9.2 The Pythagorean Theorem

'hat you should learn

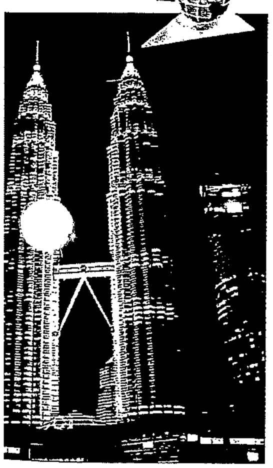
GOAL 1 Prove the
thagorean Theorem.

GOAL 2 Use the
thagorean Theorem to
lve **real-life** problems,
ch as determining how far
adder will reach in **Ex. 32.**

'hy you should learn it

To measure **real-life**
ngths indirectly, such as
e length of the support
am of a skywalk in
ample 4.

REAL LIFE



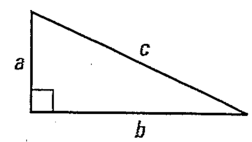
GOAL 1 PROVING THE PYTHAGOREAN THEOREM

In this lesson, you will study one of the most famous theorems in mathematics—the *Pythagorean Theorem.* The relationship it describes has been known for thousands of years.

THEOREM

THEOREM 9.4 ***Pythagorean Theorem***

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



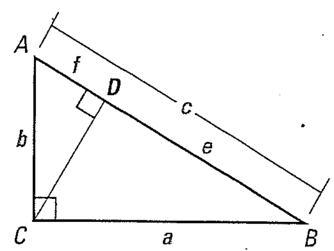
$c^2 = a^2 + b^2$

PROVING THE PYTHAGOREAN THEOREM There are many different proofs of the Pythagorean Theorem. One is shown below. Other proofs are found in Exercises 37 and 38 on page 540, and in the *Math and History* feature on page 557.

GIVEN In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE $a^2 + b^2 = c^2$

Plan for Proof Draw altitude $\overline{CD}$ to the hypotenuse. Then apply Geometric Mean Theorem 9.3, which states that when the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.



Proof

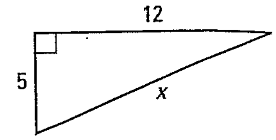
Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	**1.** Perpendicular Postulate
2. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**2.** Geometric Mean Theorem 9.3
3. $ce = a^2$ and $cf = b^2$	**3.** Cross product property
4. $ce + cf = a^2 + b^2$	**4.** Addition property of equality
5. $c(e + f) = a^2 + b^2$	**5.** Distributive property
6. $e + f = c$	**6.** Segment Addition Postulate
7. $c^2 = a^2 + b^2$	**7.** Substitution property of equality

GOAL 2 USING THE PYTHAGOREAN THEOREM

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$. For example, the integers 3, 4, and 5 form a Pythagorean triple because $5^2 = 3^2 + 4^2$.

EXAMPLE 1 *Finding the Length of a Hypotenuse*

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



SOLUTION

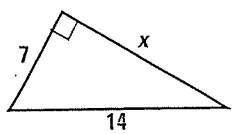
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$x^2 = 5^2 + 12^2$	**Substitute.**
$x^2 = 25 + 144$	**Multiply.**
$x^2 = 169$	**Add.**
$x = 13$	**Find the positive square root.**

▶ Because the side lengths 5, 12, and 13 are integers, they form a Pythagorean triple.

Many right triangles have side lengths that do not form a Pythagorean triple, as shown in Example 2.

EXAMPLE 2 *Finding the Length of a Leg*

Find the length of the leg of the right triangle.



SOLUTION

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$14^2 = 7^2 + x^2$	**Substitute.**
$196 = 49 + x^2$	**Multiply.**
$147 = x^2$	**Subtract 49 from each side.**
$\sqrt{147} = x$	**Find the positive square root.**
$\sqrt{49} \cdot \sqrt{3} = x$	**Use product property.**
$7\sqrt{3} = x$	**Simplify the radical.**

STUDENT HELP

Skills Review
For help with simplifying radicals, see p. 799.

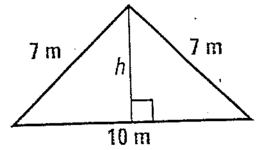
In Example 2, the side length was written as a radical in simplest form. In real-life problems, it is often more convenient to use a calculator to write a decimal approximation of the side length. For instance, in Example 2, $x = 7 \cdot \sqrt{3} \approx 12.1$.

EXAMPLE 3 *Finding the Area of a Triangle*

STUDENT HELP

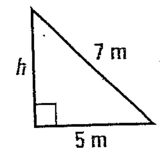
Look Back
For help with finding the area of a triangle, see p. 51.

Find the area of the triangle to the nearest tenth of a square meter.



SOLUTION

You are given that the base of the triangle is 10 meters, but you do not know the height h.



Because the triangle is isosceles, it can be divided into two congruent right triangles with the given dimensions. Use the Pythagorean Theorem to find the value of h.

$7^2 = 5^2 + h^2$ **Pythagorean Theorem**

$49 = 25 + h^2$ **Multiply.**

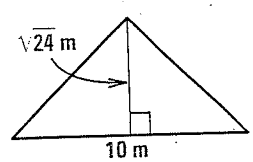
$24 = h^2$ **Subtract 25 from both sides.**

$\sqrt{24} = h$ **Find the positive square root.**

Now find the area of the original triangle.

$$\text{Area} = \frac{1}{2}bh$$

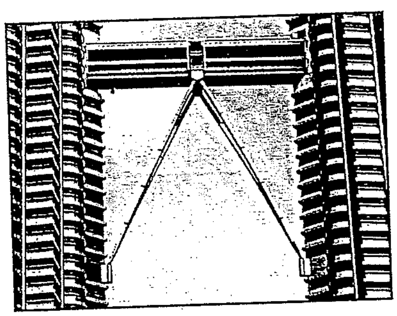
$$= \frac{1}{2}(10)(\sqrt{24}) \approx 24.5 \text{ m}^2$$

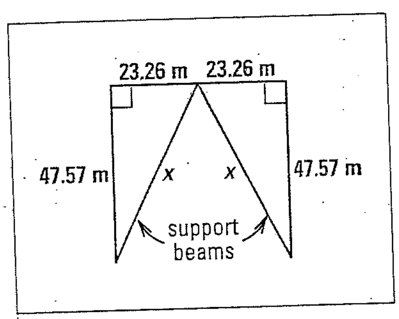


▶ The area of the triangle is about 24.5 m^2.

EXAMPLE 4 *Indirect Measurement*

SUPPORT BEAM The skyscrapers shown on page 535 are connected by a skywalk with support beams. You can use the Pythagorean Theorem to find the approximate length of each support beam.





Each support beam forms the hypotenuse of a right triangle. The right triangles are congruent, so the support beams are the same length.

$x^2 = (23.26)^2 + (47.57)^2$ **Pythagorean Theorem**

$x = \sqrt{(23.26)^2 + (47.57)^2}$ **Find the positive square root.**

$x \approx 52.95$ **Use a calculator to approximate.**

▶ The length of each support beam is about 52.95 meters.

FOCUS ON PEOPLE

REAL LIFE

CESAR PELLI is an architect who designed the twin skyscrapers shown on page 535. These 1483 foot buildings tower over the city of Kuala Lumpur, Malaysia.

GUIDED PRACTICE

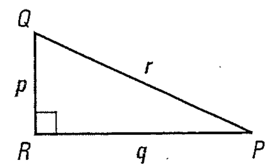
Vocabulary Check ✓

1. State the Pythagorean Theorem in your own words.

Concept Check ✓

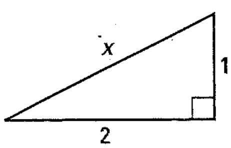
2. Which equations are true for $\triangle PQR$?

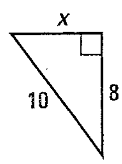
   A. $r^2 = p^2 + q^2$
   B. $q^2 = p^2 + r^2$
   C. $p^2 = r^2 - q^2$
   D. $r^2 = (p + q)^2$
   E. $p^2 = q^2 + r^2$

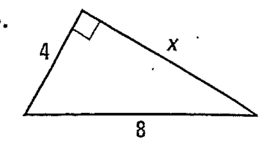


Skill Check ✓

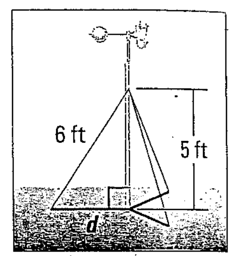
Find the unknown side length. Tell whether the side lengths form a Pythagorean triple.

3. 

4. 

5. 

6. **ANEMOMETER** An anemometer (an uh MAHM ih tur) is a device used to measure windspeed. The anemometer shown is attached to the top of a pole. Support wires are attached to the pole 5 feet above the ground. Each support wire is 6 feet long. How far from the base of the pole is each wire attached to the ground?

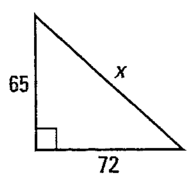


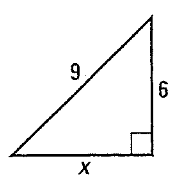
PRACTICE AND APPLICATIONS

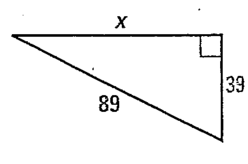
STUDENT HELP

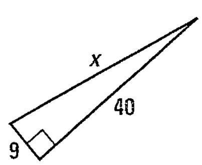
→ Extra Practice to help you master skills is on p. 819.

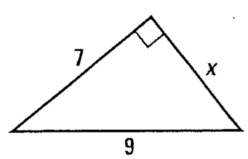
FINDING SIDE LENGTHS **Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple.**

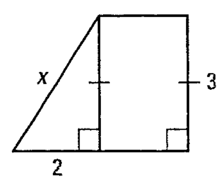
7. 

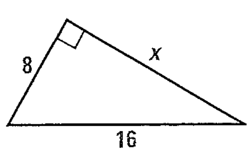
8. 

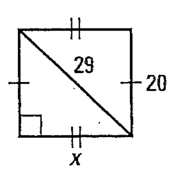
9. 

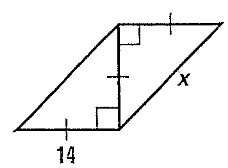
10. 

11. 

12. 

13. 

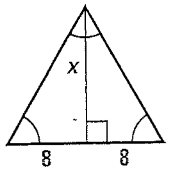
14. 

15. 

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 7–24
Example 2: Exs. 7–24
Example 3: Exs. 25–30
Example 4: Exs. 31–36

FINDING LENGTHS Find the value of *x*. Simplify answers that are radicals.

16.


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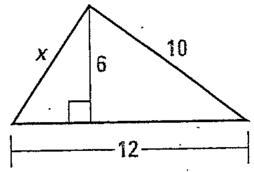

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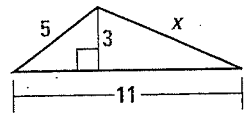

PYTHAGOREAN TRIPLES The variables *r* and *s* represent the lengths of the legs of a right triangle, and *t* represents the length of the hypotenuse. The values of *r*, *s*, and *t* form a Pythagorean triple. Find the unknown value.

19. $r = 12, s = 16$

20. $r = 9, s = 12$

21. $r = 18, t = 30$

22. $s = 20, t = 101$

23. $r = 35, t = 37$

24. $t = 757, s = 595$

DENT HELP

k Back
help with finding
as of quadrilaterals,
pp. 372–375.

FINDING AREA Find the area of the figure. Round decimal answers to the nearest tenth.

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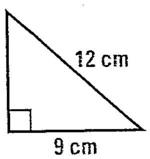

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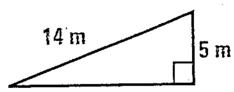

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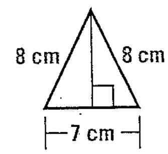

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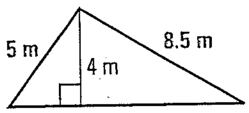

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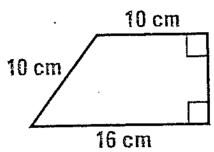

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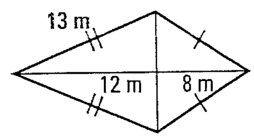
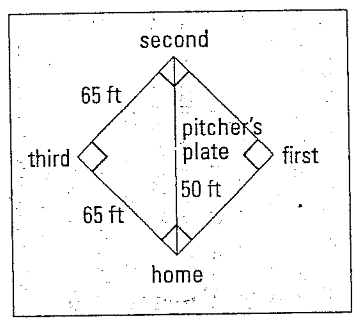
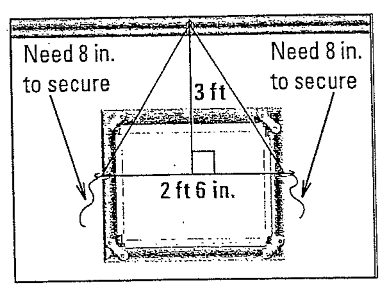


31. **SOFTBALL DIAMOND** In slow-pitch softball, the distance between consecutive bases is 65 feet. The pitcher's plate is located on a line between second base and home plate, 50 feet from home plate. How far is the pitcher's plate from second base? Justify your answer.



32. **SAFETY** The distance of the base of a ladder from the wall it leans against should be at least $\frac{1}{4}$ of the ladder's total length. Suppose a 10 foot ladder is placed according to these guidelines. Give the minimum distance of the base of the ladder from the wall. How far up the wall will the ladder reach? Explain. Include a sketch with your explanation.

33. **ART GALLERY** You want to hang a painting 3 feet from a hook near the ceiling of an art gallery, as shown. In addition to the length of wire needed for hanging, you need 16 inches of wire to secure the wire to the back of the painting. Find the total length of wire needed to hang the painting.



NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 535–541

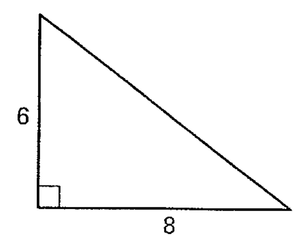
GOAL Use the Pythagorean Theorem to solve problems

> **VOCABULARY**
>
> A **Pythagorean triple** is a set of three positive integers a, b, and c, that satisfy the equation $c^2 = a^2 + b^2$.
>
> **Theorem 9.4 Pythagorean Theorem**
> In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

EXAMPLE 1 *Find the Length of a Hypotenuse*

Find the length of the hypotenuse of the right triangle.

Tell whether the side lengths form a Pythagorean triple.



SOLUTION

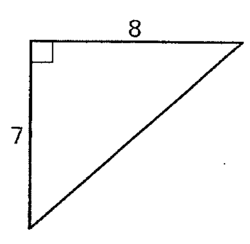
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$x^2 = 6^2 + 8^2$	Substitute.
$x^2 = 36 + 64$	Multiply.
$x^2 = 100$	Add.
$x = 10$	Find the positive square root.

Because the side lengths 6, 8, and 10 are integers, they form a Pythagorean triple.

Exercises for Example 1

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.

1.



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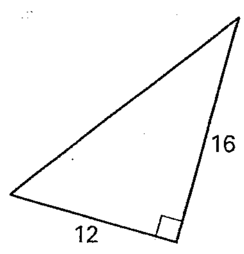

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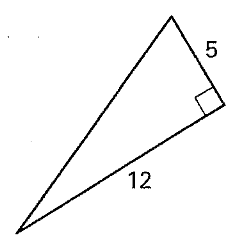
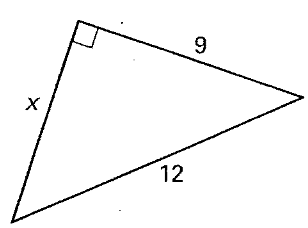


EXAMPLE 2 *Finding the Length of a Leg*

Find the length of the leg of the right triangle.



NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 535–541

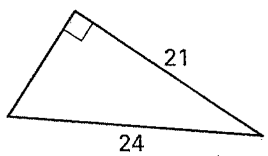
SOLUTION

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$12^2 = 9^2 + x^2$	Substitute.
$144 = 81 + x^2$	Multiply.
$63 = x^2$	Subtract 81 from each side.
$\sqrt{63} = x$	Find the positive square root.

Exercises for Example 2

Find the unknown side length. Round to the nearest tenth, if necessary.

4.



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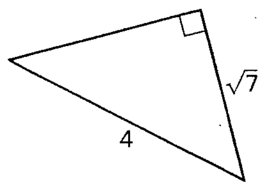

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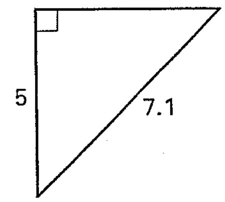
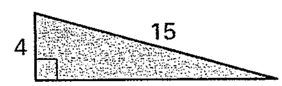


EXAMPLE 3 *Finding the Area of a Triangle*

Find the area of the triangle to the nearest tenth.



SOLUTION

In this case, the side of length 4 can be used as the height and the side of unknown length can be used as the base. To find the length of the unknown side, use the Pythagorean Theorem.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$15^2 = 4^2 + b^2$	Substitute.
$\sqrt{209} = b$	Solve for b.

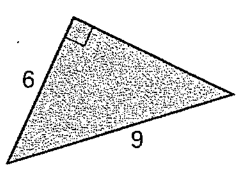
Now find the area of the triangle.

$$A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{209})(4) \approx 28.9 \text{ square units}$$

Exercises for Example 3

Find the area of the triangle to the nearest tenth.

7.



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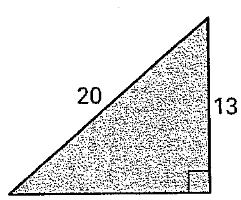

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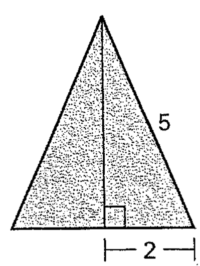


Lesson 9.2

LESSON 9.2

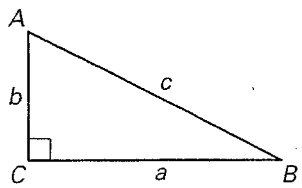
NAME ______________________ DATE ________

Practice A

For use with pages 535–541

Use the labeled triangles to state the Pythagorean Theorem.

1.



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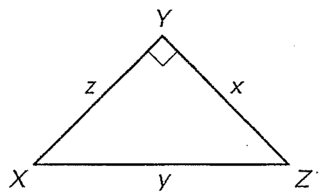

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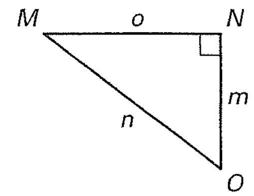


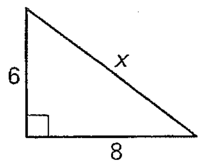
Simplify the radical.

4. $\sqrt{12}$ **5.** $\sqrt{48}$ **6.** $\sqrt{20}$

7. $\sqrt{18}$ **8.** $\sqrt{60}$ **9.** $\sqrt{75}$

Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple.

10.



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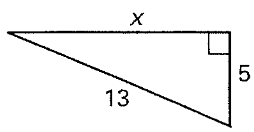

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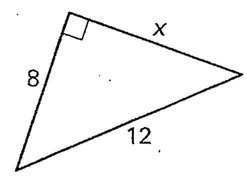

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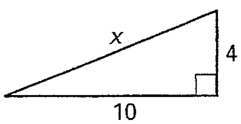

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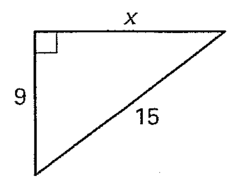

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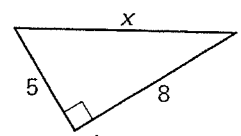
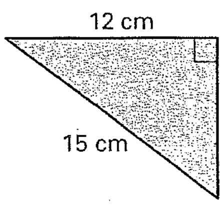


Find the area of the figure. Round decimal answers to the nearest tenth.

16.



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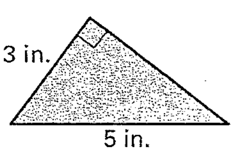
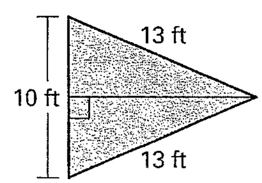


18.



Solve. Round your answer to the nearest tenth.

19. A 48-inch wide screen television means that the measure along the diagonal is 48 inches. If the screen is a square, what are the dimensions of the length and width?

20. The doorway of the family room measures $6\frac{1}{2}$ feet by 3 feet. What is the length of the diagonal of the doorway?

21. You place a 10-foot ladder against a wall. If the base of the ladder is 3 feet from the wall, how high up the wall does the top of the ladder reach?

9.3 The Converse of the Pythagorean Theorem

What you should learn

GOAL 1 Use the Converse of the Pythagorean Theorem to solve problems.

GOAL 2 Use side lengths to classify triangles by their angle measures.

Why you should learn it

▼ To determine whether real-life angles are right angles, such as the four angles formed by the foundation of a building in **Example 3**.

REAL LIFE

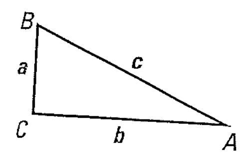
GOAL 1 USING THE CONVERSE

In Lesson 9.2, you learned that if a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. The Converse of the Pythagorean Theorem is also true, as stated below. Exercise 43 asks you to prove the Converse of the Pythagorean Theorem.

THEOREM

THEOREM 9.5 ***Converse of the Pythagorean Theorem***

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



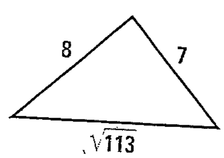
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

You can use the Converse of the Pythagorean Theorem to verify that a given triangle is a right triangle, as shown in Example 1.

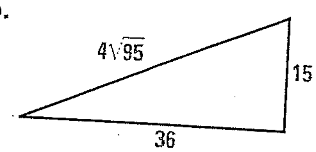
EXAMPLE 1 *Verifying Right Triangles*

The triangles below appear to be right triangles. Tell whether they are right triangles.

a.



b.



SOLUTION

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(\sqrt{113})^2 \stackrel{?}{=} 7^2 + 8^2$

$113 \stackrel{?}{=} 49 + 64$

$113 = 113$ ✓

The triangle is a right triangle.

b. $(4\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$4^2 \cdot (\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$16 \cdot 95 \stackrel{?}{=} 225 + 1296$

$1520 \neq 1521$

The triangle is not a right triangle.

GOAL 2 CLASSIFYING TRIANGLES

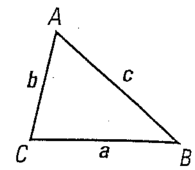
Sometimes it is hard to tell from looking whether a triangle is obtuse or acute. The theorems below can help you tell.

THEOREMS

THEOREM 9.6

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

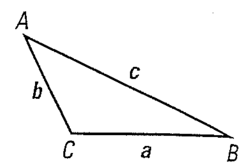


$c^2 < a^2 + b^2$

THEOREM 9.7

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

EXAMPLE 2 *Classifying Triangles*

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*.

a. 38, 77, 86

b. 10.5, 36.5, 37.5

SOLUTION

You can use the Triangle Inequality to confirm that each set of numbers can represent the side lengths of a triangle.

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

a.

$c^2 \underline{\ ?\ } a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$86^2 \underline{\ ?\ } 38^2 + 77^2$	Substitute.
$7396 \underline{\ ?\ } 1444 + 5929$	Multiply.
$7396 > 7373$	c^2 is greater than $a^2 + b^2$.

▶ Because $c^2 > a^2 + b^2$, the triangle is obtuse.

b.

$c^2 \underline{\ ?\ } a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$37.5^2 \underline{\ ?\ } 10.5^2 + 36.5^2$	Substitute.
$1406.25 \underline{\ ?\ } 110.25 + 1332.25$	Multiply.
$1406.25 < 1442.5$	c^2 is less than $a^2 + b^2$.

▶ Because $c^2 < a^2 + b^2$, the triangle is acute.

STUDENT HELP

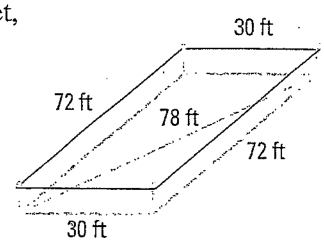
Look Back
For help with the Triangle Inequality, see p. 297.

EXAMPLE 3 *Building a Foundation*

CONSTRUCTION You use four stakes and string to mark the foundation of a house. You want to make sure the foundation is rectangular.

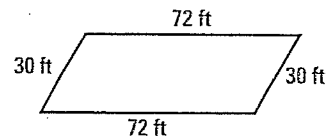
a. A friend measures the four sides to be 30 feet, 30 feet, 72 feet, and 72 feet. He says these measurements prove the foundation is rectangular. Is he correct?

b. You measure one of the diagonals to be 78 feet. Explain how you can use this measurement to tell whether the foundation will be rectangular.

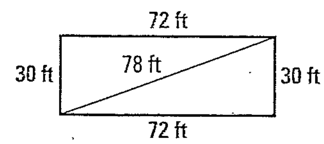


SOLUTION

a. Your friend is not correct. The foundation could be a nonrectangular parallelogram, as shown at the right.



b. The diagonal divides the foundation into two triangles. Compare the square of the length of the longest side with the sum of the squares of the shorter sides of one of these triangles. Because $30^2 + 72^2 = 78^2$, you can conclude that both the triangles are right triangles.



▶ The foundation is a parallelogram with two right angles, which implies that it is rectangular.

TUDENT HELP

ook Back
ɔr help with classifying ɹadrilaterals, see hapter 6.

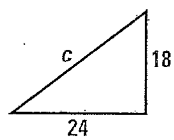
JIDED PRACTICE

ɔcabulary Check ✓

1. State the Converse of the Pythagorean Theorem in your own words.

Concept Check ✓

2. Use the triangle shown at the right. Find values for c so that the triangle is acute, right, and obtuse.



Skill Check ✓

In Exercises 3–6, match the side lengths with the appropriate description.

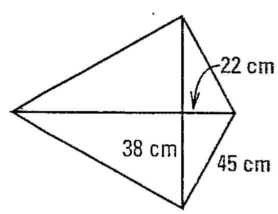
3. 2, 10, 11 **A.** right triangle

4. 13, 5, 7 **B.** acute triangle

5. 5, 11, 6 **C.** obtuse triangle

6. 6, 8, 10 **D.** not a triangle

7. **KITE DESIGN** You are making the diamond-shaped kite shown at the right. You measure the crossbars to determine whether they are perpendicular. Are they? Explain.



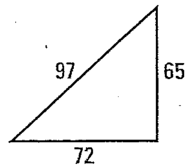
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on pp. 819 and 820.

VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

8.



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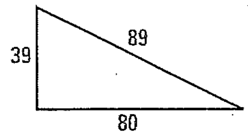

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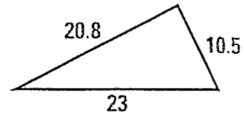

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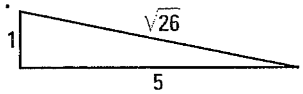

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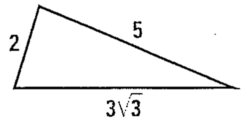

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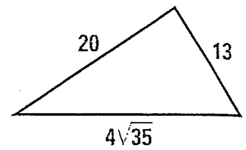


CLASSIFYING TRIANGLES **Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse.***

14. 20, 99, 101

15. 21, 28, 35

16. 26, 10, 17

17. 2, 10, 12

18. 4, $\sqrt{67}$, 9

19. $\sqrt{13}$, 6, 7

20. 16, 30, 34

21. 10, 11, 14

22. 4, 5, 5

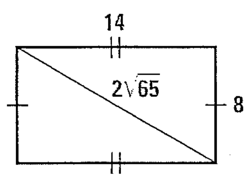
23. 17, 144, 145

24. 10, 49, 50

25. $\sqrt{5}$, 5, 5.5

CLASSIFYING QUADRILATERALS **Classify the quadrilateral. Explain how you can prove that the quadrilateral is that type.**

26.



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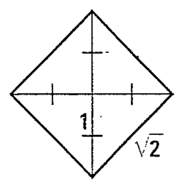

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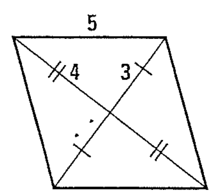
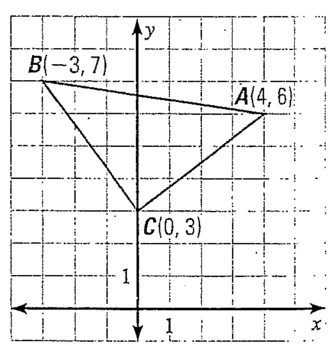


CHOOSING A METHOD **In Exercises 29–31, you will use two different methods for determining whether $\triangle ABC$ is a right triangle.**

29. ***Method 1*** Find the slope of $\overline{AC}$ and the slope of $\overline{BC}$. What do the slopes tell you about $\angle ACB$? Is $\triangle ABC$ a right triangle? How do you know?

30. ***Method 2*** Use the Distance Formula and the Converse of the Pythagorean Theorem to determine whether $\triangle ABC$ is a right triangle.

31. Which method would you use to determine whether a given triangle is right, acute, or obtuse? Explain.



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–13, 30
Example 2: Exs. 14–28, 31–35
Example 3: Exs. 39, 40

USING ALGEBRA **Graph points *P, Q,* and *R*. Connect the points to form $\triangle PQR$. Decide whether $\triangle PQR$ is *right, acute,* or *obtuse.***

32. $P(-3, 4)$, $Q(5, 0)$, $R(-6, -2)$

33. $P(-1, 2)$, $Q(4, 1)$, $R(0, -1)$

NAME ______________________________ DATE ____________

Reteaching with Practice

For use with pages 543–549

GOAL **Use the converse of the Pythagorean Theorem to solve problems and use side lengths to classify triangles by their angle measures**

Theorem 9.5 Converse of the Pythagorean Theorem
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

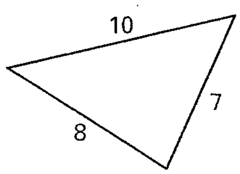
Theorem 9.6
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

Theorem 9.7
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

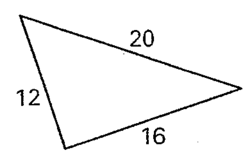
EXAMPLE 1 *Verifying Right Triangles*

The triangles below appear to be right triangles. Tell whether they are right triangles.

a.



b.



SOLUTION

Let c represent the length of the longest side of the triangle (you do not want to call this the "hypotenuse" because you do not yet know if the triangle is a right triangle). Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $10^2 \stackrel{?}{=} 8^2 + 7^2$

$100 \stackrel{?}{=} 64 + 49$

$100 \neq 113$

The triangle is not a right triangle.

b. $20^2 \stackrel{?}{=} 12^2 + 16^2$

$400 \stackrel{?}{=} 144 + 256$

$400 = 400$

The triangle is a right triangle.

NAME ______________________ DATE __________

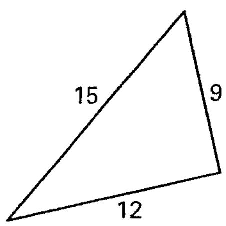
Reteaching with Practice

For use with pages 543–549

Exercises for Example 1

In Exercises 1–3, determine if the triangles are right triangles.

1.



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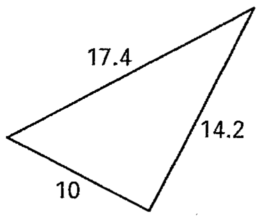

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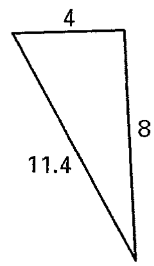


EXAMPLE 2 Classifying Triangles

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*.

a. 58, 69, 80 **b.** 11, 30, 39

SOLUTION

You can use the Triangle Inequality to confirm that each set of numbers can represent the side lengths of a triangle.

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

a. $c^2 \underline{?} a^2 + b^2$ Compare c^2 with $a^2 + b^2$.

$80^2 \underline{?} 58^2 + 69^2$ Substitute.

$6400 \underline{?} 3364 + 4761$ Multiply.

$6400 < 8125$ c^2 is less than $a^2 + b^2$.

Because $c^2 < a^2 + b^2$, the triangle is acute.

b. $c^2 \underline{?} a^2 + b^2$ Compare c^2 with $a^2 + b^2$.

$39^2 \underline{?} 11^2 + 30^2$ Substitute.

$1521 \underline{?} 121 + 900$ Multiply.

$1521 > 1021$ c^2 is greater than $a^2 + b^2$.

Because $c^2 > a^2 + b^2$, the triangle is obtuse.

Exercises for Example 2

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse*.

4. $5, \sqrt{56}, 9$ **5.** 23, 44, 70 **6.** 12, 80, 87 **7.** 4, 7, 10

LESSON 9.3

NAME ______________________________ DATE __________

Practice A

For use with pages 543–549

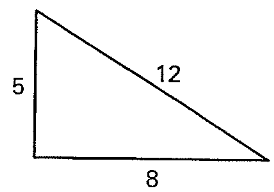
Decide whether the numbers can represent the side lengths of a triangle.

1. 5, 4, 3 **2.** 5, 6, 7 **3.** 5, 5, 10

4. 5, 10, 10 **5.** 5, 10, 15 **6.** 5, 15, 15

Tell whether the triangle is a right triangle.

7.



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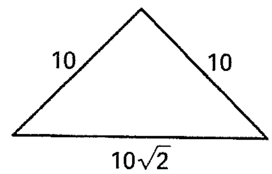

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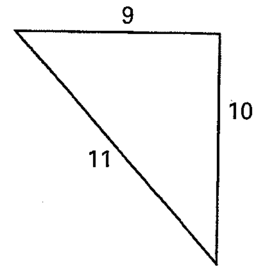

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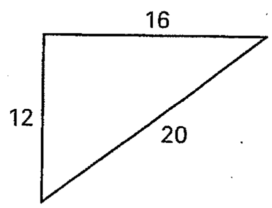
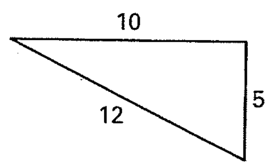


11.



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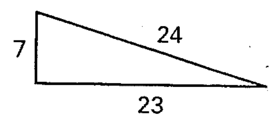


Classify the triangles with the given side lengths as *right, acute,* or *obtuse*.

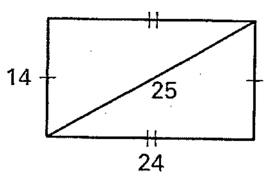
13. 6, 8, 10 **14.** 6, 6, 10 **15.** 6, 10, 10

16. $\sqrt{6}, \sqrt{8}, \sqrt{10}$ **17.** 0.6, 0.8, 1.0 **18.** 7, 9, 11

Lesson 9.3

Classify the quadrilateral. Explain how you can prove that the quadrilateral is that type.

19.



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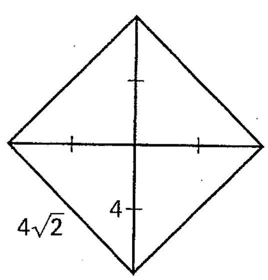

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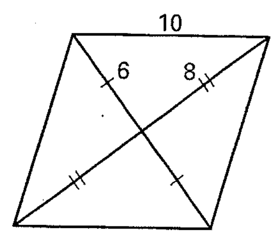
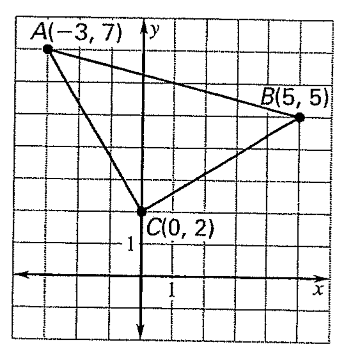


In Exercises 22–24, you will use two different methods for determining whether $\triangle ABC$ is a right triangle.



22. *Method 1* Find the slope of $\overline{AC}$ and the slope of $\overline{BC}$. What do the slopes tell you about $\angle ACB$? Is $\triangle ABC$ a right triangle? How do you know?

23. *Method 2* Use the Distance Formula and the Converse of the Pythagorean Theorem to determine whether $\triangle ABC$ is a right triangle.

24. Which method would you use to determine whether a given triangle is right, acute, or obtuse? Explain.

9.4

Special Right Triangles

GOAL 1 SIDE LENGTHS OF SPECIAL RIGHT TRIANGLES

…you should learn

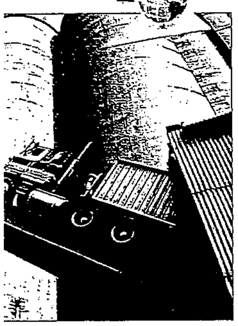
1 Find the side
…s of special right
…es.

2 Use special right
…es to solve **real-life**
…ns, such as finding the
…ngths of the triangles
…spiral quilt design
… **31–34.**

…you should learn it

…use special right
…es to solve **real-life**
…ms, such as finding the
… of a tipping platform
…**mple 4.**

REAL LIFE

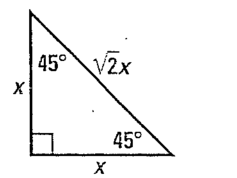


Right triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called **special right triangles**. In the Activity on page 550, you may have noticed certain relationships among the side lengths of each of these special right triangles. The theorems below describe these relationships. Exercises 35 and 36 ask you to prove the theorems.

THEOREMS ABOUT SPECIAL RIGHT TRIANGLES

THEOREM 9.8 *45°-45°-90° Triangle Theorem*

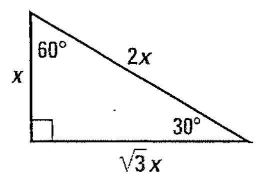
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



Hypotenuse = $\sqrt{2}$ · leg

THEOREM 9.9 *30°-60°-90° Triangle Theorem*

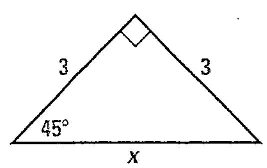
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Hypotenuse = 2 · shorter leg
Longer leg = $\sqrt{3}$ · shorter leg

EXAMPLE 1 *Finding the Hypotenuse in a 45°-45°-90° Triangle*

Find the value of x.



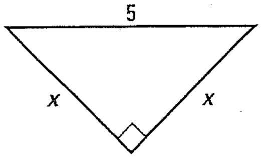
SOLUTION

By the Triangle Sum Theorem, the measure of the third angle is 45°. The triangle is a 45°-45°-90° right triangle, so the length x of the hypotenuse is $\sqrt{2}$ times the length of a leg.

Hypotenuse $= \sqrt{2} \cdot$ leg		45°-45°-90° Triangle Theorem
$x = \sqrt{2} \cdot 3$		Substitute.
$x = 3\sqrt{2}$		Simplify.

EXAMPLE 2 *Finding a Leg in a 45°-45°-90° Triangle*

Find the value of x.



SOLUTION

Because the triangle is an isosceles right triangle, its base angles are congruent. The triangle is a 45°-45°-90° right triangle, so the length of the hypotenuse is $\sqrt{2}$ times the length x of a leg.

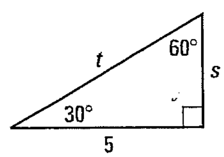
$\text{Hypotenuse} = \sqrt{2} \cdot \text{leg}$	45°-45°-90° Triangle Theorem
$5 = \sqrt{2} \cdot x$	Substitute.
$\frac{5}{\sqrt{2}} = \frac{\sqrt{2}x}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$\frac{5}{\sqrt{2}} = x$	Simplify.
$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = x$	Multiply numerator and denominator by $\sqrt{2}$.
$\frac{5\sqrt{2}}{2} = x$	Simplify.

EXAMPLE 3 *Side Lengths in a 30°-60°-90° Triangle*

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Find the values of s and t.



SOLUTION

Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length s of the shorter leg.

$\text{Longer leg} = \sqrt{3} \cdot \text{shorter leg}$	30°-60°-90° Triangle Theorem
$5 = \sqrt{3} \cdot s$	Substitute.
$\frac{5}{\sqrt{3}} = \frac{\sqrt{3} \cdot s}{\sqrt{3}}$	Divide each side by $\sqrt{3}$.
$\frac{5}{\sqrt{3}} = s$	Simplify.
$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{5}{\sqrt{3}} = s$	Multiply numerator and denominator by $\sqrt{3}$.
$\frac{5\sqrt{3}}{3} = s$	Simplify.

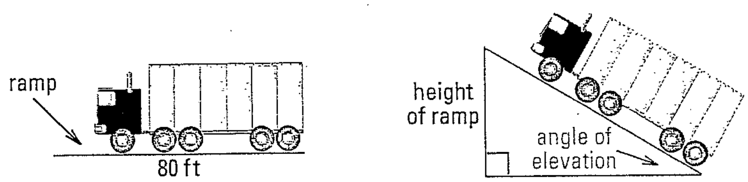
The length t of the hypotenuse is twice the length s of the shorter leg.

$\text{Hypotenuse} = 2 \cdot \text{shorter leg}$	30°-60°-90° Triangle Theorem
$t = 2 \cdot \frac{5\sqrt{3}}{3}$	Substitute.
$t = \frac{10\sqrt{3}}{3}$	Simplify.

GOAL 2 Using Special Right Triangles in Real Life

EXAMPLE 4 *Finding the Height of a Ramp*

TIPPING PLATFORM A tipping platform is a ramp used to unload trucks, as shown on page 551. How high is the end of an 80 foot ramp when it is tipped by a 30° angle? by a 45° angle?



SOLUTION

When the angle of elevation is 30°, the height h of the ramp is the length of the shorter leg of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ **30°-60°-90° Triangle Theorem**

$40 = h$ **Divide each side by 2.**

When the angle of elevation is 45°, the height of the ramp is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = \sqrt{2} \cdot h$ **45°-45°-90° Triangle Theorem**

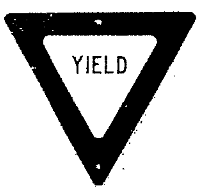
$\frac{80}{\sqrt{2}} = h$ **Divide each side by $\sqrt{2}$.**

$56.6 \approx h$ **Use a calculator to approximate.**

▶ When the angle of elevation is 30°, the ramp height is 40 feet. When the angle of elevation is 45°, the ramp height is about 56 feet 7 inches.

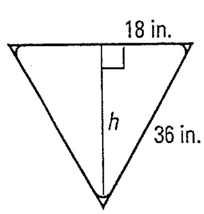
EXAMPLE 5 *Finding the Area of a Sign*

ROAD SIGN The road sign is shaped like an equilateral triangle. Estimate the area of the sign by finding the area of the equilateral triangle.



SOLUTION

First find the height h of the triangle by dividing it into two 30°-60°-90° triangles. The length of the longer leg of one of these triangles is h. The length of the shorter leg is 18 inches.



$h = \sqrt{3} \cdot 18 = 18\sqrt{3}$ **30°-60°-90° Triangle Theorem**

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)\left(18\sqrt{3}\right) \approx 561.18$$

▶ The area of the sign is about 561 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is meant by the term *special right triangles*?

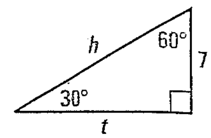
Concept Check ✓

2. **CRITICAL THINKING** Explain why any two 30°-60°-90° triangles are similar.

Use the diagram to tell whether the equation is *true* or *false*.

3. $t = 7\sqrt{3}$ **4.** $t = \sqrt{3h}$ **5.** $h = 2t$

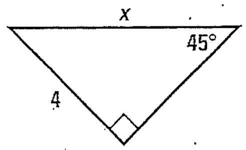
6. $h = 14$ **7.** $7 = \frac{h}{2}$ **8.** $7 = \frac{t}{\sqrt{3}}$



Skill Check ✓

Find the value of each variable. Write answers in simplest radical form.

9.



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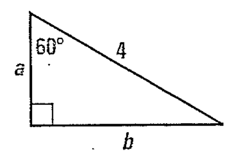

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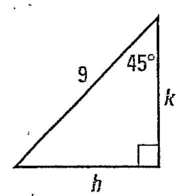

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 820.

USING ALGEBRA **Find the value of each variable. Write answers in simplest radical form.**

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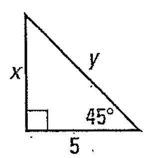

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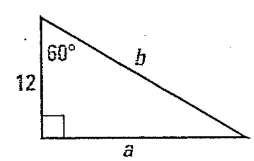

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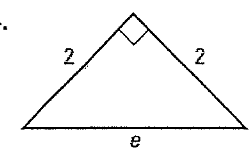

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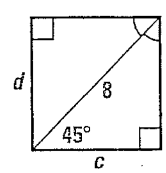

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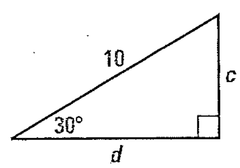

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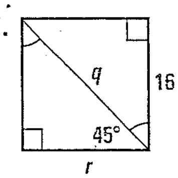

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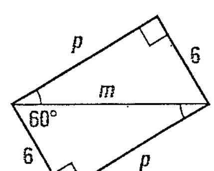

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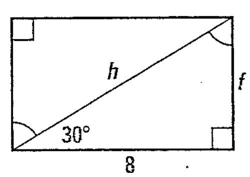

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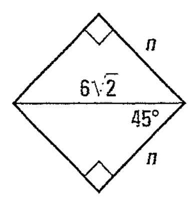

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–23
Example 2: Exs. 12–23
Example 3: Exs. 12–23
Example 4: Exs. 28–29, 34
Example 5: Exs. 24–27

FINDING LENGTHS **Sketch the figure that is described. Find the requested length. Round decimals to the nearest tenth.**

21. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude of the triangle.

22. The perimeter of a square is 36 inches. Find the length of a diagonal.

23. The diagonal of a square is 26 inches. Find the length of a side.