Mini Mathletes 2018 - Written Test Solution Key

1 Solutions

1. **(D)** If Becky followed order of operations, she would do multiplication and division first so $2 \times 4 - 6 \div 2 = 8 - 3 = 5$. Then the difference between 5 and Becky's answer is $5 - 1 = 4$.

2. **(E)** One quarter is 25 cents so 6 quarters is $6 \cdot 25 = 150$ centers. Since one dime is 10 cents, the number of dimes that equals 150 cents is $\frac{150}{10} = 15$.

3. **(D)** Creating a common denominator of 28, we see that $\frac{5}{7} - \frac{1}{4} = \frac{5 \cdot 4}{28} - \frac{7 \cdot 1}{28} = \frac{20-7}{28} = \frac{13}{28}$.

4. **(C)** Since the opposite side of the pool is $\frac{1}{5}$th as deep, the depth of the opposite side is $75 \div 5 = 15$ inches.

5. **(B)** The sum of the angles in a triangle is $180°$ so the measure of the third angle of the triangle is $180° - 24° - 63° = 87°$.

6. **(A)** The perimeter of the other triangle is $9 \times 4 = 36$. Since an equilateral triangle has three equal sides, the side length of the triangle is $36 \div 3 = 12$ inches.

7. **(D)** Since the water rises at a rate of 18 inches per hours, it will take $72 \div 18 = 4$ hours for the height of the water to increase by 72 inches. Since there are 60 minutes in a hour, our answer is $4 \times 60 = 240$ minutes.

8. **(C)** The area of one paper is 5in $\times$ 7 in $= 35$ in^2 so the number of papers that can be covered by one can of paint is $450 \div 35 \approx 12.8$. Therefore, the number of papers that can be fully covered is 12.

9. **(C)** The highest number of socks Maddie can pick without having a pair is 2: when Maddie picks one white sock and one black sock. When she picks another sock, she is guaranteed to have a pair so the number of socks Maddie needs to pick is 3.

10. **(B)** There are 100 whole numbers between 1 and 100 inclusive and of those, exactly $100 \div 5 = 20$ are multiples of 5. Therefore, the probability that a randomly chosen whole number between 1 and 100 inclusive is a multiple of 5 is $\frac{20}{100} = \frac{1}{5}$.

11. **(C)** Let the side length of the cube be s. A cube has 6 faces with area s^2 so the surface area of the cube is $6s^2$. A cube has a volume of s^3 so we have $6s^2 = s^3$. Therefore, $s = 6$ so the side length of the cube is 6.

12. **(D)** When Allie multiplies her negative number by a positive number, the answer muse be a negative number. Then when she multiplies her answer by 2, the result must be a even negative number. Finally, when she subtracts a odd number from her last number, the result must be a negative odd number. The only answer choice that is a negative odd number is -97.

13. **(A)** By the rule, $3\#2 = (3+2)(3-2) = 5 \times 1 = 5$ so our answer is $5\#1 = (5+1)(5-1) = 6 \times 4 = 24$.

14. **(A)** The day of the week repeats every 7 days so May 19th, May 26th, June 2nd, June 9th, June 16th, and June 23th will all be Saturdays. Therefore, June 25th will be a Monday.

15. **(E)** If the two sides of the rectangle are a and b, then the perimeter is $2a + 2b = 42$ so $a + b = 21$. The area of the tennis court is maximized when the two sides of the court are as close together as possible. Therefore, $a = 10$ and $b = 11$ so the area of the tennis court is $10 \times 11 = 110$.

16. **(B)** If a square has area 100 then the square has sides of length $\sqrt{100} = 10$. If one side decreased by 20% then it becomes $10 \cdot \frac{80}{100} = 8$. If one side increased by 20% then it becomes $10 \cdot \frac{120}{100} = 12$ so the new rectangle has area $8 \times 12 = 96$. Then the difference between the area of the square and the area of the rectangle is $100 - 96 = 4$.

17. **(B)** The total amount of time Shreya's homework will take is $45 + 18 + 67 = 130$ minutes. This is 2 hours and 10 minutes so if she starts at $6:00$, she will end at $8:10$.

18. **(E)** If we need $3\frac{1}{2}$ cups of pineapple juice for every cup of apple juice, then for 1 cup of pineapple juice we need $\frac{1}{\frac{7}{2}} = \frac{2}{7}$ cups of apple juice. Therefore, for 6 cups of pineapple juice, we need $6 \times \frac{2}{7} = \frac{12}{7}$ cups of apple juice. Since we already have 1 cup of apple juice, we need $\frac{12}{7} - 1 = \frac{5}{7}$ more cups of apple juice.

19. **(E)** The area of a circle is πr^2 where r is the radius so if the area of the garden is 25π, then the radius of the garden is $\sqrt{25} = 5$. The area of the sidewalk is the area of the sidewalk and the garden minus the area of the garden. Since the sidewalk is 1 foot wide, the circle formed by the garden and the sidewalk has radius $5 + 1 = 6$ so the area of the circle is $6^2\pi = 36\pi$. Then the area of just the sidewalk part is $36\pi - 25\pi = 11\pi$.

20. **(A)** The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19 so the whole numbers less that twenty that can be written as the product of two different prime numbers are $2 \times 3 = 6$, $2 \times 5 = 10$, $2 \times 7 = 14$, and $3 \times 5 = 15$. This gives us a total of 4 numbers.

21. **(B)** Let Mandy's current age be m and let Annie's current age by a. One year ago, Mandy's age was $m-1$ and Annie's age was $a-1$ so $m-1 = 4 \cdot (a-1) \implies m = 4a - 3$. In three years, Mandy's age will be $m+3$ and Annie's age will be $a+3$ so $m+3 = 2 \cdot (a+3) \implies m = 2a+3$. Setting the expressions for m equal to each other, we obtain $4a - 3 = 2a + 3$ so $a = 3$. Therefore, Annie is currently 3 years old.

22. **(D)** Encompass the figure in a rectangle so that the sides of the rectangle lie on the outside sides of the figure. Then the area of the figure is equal to the area of the rectangle minus the areas of the four small rectangles cut out of the rectangle's corners. The rectangle has a height of $2+4+3 = 9$ and a width of $5+7+3 = 15$ so the area of the rectangle is $9 \times 15 = 135$. The four small rectangles have areas $3 \times 2 = 6$, $2 \times 2 = 4$, $5 \times 2 = 10$, and $3 \times 3 = 9$. Therefore, the area of the figure is $135 - 6 - 4 - 10 - 9 = 106$.

23. **(E)** Let b be the number of bicycles and let t be the number of tricycles. Then $b + t = 59$ and the total number of wheels is $2b + 3t = 134$. From the first equation, we have $t = 59 - b$. Substituting into the second equation, we have $2b + 3 \cdot (59 - b) = 134$ so $177 - b = 134$. Therefore, the number of bicycles is $177 - 134 = 43$.

24. **(C)** If she works for t hours, then the amount she charges is $23 + 12t$. Now we test each answer choice. If $23 + 12t = 59$, then $t = \frac{36}{12} = 3$. If $23 + 12t = 95$, then $t = \frac{72}{12} = 6$. If $23 + 12t = 163$ then $t = \frac{140}{12}$ which is not a whole number. Therefore, she cannot charge 163 dollars.

25. **(A)** Say Billy got b dollars for his birthday. After Billy spent 50% of his money on shoes, he has $\frac{b}{2}$ dollars left. Of the $\frac{b}{2}$ dollars remaining, he spent $\frac{1}{4}$th of it on shoes, so we has $\frac{3}{4}$ths of the $\frac{b}{2}$ dollars left which is $\frac{3b}{8}$ dollars. Therefore, $\frac{3b}{8} = 12$ so Billy started with $\frac{8}{3} \times 12 = 32$ dollars. The product of the digits of 32 is $3 \cdot 2 = 6$.

26. **(B)** Ignoring the letters that are the same, there are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways to order the letters in SPLASH. Since there are two Ss, we are double counting each permutation so we have to divide by 2. Therefore, the answer is $\frac{720}{2} = 360$.

27. **(D)** Let c be the number of questions Isabel answers correctly and let i be the number of questions she answers incorrectly. Since there are 20 questions, $i + c$ and her total score is $5c - 2i = 72$. From the first equation, $i = 20 - c$. Substituting into the second equation, we have $5c - 2 \cdot (20 - c) = 72$ so $7c - 40 = 72$. Therefore, the number of questions she answers correctly is $\frac{112}{7} = 16$ so the percent of questions she answered correctly is $\frac{16}{20} = 80\%$.

28. **(E)** There are 3 ways to select the size of the pizza. If she picks 1 topping, then there are 4 ways to choose which topping. If she chooses 2 toppings, then there are 6 ways to choose which two toppings. If she chooses 3 toppings, then there are 4 ways to choose which three toppings. Therefore, the total number of pizzas Jenny can make is $3 \times (4 + 6 + 4) = 3 \times 14 = 42$.

29. **(B)** The volume of a cylinder is $\pi r^2 h$ where r is the radius of the circular base and h is the height of the cylinder so the volume of a semi-circular cylinder is $\frac{1}{2} \cdot \pi r^2 h$. Since semi-circular cylindar has a base radius of 5 and a height of 10, the volume is $\frac{1}{2} \cdot 5^2 \cdot 10\pi = 125\pi$. Therefore, the volume of the air in the tunnel is equal to the volume of the tunnel minus the volume of the hamster or $125\pi - 5\pi = 120\pi$.

30. **(D)** If Anna and Carl can sit next to each other, the number of ways to seat them in a row is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. If Anna and Carl sit next two each other, then there are 5 ways to choose which two spots in the row Anna and Carl will be in and 2 ways to decide which of them will be first. After Anna and Carl are seated, there are $4 \times 3 \times 2 \times 1 = 24$ ways to seat the rest of the four people. Then the total number of ways to seat them so that Anna and Carl do sit next to each other is $5 \cdot 2 \cdot 24 = 240$. Therefore, the number of ways for them to sit such that Anna and Carl do NOT sit next to each other is $720 - 240 = 480$.