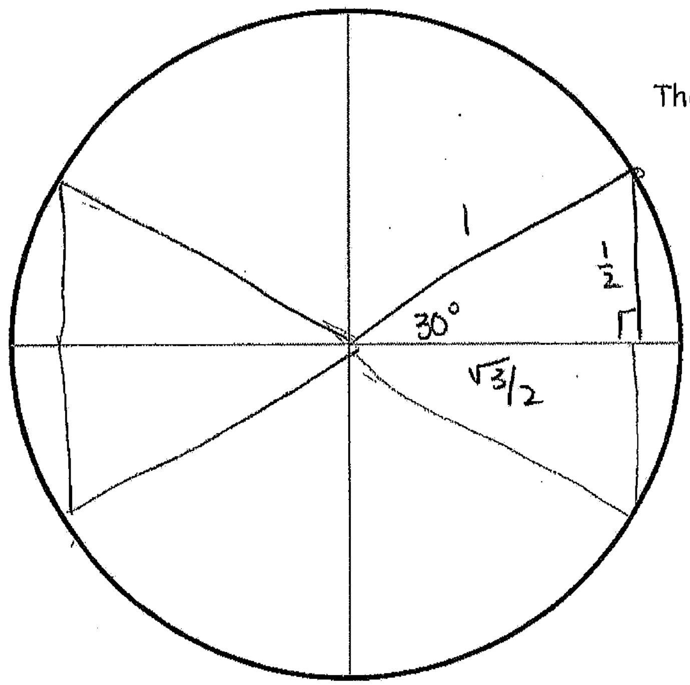


How can we make the hypotenuse = 1?

divide all sides by 2



Now draw θ = 30° on the unit circle:



The coordinates for 30° are:

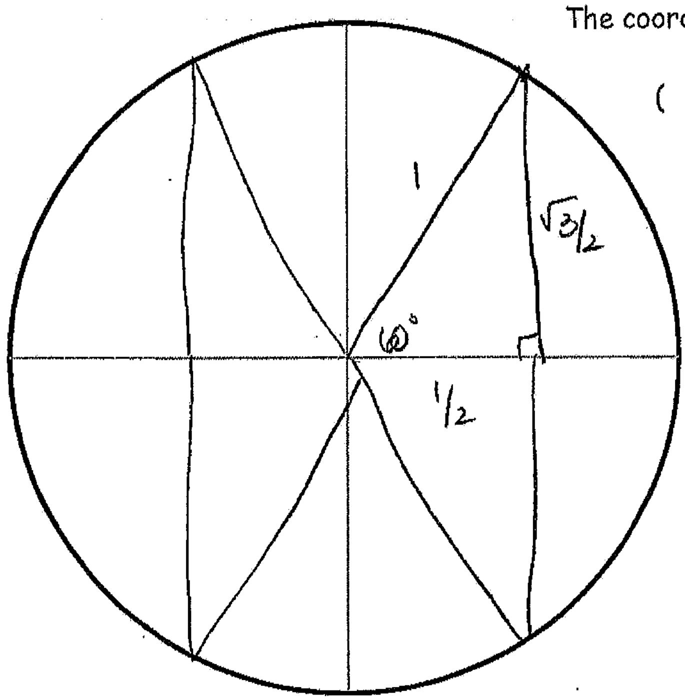
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

sin 30° = $y = \frac{1}{2}$ cos 30° = $x = \frac{\sqrt{3}}{2}$

csc 30° = $\frac{2}{1} = 2$ sec 30° = $\frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3}$

tan 30° = $\frac{y}{x} = \frac{1/\cancel{2}}{\sqrt{3}/\cancel{2}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$ cot 30° = $\frac{x}{y} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \sqrt{3}$

Now draw θ = 60° on the unit circle:



The coordinates for 60° are:

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

sin 60° = $y = \frac{\sqrt{3}}{2}$ cos 60° = $x = \frac{1}{2}$

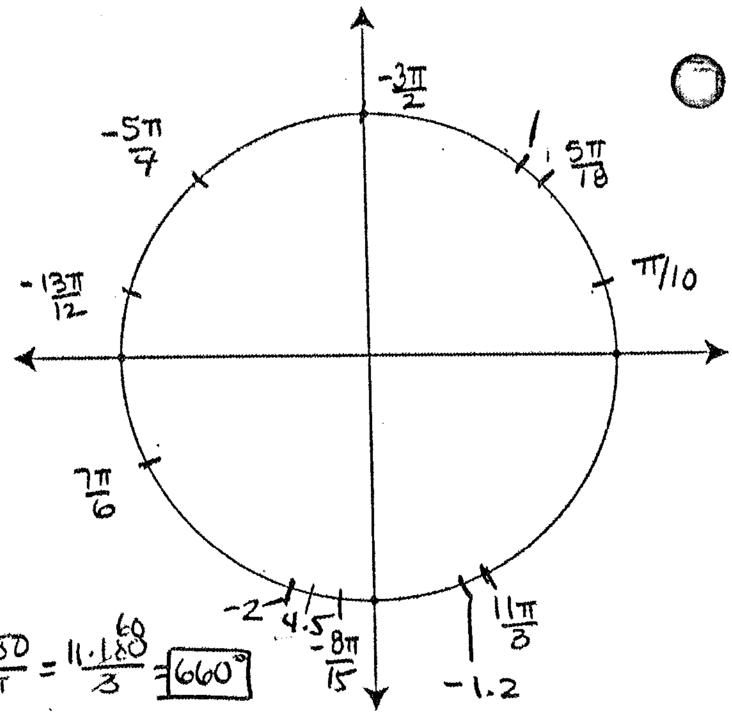
csc 60° = $\frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3}$ sec 60° = $\frac{2}{1} = 2$

tan 60° = $\frac{y}{x} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \sqrt{3}$ cot 60° = $\frac{x}{y} = \frac{1/\cancel{2}}{\sqrt{3}/\cancel{2}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$

III. Practice

Plot the approximate location of each radian angle on the circle to the right, measured in standard position.

1. $\frac{7\pi}{6}$ radians Q3
2. $\frac{11\pi}{3}$ radians Q4
3. $-\frac{5\pi}{4}$ radians Q2
4. $-\frac{3\pi}{2}$ radians +y axis
5. $-\frac{13\pi}{12}$ radians Q2
6. 1 radian Q1
7. 4.5 radians Q3
8. -2 radians Q3
9. -1.2 radians Q4
10. $\frac{\pi}{10}$ radians Q1
11. $\frac{5\pi}{18}$ radians Q1
12. $-\frac{8\pi}{15}$ radians Q4



Convert each of the radian measurements above (#1 – 12) into degree measurements.

1b. $\frac{7\pi}{6} \cdot \frac{180}{\pi} = \frac{7 \cdot \overset{30}{\cancel{180}}}{\cancel{6}} = \boxed{210^\circ}$

2b. $\frac{11\pi}{3} \cdot \frac{180}{\pi} = \frac{11 \cdot \overset{60}{\cancel{180}}}{\cancel{3}} = \boxed{660^\circ}$

3b. $-\frac{5\cancel{\pi}}{4} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{-5 \cdot \overset{45}{\cancel{180}}}{\cancel{4}} = \boxed{-225^\circ}$

4b. $-\frac{3\cancel{\pi}}{2} \cdot \frac{180}{\cancel{\pi}} = -\frac{3 \cdot \overset{90}{\cancel{180}}}{\cancel{2}} = \boxed{-270^\circ}$

5b. $-\frac{13\cancel{\pi}}{12} \cdot \frac{180}{\cancel{\pi}} = \frac{-13 \cdot \overset{15}{\cancel{180}}}{\cancel{12}}$

$= \boxed{-195^\circ}$

6b. $1 \cdot \frac{180}{\pi} = \boxed{\frac{180^\circ}{\pi} \approx 57.296^\circ}$

7b. $4.5 \cdot \frac{180}{\pi} = \boxed{\frac{810^\circ}{\pi} \approx 257.831^\circ}$

8b. $-2 \cdot \frac{180^\circ}{\pi} = \boxed{\frac{-360^\circ}{\pi} \approx -114.592^\circ}$

9b. $-1.2 \cdot \frac{180}{\pi} = \boxed{\frac{-216^\circ}{\pi} \approx -68.755^\circ}$

10b. $\frac{\cancel{\pi}}{10} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{180}{10} = \boxed{18^\circ}$

11b. $\frac{5\cancel{\pi}}{18} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{5 \cdot \overset{10}{\cancel{180}}}{\cancel{18}}$

$= \boxed{50^\circ}$

12b. $-\frac{8\cancel{\pi}}{15} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{-8 \cdot \overset{12}{\cancel{180}}}{\cancel{15}}$

$= \boxed{-96^\circ}$

How were your approximations of the locations of the angles?

Accelerated Precalculus **Date:** Key

Unit Circle Trigonometry

typically add $2\pi k$ ← subtract $2\pi k$ (k for multiple times)

State two co-terminal angles, one positive and one negative, with the given angle.

answers can vary $\pm 2\pi k$ →

a. $\frac{7\pi}{10} + \frac{20\pi}{10} = \boxed{\frac{27\pi}{10}}$; $\frac{7\pi}{10} - \frac{20\pi}{10} = \boxed{\frac{-13\pi}{10}}$

b. $-\frac{2\pi}{9} + \frac{18\pi}{9} = \boxed{\frac{16\pi}{9}}$; $-\frac{2\pi}{9} - \frac{18\pi}{9} = \boxed{\frac{-20\pi}{9}}$

c. $\frac{21\pi}{5} - \frac{10\pi}{5} = \boxed{\frac{11\pi}{5}}$; $\frac{21\pi}{5} - \frac{30\pi}{5} = \boxed{\frac{-9\pi}{5}}$ (6π, $3(2\pi)$)

d. $-\frac{17\pi}{7} + \frac{28\pi}{7} = \boxed{\frac{11\pi}{7}}$ ($2(2\pi)$) ; $-\frac{17\pi}{7} + \frac{14\pi}{7} = \boxed{\frac{-3\pi}{7}}$

State the co-terminal angle between $[0, 2\pi)$ with the given angle.

only 1 answer →

e. $\frac{41\pi}{3} - \frac{6\pi}{3}(6)$; $\frac{41\pi}{3} - \frac{36\pi}{3} = \boxed{\frac{5\pi}{3}}$

f. $-\frac{31\pi}{8} + \frac{16\pi}{8}(2)$; $-\frac{31\pi}{8} + \frac{32\pi}{8} = \boxed{\frac{\pi}{8}}$

g. $\frac{59\pi}{10} - \frac{20\pi}{10}(2)$; $\frac{59\pi}{10} - \frac{40\pi}{10} = \boxed{\frac{19\pi}{10}}$

h. $-\frac{33\pi}{7} + \frac{14\pi}{7}(3)$; $-\frac{33\pi}{7} + \frac{42\pi}{7} = \boxed{\frac{9\pi}{7}}$

Simplify the following using what you know about trigonometry and the unit circle.

1. $\cos\frac{17\pi}{3}$ (-4π) $= \cos\frac{5\pi}{3} = \boxed{\frac{1}{2}}$

2. $\cos 6\pi$ (-6π) $= \cos 0 = \boxed{1}$

3. $\sin\frac{19\pi}{6}$ (-2π) $= \sin\frac{7\pi}{6} = \boxed{-\frac{1}{2}}$

4. $\sin\frac{21\pi}{4}$ (-4π) $= \sin\frac{5\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

5. $\cos\left(-\frac{17\pi}{6}\right)$ ($+4\pi$) $= \cos\frac{7\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$

6. $\cos\left(-\frac{5\pi}{3}\right)$ ($+2\pi$) $= \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

7. $\sin(-7\pi)$ ($+8\pi$) $= \sin\pi = \boxed{0}$

8. $\sin\left(-\frac{23\pi}{3}\right)$ ($+8\pi$) $= \sin\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$

9. $\sec\frac{17\pi}{4}$ (-4π) $= \sec\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$

10. $\sec\frac{9\pi}{2}$ (-4π) $= \sec\frac{\pi}{2} = \frac{1}{0} = \boxed{\text{undefined}}$

11. $\csc\frac{14\pi}{3}$ (-4π) $= \csc\frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

12. $\csc\frac{25\pi}{6}$ (-4π) $= \csc\frac{\pi}{6} = \frac{2}{1} = \boxed{2}$

13. $\sec\left(-\frac{13\pi}{6}\right)$ ($+4\pi$) $= \sec\frac{11\pi}{6} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

14. $\sec\left(-\frac{17\pi}{3}\right)$ ($+6\pi$) $= \sec\frac{\pi}{3} = \frac{2}{1} = \boxed{2}$

15. $\csc\left(-\frac{5\pi}{2}\right)$ (4π) $= \csc\frac{3\pi}{2} = \frac{1}{-1} = \boxed{-1}$

16. $\csc\left(-\frac{3\pi}{4}\right)$ ($+2\pi$) $= \csc\frac{5\pi}{4} = -\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

17. $\tan\frac{15\pi}{4}$ (-2π) $= \tan\frac{7\pi}{4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$

18. $\tan\frac{29\pi}{6}$ (-4π) $= \tan\frac{5\pi}{6} = \frac{1/2}{-\sqrt{3}/2} = \boxed{-\frac{\sqrt{3}}{3}}$

19. $\cot\frac{11\pi}{3}$ (-2π) $= \cot\frac{5\pi}{3} = \frac{1/2}{-\sqrt{3}/2} = \boxed{-\frac{\sqrt{3}}{3}}$

20. $\cot 8\pi$ (-8π) $= \cot 0 = \frac{1}{0} = \boxed{\text{undefined}}$

21. $\tan\left(-\frac{17\pi}{6}\right) = \tan\frac{7\pi}{6}$
$+4\pi$
$= \frac{-1/2}{-\sqrt{3}/2}$
$= \boxed{\frac{-\sqrt{3}}{3}}$

22. $\tan(-9\pi) = \tan\pi$
$+10\pi$
$= \frac{0}{-1}$
$= \boxed{0}$

23. $\cot\left(-\frac{9\pi}{4}\right) = \cot\frac{3\pi}{4}$
$+3\pi$
$= \frac{\sqrt{2}/2}{-\sqrt{2}/2}$
$= \boxed{-1}$

24. $\cot\left(-\frac{14\pi}{3}\right) = \cot\frac{4\pi}{3}$
$+6\pi$
$= \frac{-1/2}{-\sqrt{3}/2}$
$= \boxed{\frac{\sqrt{3}}{3}}$

$+2\pi k,\ k \in \mathbb{Z}$

State all angles that solve each of the following equations.

25. $\cos\theta = \frac{\sqrt{3}}{2}$ Q1, Q4

$\boxed{\theta = \frac{\pi}{6} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{11\pi}{6} + 2\pi k, k\in\mathbb{Z}}$

26. $\cos\theta = -\frac{\sqrt{3}}{2}$ Q2, Q3

$\boxed{\theta = \frac{5\pi}{6} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{7\pi}{6} + 2\pi k, k\in\mathbb{Z}}$

27. $\sin\theta = \frac{1}{2}$ Q1, Q2

$\boxed{\theta = \frac{\pi}{6} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{5\pi}{6} + 2\pi k, k\in\mathbb{Z}}$

28. $\sin\theta = -1$ $(0,-1)$

$\boxed{\theta = \frac{3\pi}{2} + 2\pi k, k\in\mathbb{Z}}$

29. $\cos\theta = 1$ $(1,0)$

$\boxed{\theta = 2\pi k, k\in\mathbb{Z}}$

30. $\cos\theta = -\frac{1}{2}$ Q2, Q3

$\boxed{\theta = \frac{2\pi}{3} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{4\pi}{3} + 2\pi k, k\in\mathbb{Z}}$

31. $\sin\theta = -\frac{\sqrt{2}}{2}$ Q3, Q4

$\boxed{\theta = \frac{5\pi}{4} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{7\pi}{4} + 2\pi k, k\in\mathbb{Z}}$

32. $\sin\theta = -\frac{\sqrt{3}}{2}$ Q3, Q4

$\boxed{\theta = \frac{4\pi}{3} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{5\pi}{3} + 2\pi k, k\in\mathbb{Z}}$

33. $\sec\theta = 1$
$\cos\theta = 1$ $(1,0)$

$\boxed{\theta = 2\pi k, k\in\mathbb{Z}}$

34. $\sec\theta = 2$
$\cos\theta = \frac{1}{2}$ Q1, Q4

$\boxed{\theta = \frac{\pi}{3} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{5\pi}{3} + 2\pi k, k\in\mathbb{Z}}$

35. $\csc\theta = \frac{2\sqrt{3}}{3}$
$\sin\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$
Q1, Q2

$\boxed{\theta = \frac{\pi}{3} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{2\pi}{3} + 2\pi k, k\in\mathbb{Z}}$

36. $\csc\theta = \sqrt{2}$
$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
Q1, Q2

$\boxed{\theta = \frac{\pi}{4} + 2\pi k, k\in\mathbb{Z} \text{ or } \theta = \frac{3\pi}{4} + 2\pi k, k\in\mathbb{Z}}$

37. $\tan\theta = -1$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ or $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
Q2 Q4

$\boxed{\theta = \frac{3\pi}{4} + \pi k, k\in\mathbb{Z}}$

(The angles are exactly π apart)

38. $\tan\theta = \sqrt{3}/1 = \frac{\sqrt{3}/2}{1/2}$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
Q1 or Q3

$\boxed{\theta = \frac{\pi}{3} + \pi k, k\in\mathbb{Z}}$

39. $\cot\theta = -\frac{\sqrt{3}}{3} = \frac{-1/2}{\sqrt{3}/2}$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
Q2 Q4

$\boxed{\theta = \frac{2\pi}{3} + \pi k, k\in\mathbb{Z}}$

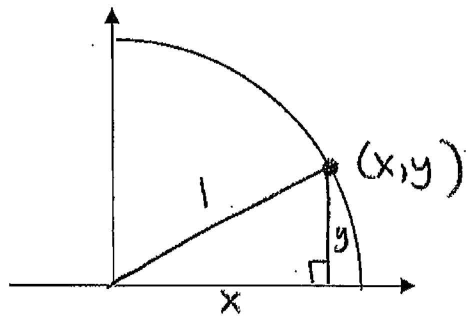
40. $\csc\theta = 1$ $(0,1)$
$\sin\theta = 1$

$\boxed{\theta = \frac{\pi}{2} + 2\pi k, k\in\mathbb{Z}}$

Accelerated Pre-Calculus Name Key

Pythagorean Identities

Use the following to develop the Pythagorean Identities.



1a. On the arc mark a point and label it (x, y).

b. Connect the origin to your point.

c. From your point draw a segment perpendicular to the x-axis, forming a right triangle.

d. Given that the radius = 1, use the Pythagorean Theorem to write an equation representing the sides of the triangle.

$x^2+y^2=1$

2a. Using your equation from #1d, substitute *x* and *y* with cos Θ and sin Θ, respectively. This is one of the Pythagorean Identities.

* $\cos^2\theta+\sin^2\theta=1$

b. Solve for $\sin^2 x$, and write this new form: $\sin^2\theta=1-\cos^2\theta$

c. Solve for $\cos^2 x$, and write this form: $\cos^2\theta=1-\sin^2\theta$

3a. Using your equation from #2, divide **each term** by $\cos^2 \Theta$.

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

b. Simplify. Your equation should *not* contain any fractions. This is another Pythagorean Identity.

* $1+\tan^2\theta=\sec^2\theta$

c. Give 2 additional forms (see #2b-c): $1=\sec^2\theta-\tan^2\theta$ $\tan^2\theta=\sec^2\theta-1$

4a. Using your equation again from #2, divide **each term** by $\sin^2 \Theta$.

$$\frac{\cos^2\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

b. Simplify. Your equation should *not* contain any fractions. This is also a Pythagorean Identity.

* $\cot^2\theta+1=\csc^2\theta$

c. Give 2 additional forms: $1=\csc^2\theta-\cot^2\theta$ $\cot^2\theta=\csc^2\theta-1$

Simplify each, using a Pythagorean Identity.

1. $\frac{1+\tan^2 x}{\csc^2 x} = \frac{\sec^2 x}{\csc^2 x}$ → or just use reciprocals

$= \sec^2 x \cdot \frac{1}{\csc^2 x}$

$= \frac{1}{\cos^2 x} \cdot \sin^2 x = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$

2. $\frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x}$

$= \boxed{\sin x}$

3. $\sin x - \sin^3 x = \sin x(1-\sin^2 x)$ gcf!

$= \boxed{\sin x \cos^2 x}$

4. $\cos^3 x + \cos x \sin^2 x = \cos x(\cos^2 x + \sin^2 x)$ gcf!

$= \cos x \ (1)$

$= \boxed{\cos x}$

5. $\frac{1-\sin^2 x}{\csc^2 x-1} = \frac{\cos^2 x}{\cot^2 x}$ → Reciprocals!

$= \cos^2 x \cdot \tan^2 x$

$= \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$

$= \boxed{\sin^2 x}$

6. $\frac{\sin^2 x + \tan^2 x + \cos^2 x}{\sec x} = \frac{\tan^2 x + 1}{\sec x}$

$= \frac{\sec^2 x}{\sec x}$

$= \boxed{\sec x}$

7. $\frac{\cot^2 x}{1+\csc x} = \frac{\csc^2 x - 1}{1+\csc x}$ → FACTOR!

$= \frac{\cancel{(\csc x + 1)}(\csc x - 1)}{\cancel{1+\csc x}}$

$= \boxed{\csc x - 1}$

8. $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$ FOIL FOIL

$= \cos^2 x - \cancel{2\sin x\cos x} + \sin^2 x$

$+ \cos^2 x + \cancel{2\sin x \cos x} + \sin^2 x$

$= \cos^2 x + \sin^2 x + \cos^2 x + \sin^2 x$

$= 1 + 1$

$= \boxed{2}$

9. $\frac{(\sec x+1)(\sec x-1)}{\sin^2 x} = \frac{\sec^2 x - 1}{\sin^2 x}$ FOIL conjugates

$= \frac{\tan^2 x}{\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

10. $\frac{1-\sin^2 x}{1-\cos^2 x} * \tan x = \frac{\cos^2 x}{\sin^2 x} \cdot \cancel{\frac{\sin x}{\cos x}}$

$= \frac{\cos x}{\sin x}$

$= \boxed{\cot x}$

Answer Bank: 2 ~~$\sin x$~~ 4 ~~$\cos x$~~ 5 ~~$\sin^2 x$~~ 1 ~~$\tan^2 x$~~ 10 ~~$\cot x$~~

9 ~~$\sec^2 x$~~ 7 ~~$\csc x - 1$~~ 3 ~~$\sin x \cos^2 x$~~ 8 ~~2~~ 6 ~~$\sec x$~~

Accelerated Pre-Calculus

Date: Key

Double Angle Identities

Double angle identities: sin 2θ, cos 2θ, and tan 2θ. Where do they come from? **Derive each.**

For sin 2θ, use the angle sum formula: sin (x + y) = sin x cos y + cos x sin y

$$\sin 2\theta = \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= \underline{\sin\theta\cos\theta} + \underline{\sin\theta\cos\theta} \quad \text{like terms!}$$
$$= 2\sin\theta\cos\theta$$

***sin 2θ =** $2\sin\theta\cos\theta$

For cos 2θ, use the angle sum formula: cos (x + y) = cos x cos y – sin x sin y

$$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= \cos^2\theta - \sin^2\theta$$

***cos 2θ =** $\cos^2\theta - \sin^2\theta$

There are *two additional forms* for cos 2θ.
To find one, replace $\cos^2\theta$ with **$1-\sin^2\theta$**.

or cos 2θ = $1-2\sin^2\theta$

$$= \underline{(1-\sin^2\theta)} - \underline{\sin^2\theta}$$
$$= 1-2\sin^2\theta$$

To find the other, replace $\sin^2\theta$ with **$1-\cos^2\theta$**.

or cos 2θ = $2\cos^2\theta - 1$

$$= \cos^2\theta - (1-\cos^2\theta)$$
$$= \underline{\cos^2\theta} - 1 + \underline{\cos^2\theta}$$
$$= 2\cos^2\theta - 1$$

For tan 2θ, use the angle sum formula: $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

***tan 2θ =** $\dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\tan 2\theta = \frac{\tan\theta + \tan\theta}{1-\tan\theta\tan\theta}$$
$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

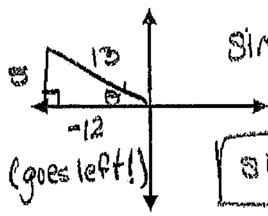
$\frac{\sin 2\theta}{\cos 2\theta}$ may be easier when you already have sin2θ and cos2θ.

Practice *hypotenuse is never negative!

a. Draw and label the reference triangle in the appropriate quadrant for the given trig information about theta. (You will need to determine the length of the third side of the triangle.)

b. Find the exact values of sin 2θ, cos 2θ, and tan 2θ using the double-angle formulas.

1. $\sin\theta = \frac{5 \text{ opp}}{13 \text{ hyp}}$ and $\frac{\pi}{2} < \theta < \pi$ Q2



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

$\boxed{\sin 2\theta = \frac{-120}{169}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

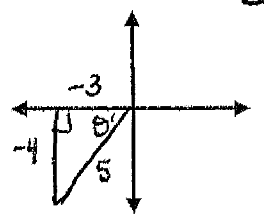
$\boxed{\cos 2\theta = \frac{119}{169}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$= \frac{-120/169}{119/169}$

$\boxed{\tan 2\theta = \frac{-120}{119}}$

2. $\tan\theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$ Q3



$\sin 2\theta = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$\boxed{\sin 2\theta = \frac{24}{25}}$

$\cos 2\theta = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$

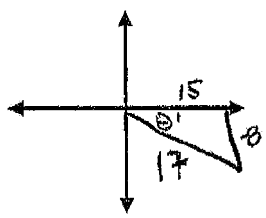
$= \frac{9}{25} - \frac{16}{25}$

$\boxed{\cos 2\theta = -\frac{7}{25}}$

$\tan 2\theta = \frac{24/25}{-7/25}$

$\boxed{\tan 2\theta = \frac{24}{-7}}$

3. $\cos\theta = \frac{15}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$ Q4



$\sin 2\theta = 2\left(-\frac{8}{17}\right)\left(\frac{15}{17}\right)$

$\boxed{\sin 2\theta = \frac{-240}{289}}$

$\cos 2\theta = \left(\frac{15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2$

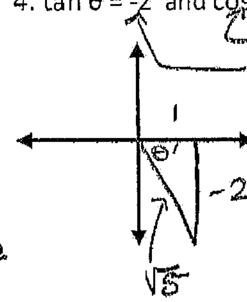
$= \frac{225}{289} - \frac{64}{289}$

$\boxed{\cos 2\theta = \frac{161}{289}}$

$\tan 2\theta = \frac{-240/289}{161/289}$

$\boxed{\tan 2\theta = \frac{-240}{161}}$

4. $\tan\theta = -2$ and $\cos\theta > 0$ Q1 or Q3 must be Q3!



$\sin 2\theta = 2\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)$

$\boxed{\sin 2\theta = -\frac{4}{5}}$

$\cos 2\theta = \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{-2}{\sqrt{5}}\right)^2$

$= \frac{1}{5} - \frac{4}{5}$

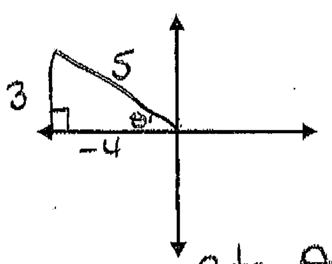
$\boxed{\cos 2\theta = -3/5}$

$\tan 2\theta = \frac{-4/5}{-3/5}$

$\boxed{\tan 2\theta = \frac{4}{3}}$

5. $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$ Q2

Using the tan 2θ formula, find <u>only</u> tan 2θ

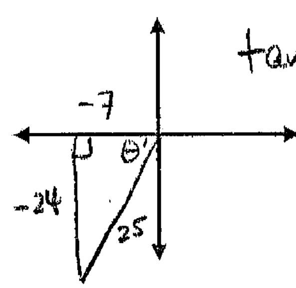


$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{3}{-4}\right)}{1-\left(\frac{3}{-4}\right)^2}$

$= \frac{-\frac{3}{2}}{1-\frac{9}{16}} = \frac{-\frac{24}{16}}{\frac{16}{16}-\frac{9}{16}} = \boxed{\frac{-24}{7}}$

6. $\sin\theta < 0$ and $\cos\theta = -\frac{7}{25}$ both neg in Q3

Using the tan 2θ formula, find <u>only</u> tan 2θ



$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{24}{7}\right)}{1-\left(\frac{24}{7}\right)^2}$

$= \frac{\frac{48}{7}}{1-\frac{576}{49}}$

$= \frac{\frac{336}{49}}{\frac{49}{49}-\frac{576}{49}} = \boxed{\frac{336}{-527}}$

Accelerated Precalculus

Quiz: Trigonometry Review

Name: Key

Date: ______________ Period: ______

Evaluate.

1. $\tan\frac{4\pi}{3} = \frac{-\sqrt{3}/2}{-1/2} = \boxed{\sqrt{3}}$

2. $\sec\frac{3\pi}{4}$ $-\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

3. $\sin\left(-\frac{5\pi}{6}\right) = \sin\frac{7\pi}{6} = \boxed{-\frac{1}{2}}$

4. $\csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

5. $\cot(-\pi) = \frac{-1}{0} = \boxed{\text{und}}$

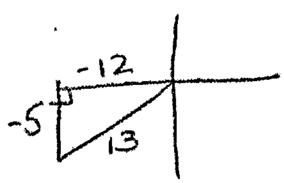
6. $\cos\frac{23\pi}{4} = \cos\frac{7\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

7. $\cot\frac{13\pi}{3} = \cot\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{3}}$

8. $\csc\left(-\frac{\pi}{2}\right) = \frac{1}{-1} = \boxed{-1}$

9. $\sec\frac{7\pi}{2} = \sec\frac{3\pi}{2} = \frac{1}{0} = \boxed{\text{und}}$

10. If $\sec\theta = -\frac{13}{12}$ and $\underbrace{\tan\theta > 0}_{Q1, Q3}$, find $\sin 2\theta, \cos 2\theta, \tan 2\theta$.



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)$
$= \boxed{\frac{120}{169}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)$
$= \frac{144}{169} - \frac{25}{169}$
$= \boxed{\frac{119}{169}}$

$\tan 2\theta = \boxed{\frac{120}{119}}$

Solve each equation for the variable on the interval from $[0, 2\pi)$.

11. $4\cos^2\theta - 4\cos\theta = 3$

$4\cos^2\theta - 4\cos\theta - 3 = 0$
$(2\cos\theta - 3)(2\cos\theta + 1) = 0$
$2\cos\theta - 3 = 0$ $\quad 2\cos\theta + 1 = 0$
$\cos\theta = 1.5$ $\quad \cos\theta = -\frac{1}{2}$
Not Possible $\quad \boxed{\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}}$

12. $1 = \cot^2\theta + \csc\theta$

$1 = (\csc^2\theta - 1) + \csc\theta$
$0 = \csc^2\theta + \csc\theta - 2$
$0 = (\csc\theta - 1)(\csc\theta + 2)$
$\csc\theta - 1 = 0$ $\quad \csc\theta + 2 = 0$
$\csc\theta = 1$ $\quad \csc\theta = -2$
$\sin\theta = 1$ $\quad \sin\theta = -\frac{1}{2}$
$\boxed{\theta = \frac{\pi}{2}}$ $\quad \boxed{\theta = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$

13. $\sin 2x\cos x = \sin x$

$2\sin x\cos x\cos x = \sin x$
$2\sin x\cos^2 x = \sin x$
$\sin x = 0$ or $2\cos^2 x = 1$
$\boxed{x = 0 \text{ or } \pi}$ $\quad \cos^2 x = \frac{1}{2}$
$\cos x = \frac{\pm\sqrt{2}}{2}$
$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

Accelerated Pre-Calculus

Name Key

Solving Trig Equations Review

Date ________________ Period ______

Solve each equation for the variable on the interval from [0, 2π). Use Pythagorean and/or Double Angle Identities to rewrite as needed.

1. $2\sin^2 x = 1$

$\frac{2\sin^2 x}{2} = \frac{1}{2}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

2. $2\sin^2 x + 3\sin x + 1 = 0$

① factor
② set factors = 0

$2\sin x \cdot 1\sin x$ $\quad 1 \cdot 1$

$(2\sin x+1)(1\sin x+1)=0$

$2\sin x+1=0$ or $\sin x+1=0$

$2\sin x=-1$ $\quad \sin x=-1$

$\sin x=-\frac{1}{2}$ $\quad \boxed{x=\frac{3\pi}{2}}$

$\boxed{x=\frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$

3. $\sin^2 x + 1 = \cos x$

$(1-\cos^2 x)+1=\cos x$

$0=\cos^2 x+\cos x-2$

$0=(\cos x+2)(\cos x-1)$

$\cos x+2=0$ or $\cos x-1=0$

$\cos x=-2$ $\quad \cos x=1$

Not Possible $\quad \boxed{x=0 \text{ rad,}}$

4. $\sec^2 x + \tan x - 1 = 0$

$(1+\tan^2 x)+\tan x-1=0$

$\tan^2 x+\tan x=0$

$\tan x\,(\tan x+1)=0$

$\tan x=0$ or $\tan x+1=0$

$\boxed{x=0 \text{ or } \pi}$ $\quad \tan x=-1$

$\boxed{x=\frac{3\pi}{4} \text{ or } \frac{7\pi}{4}}$

5. $\cot^2 x - \csc x = 1$

$(\csc^2 x-1)-\csc x=1$

$\csc^2 x-\csc x-2=0$

$(\csc x-2)(\csc x+1)=0$

$\csc x-2=0$ or $\csc x+1=0$

$\csc x=2$ $\quad \csc x=-1$

$\sin x=\frac{1}{2}$ $\quad \sin x=-1$

$\boxed{x=\frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$ $\quad \boxed{x=\frac{3\pi}{2}}$

6. $5\sin x - 5 = \cos^2 x$

$5\sin x-5=(1-\sin^2 x)$

$\sin^2 x+5\sin x-6=0$

$(\sin x+6)(\sin x-1)=0$

$\sin x+6=0$ or $\sin x-1=0$

$\sin x=-6$ $\quad \sin x=1$

Not Possible $\quad \boxed{x=\frac{\pi}{2}}$

7. $\tan^2 x + 3\sec x + 3 = 0$

$(\sec^2 x - 1) + 3\sec x + 3 = 0$

$\sec^2 x + 3\sec x + 2 = 0$

$(\sec x + 2)(\sec x + 1) = 0$

$\sec x + 2 = 0$ or $\sec x + 1 = 0$

$\sec x = -2$ $\quad$ $\sec x = -1$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = -1$

$\boxed{x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}}$ $\boxed{x = \pi}$

8. $3\sin x = 2\cos^2 x - 3\cancel{\cot x}\cancel{\tan x}$

$3\sin x = 2(1 - \sin^2 x) - 3$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = -1$ $\quad$ $\sin x = -1$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$ $\boxed{x = \frac{3\pi}{2}}$

9. $\cos 2\theta - 1 = 0$

$(2\cos^2\theta - 1) - 1 = 0$

$2\cos^2\theta - 2 = 0$

$2\cos^2\theta = 2$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\boxed{\theta = 0 \text{ or } \pi}$

10. $\sin 2\theta - \cos\theta = 0$

$(2\sin\theta\cos\theta) - \cos\theta = 0$

$\cos\theta\,(2\sin\theta - 1) = 0$

$\cos\theta = 0$ or $2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$ $\boxed{\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$

11. $\cos 2\theta + \cos\theta = 0$

$(2\cos^2\theta - 1) + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2}$ $\quad$ $\cos\theta = -1$

$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}}$ $\boxed{\theta = \pi}$

12. $\sin\theta = \cos 2\theta - 1$

$\sin\theta = (1 - 2\sin^2\theta) - 1$

$2\sin^2\theta + \sin\theta = 0$

$\sin\theta\,(2\sin\theta + 1) = 0$

$\sin\theta = 0$ $\quad$ $2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$\boxed{\theta = 0 \text{ or } \pi}$ $\boxed{\theta = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$

13. $\sin 2\theta - \tan\theta = 0$

$(2\sin\theta\cos\theta) - \left(\frac{\sin\theta}{\cos\theta}\right) = 0$

$\sin\theta\left(2\cos\theta - \frac{1}{\cos\theta}\right) = 0$

$\sin\theta = 0$ or $2\cos\theta - \frac{1}{\cos\theta} = 0$

$\boxed{\theta = 0 \text{ or } \pi}$

$2\cos\theta = \frac{1}{\cos\theta}$

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

14. $5\sin\theta - 2\cos^2\theta = 1$

$5\sin\theta - 2(1 - \sin^2\theta) = 1$

$2\sin^2\theta + 5\sin\theta - 3 = 0$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta + 3 = 0$

$\sin\theta = \frac{1}{2}$ $\quad$ $\sin\theta = -3$

$\boxed{\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$ $\quad$ Not Possible

Accelerated Pre-Calculus

Name Key

Solving Trig Equations Review

Date ______________ Period ______

Solve each equation for the variable on the interval from [0, 2π). Use Pythagorean and/or Double Angle Identities to rewrite as needed.

1. $2\sin^2 x = 1$

$\frac{2\sin^2 x}{2} = \frac{1}{2}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

2. $2\sin^2 x + 3\sin x + 1 = 0$

① factor
② set factors = 0

$2\sin x \cdot 1\sin x$ $\quad$ $1 \cdot 1$

$(2\sin x + 1)(1\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = -1$ $\quad$ $\sin x = -1$

$\sin x = -\frac{1}{2}$ $\quad$ $\boxed{x = \frac{3\pi}{2}}$

$\boxed{x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$

3. $\sin^2 x + 1 = \cos x$

$(1 - \cos^2 x) + 1 = \cos x$

$0 = \cos^2 x + \cos x - 2$

$0 = (\cos x + 2)(\cos x - 1)$

$\cos x + 2 = 0$ or $\cos x - 1 = 0$

$\cos x = -2$ $\quad$ $\cos x = 1$

Not Possible $\quad$ $\boxed{x = 0 \text{ rad,}}$

4. $\sec^2 x + \tan x - 1 = 0$

$(1 + \tan^2 x) + \tan x - 1 = 0$

$\tan^2 x + \tan x = 0$

$\tan x\ (\tan x + 1) = 0$

$\tan x = 0$ or $\tan x + 1 = 0$

$\boxed{x = 0 \text{ or } \pi}$ $\quad$ $\tan x = -1$

$\boxed{x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}}$

5. $\cot^2 x - \csc x = 1$

$(\csc^2 x - 1) - \csc x = 1$

$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$ or $\csc x + 1 = 0$

$\csc x = 2$ $\quad$ $\csc x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$\boxed{x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$ $\quad$ $\boxed{x = \frac{3\pi}{2}}$

6. $5\sin x - 5 = \cos^2 x$

$5\sin x - 5 = (1 - \sin^2 x)$

$\sin^2 x + 5\sin x - 6 = 0$

$(\sin x + 6)(\sin x - 1) = 0$

$\sin x + 6 = 0$ or $\sin x - 1 = 0$

$\sin x = -6$ $\quad$ $\sin x = 1$

Not Possible $\quad$ $\boxed{x = \frac{\pi}{2}}$

7. $\tan^2 x + 3\sec x + 3 = 0$

$$(\sec^2 x - 1) + 3\sec x + 3 = 0$$
$$\sec^2 x + 3\sec x + 2 = 0$$
$$(\sec x + 2)(\sec x + 1) = 0$$
$$\sec x + 2 = 0 \quad \text{or} \quad \sec x + 1 = 0$$
$$\sec x = -2 \qquad \sec x = -1$$
$$\cos x = -\frac{1}{2} \qquad \cos x = -1$$
$$\boxed{x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}} \quad \boxed{x = \pi}$$

8. $3\sin x = 2\cos^2 x - 3\cancel{\cot x}\cancel{\tan x}$

$$3\sin x = 2(1 - \sin^2 x) - 3$$
$$2\sin^2 x + 3\sin x + 1 = 0$$
$$(2\sin x + 1)(\sin x + 1) = 0$$
$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$
$$2\sin x = -1 \qquad \sin x = -1$$
$$\sin x = -\frac{1}{2} \qquad \boxed{x = \frac{3\pi}{2}}$$
$$\boxed{x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$$

9. $\cos 2\theta - 1 = 0$

$$(2\cos^2\theta - 1) - 1 = 0$$
$$2\cos^2\theta - 2 = 0$$
$$2\cos^2\theta = 2$$
$$\cos^2\theta = 1$$
$$\cos\theta = \pm 1$$
$$\boxed{\theta = 0 \text{ or } \pi}$$

10. $\sin 2\theta - \cos\theta = 0$

$$(2\sin\theta\cos\theta) - \cos\theta = 0$$
$$\cos\theta\,(2\sin\theta - 1) = 0$$
$$\cos\theta = 0 \quad \text{or} \quad 2\sin\theta - 1 = 0$$
$$\boxed{\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}} \qquad 2\sin\theta = 1$$
$$\sin\theta = \frac{1}{2}$$
$$\boxed{\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$$

11. $\cos 2\theta + \cos\theta = 0$

$$(2\cos^2\theta - 1) + \cos\theta = 0$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$
$$2\cos\theta - 1 = 0 \quad \text{or} \quad \cos\theta + 1 = 0$$
$$\cos\theta = \frac{1}{2} \qquad \cos\theta = -1$$
$$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}} \quad \boxed{\theta = \pi}$$

12. $\sin\theta = \cos 2\theta - 1$

$$\sin\theta = (1 - 2\sin^2\theta) - 1$$
$$2\sin^2\theta + \sin\theta = 0$$
$$\sin\theta\,(2\sin\theta + 1) = 0$$
$$\sin\theta = 0 \qquad 2\sin\theta + 1 = 0$$
$$\boxed{\theta = 0 \text{ or } \pi} \qquad \sin\theta = -\frac{1}{2}$$
$$\boxed{\theta = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}$$

13. $\sin 2\theta - \tan\theta = 0$

$$(2\sin\theta\cos\theta) - \left(\frac{\sin\theta}{\cos\theta}\right) = 0$$
$$\sin\theta\left(2\cos\theta - \frac{1}{\cos\theta}\right) = 0$$
$$\sin\theta = 0 \quad \text{or} \quad 2\cos\theta - \frac{1}{\cos\theta} = 0$$
$$\boxed{\theta = 0 \text{ or } \pi} \qquad 2\cos\theta = \frac{1}{\cos\theta}$$
$$2\cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{2}$$
$$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$
$$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

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$$5\sin\theta - 2(1 - \sin^2\theta) = 1$$
$$2\sin^2\theta + 5\sin\theta - 3 = 0$$
$$(2\sin\theta - 1)(\sin\theta + 3) = 0$$
$$2\sin\theta - 1 = 0 \quad \text{or} \quad \sin\theta + 3 = 0$$
$$\sin\theta = \frac{1}{2} \qquad \sin\theta = -3$$
$$\boxed{\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$$

Not Possible